Information Frictions and Real Exchange Rate Dynamics[†]

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Abstract

I provide a novel explanation for the observed large and persistent fluctuations in real exchange rates using a flexible-price model with dispersed information among firms. When firms face strategic complementarities in price-setting, the continuing uncertainty about their competitors' beliefs results in sluggish price adjustment that can generate large and long-lived real exchange rate movements. I estimate the model using real output and output deflator data from the US and the Euro Area and show, as an out-of-sample test, that the model successfully explains the observed volatility and persistence of the Euro/Dollar real exchange rate. A Bayesian model comparison strongly favors the dispersed information model relative to a sticky-price model à la Calvo. The model also accounts for the persistent effects of monetary shocks on the real exchange rate that I document using a structural VAR.

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1 Introduction

Real exchange rates have been extremely volatile and persistent since the end of the Bretton Woods system (Mussa, 1986). For many developed economies, real exchange rates are roughly four times as volatile as output, and their fluctuations exhibit a half-life in the range of three to five years. Moreover, real and nominal exchange rates are highly correlated.[1] In principle, sticky-price models can explain this correlation and the high volatility: if price levels fail to adjust, changes in nominal exchange rates following monetary shocks will readily translate in real exchange rate movements. However, such models cannot produce the observed persistence under plausible nominal rigidities, as demonstrated by Bergin and Feenstra (2001) and Chari, Kehoe, and McGrattan (2002).[2] Is it possible to reconcile, in a single framework, the enormous short-term volatility of the real exchange rate with its extremely long half-life?

I study this classic open economy question in a two-country, flexible-price model in which firms have noisy, dispersed information about the economic environment. I show analytically that when firms face strategic complementarities in price-setting, uncertainty about other firms' beliefs results in sluggish price adjustments that can generate large and long-lived real exchange rate fluctuations. The model is estimated on output and output deflator data for the US and the Euro Area using Bayesian methods. I evaluate the quantitative success of the framework by asking whether it reproduces the dynamics of the Euro/Dollar real exchange rate, which were not targeted in the estimation. I find that the estimated model successfully explains these dynamics, as captured by the unconditional volatility and half-life of the real exchange rate, as well as its correlation with the nominal exchange rate. In addition, the model accounts for the persistent effects of monetary shocks on the real exchange rate that I document using a structural VAR. Finally, I conduct a Bayesian model comparison and find that the data strongly favor the dispersed-information framework relative to a sticky-price model à la Calvo.

The main contribution of the paper is to provide a novel explanation for the observed real exchange rate dynamics, by showing that the estimated dispersed-information model captures remarkably well the volatility and persistence of the real exchange rate. Closed-economy models in which agents are imperfectly informed are known to be quantitatively successful for explaining the highly persistent effects of monetary disturbances on output and inflation (Melosi, 2014) documented by VAR studies (e.g. Christiano, Eichenbaum, and Evans, 2005). However, little is known about these models' ability to explain the behavior of international relative prices. This paper fills this gap. First, it shows analytically that the model with dispersed information can deliver highly volatile and persistent real exchange rates. Second, it demonstrates quantitatively that the estimated model successfully accounts for the observed real exchange rate behavior. The model's success stems from its ability to generate endogenous persistence in the real exchange rate fluctuations that follow monetary shocks.

The second contribution lies in the quantitative comparison between the dispersed-information

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[1]Empirical evidence for these facts is presented in Chari, Kehoe, and McGrattan (2002) and Steinsson (2008).

[2]Subsequent research addresses the persistence anomaly by introducing strategic complementarities (Bouakez, 2005), inertial Taylor rules (Benigno, 2004) and real shocks (Steinsson, 2008; Iversen and Söderström, 2014). While these features increase the persistence of the exchange rate, they are not sufficient to jointly explain the observed half life of the real exchange rate as well as its relative volatility to consumption and output.

model and an alternative benchmark sticky-price model, a comparison currently missing in the open economy literature. To this end, I also estimate a two-country sticky price model à la Calvo. The Bayesian model comparison suggests that the output and output deflator data strongly favor the dispersed-information model relative to the sticky-price model. In sample, the dispersed-information model is already clearly preferred to the Calvo model, but the model with information frictions fares even better in the out-of-sample test, generating far more realistic real exchange rate dynamics.

The information friction that I model is motivated by the mounting evidence about heterogeneity in beliefs among decision makers. To illustrate, Figure 1 depicts the times series of the interquartile range of two types of forecasts: the professional analysts' one-year ahead forecasts of CPI inflation and real GDP growth, as taken from the Federal Reserve's Survey of Professional Forecasts. The time series show that there is considerable dispersion in these forecasts. Dispersion in beliefs is pervasive in the economy. Indeed, recent survey data show that there is also widespread dispersion in firms' beliefs about both past and future macroeconomic conditions (Coibion, Gorodnichenko, and Kumar, 2015). This evidence suggests that firms have their own "window on the world" (Amato and Shin, 2006). In this environment, defending a firm's market share will entail some degree of second-guessing competitors' pricing strategies. This second-guessing game might prove particularly challenging in an open economy, in which firms face competition not only from domestic firms but also from foreign exporters.

I follow Woodford (2002) and Melosi (2014) in modeling this heterogeneity in beliefs. Specifically, firms in the model observe private, idiosyncratic signals about nominal aggregate demand and aggregate productivity in the two countries. They also face strategic complementarity in price-setting, which implies that a firm's optimal price depends positively on the prices set by competitors. With private information, strategic complementarity requires firms to respond to higher-order beliefs i.e., beliefs about other firms' beliefs about underlying economic conditions. Beliefs update slowly, as the private signals a firm receives provide relatively little information about other firms' signals. Notwithstanding the absence of nominal rigidities, slow movements in beliefs translate into *endogenously* slow-moving prices. Therefore, while nominal shocks generate swings in the nominal exchange rate, the slow price dynamics can trigger large and persistent real exchange rate fluctuations.

Despite prices' dependence on an infinite hierarchy of beliefs, I can show analytically that the volatility and persistence of the real exchange rate are higher (i) the lower the precision of firms' signals about aggregate demand and (ii) the higher the degree of strategic complementarity. Intuitively, when signals are not very precise, firms learn slowly about changes in nominal aggregate demand and sluggishly update their prices. When strategic complementarities are strong, firms fail to adjust prices quickly in an effort of keeping their own prices in line with those of their rivals. Both of these channels slow down the price adjustment, delivering volatile and persistent real exchange rates following nominal shocks. Notably, strategic complementarity depends on the degree of the economies' openness and on the substitutability between domestic- and foreign-produced goods. Thus, foreign competition provides a channel through which the adjustment of prices might be delayed, one that is naturally absent in closed-economy models.

In the empirical part of the paper, I assess whether the model can quantitatively explain the dynamics of the Euro-Dollar real exchange rate in the period 1971-2011. In the model, low enough signal precisions would be able to generate a highly volatile and persistent real exchange rate, but it is unclear which values should be considered empirically relevant, given the scarce existing evidence on these parameters. To address this shortcoming, I estimate the model parameters via Bayesian methods using real GDP and GDP deflator data for the US and the Euro Area. These data do not directly contain information on the real exchange rate, which is instead defined as the nominal exchange rate adjusted by consumption price indices.

The exclusion of the real exchange rate from the estimation allows me to conduct an out-of-sample test for my model. Specifically, I simulate the model at the estimated parameter values and ask whether it reproduces the dynamics of the Euro/Dollar real exchange rate, which were not targeted in the estimation. I show that the model successfully explains these dynamics, as measured by the volatility, persistence, and half-life of the real exchange rate. The model also delivers the hump-shaped dynamics that are a salient feature of the Euro-Dollar real exchange rate and are central to the observed half life of about 4.5 years. Additionally, the estimated signal-to-noise ratios suggest that firms' signals about nominal aggregate conditions are less precise than signals about productivity, which generates persistence of the real exchange rate from nominal shocks. Using a structural VAR approach, I show that these persistent effects of monetary shocks on real exchange rates are indeed a feature of the data.

I compare these predictions with those of a standard sticky-price model, which I estimate using the same data on real GDP and GDP deflators. The sticky-price model deviates from the dispersed-information model in only two respects: (i) all agents are perfectly informed, and (ii) firms can optimally adjust their prices only in random periods, as in Calvo (1983). Two sets of results emerge. First, the dispersed-information model fits the data significantly better than the sticky-price model, as suggested by the Bayesian model comparison. Second, the model with information frictions is more successful at explaining the out-of-sample dynamics of the real exchange rate. The estimated Calvo model generates low real exchange rate persistence following monetary shocks, confirming the intuition behind the results of Chari, Kehoe, and McGrattan (2002). When technology shocks are added to the picture, the model produces a half-life of the real exchange rate that is twice as large as in the data. Intuitively, this happens because the estimated Calvo model requires large technology shocks to account for the volatility and persistence of output and domestic price indices. However, the size of these technology shocks and their internal propagation in the sticky-price model generates counterfactual predictions for the real exchange rate.

Finally, I investigate the robustness of the predictions of the dispersed-information model to changes in the firms' information set. Specifically, I allow firms to observe noisy signals about equilibrium prices, which are relevant for their price-setting decisions. A re-estimation of the model suggests that these additional signals are relatively noisy, and therefore they carry a low weight in the firms' signal-extraction problem. The implications are that the presence of these additional signals does not substantively ameliorate the fit of the model to the data and, leaves the real exchange rate dynamics unchanged.

This paper contributes to the growing literature that focuses on the aggregate implications of

dispersed information among price setters, such as Woodford (2002), Maćkowiak and Wieder-holt (2009), Nimark (2008), and Melosi (2014), which builds on the seminal contributions of Phelps (1970) and Lucas (1972). In contrast to most of this literature, which is developed in closed economies and focuses on inflation dynamics, this paper studies the implications for international prices, where uncertainty about foreign demand as well as foreign competitors' actions, plays an important role. My analysis lends further empirical support to the dispersed-information theory, by testing its natural predictions in an open-economy environment. The paper is also naturally related to the literature that studies real exchange rate dynamics in the context of monetary models, such as Johri and Lahiri (2008) and Carvalho and Nechio (2011), in addition to the works already mentioned. Relative to this literature, this paper highlights the importance of a source of endogenous persistence in real exchange rates—dispersed information in environments with strategic complementarities—that has so far been ignored in this context.

Finally, the present study adds to the small literature that focuses on information frictions in open economies. Bacchetta and van Wincoop (2006, 2010) combine information frictions with a finance approach to study other puzzles in international macroeconomics, such as the exchange-rate disconnect and the the forward-discount puzzle. Crucini, Shintani, and Tsuruga (2010) introduce sticky information à la Mankiw and Reis (2002) in a sticky-price model to explain the volatility and persistence of deviations from the law of one price. They find that such a model can explain the empirical half life of eighteen months if information updates occur every 12 months. In contrast, this paper seeks to explain the substantially longer half-life of aggregate real exchange rates by relying only on dispersed information and Bayesian updating, which is consistent with the recent evidence on firms' behavior, as documented by Coibion, Gorodnichenko, and Kumar (2015).

The paper proceeds as follows. Section 2 develops the dispersed-information model. Section 3 provides some analytical results. Section 4 discusses the solution method. Section 5 analyzes the model's impulse responses. Section 6 contains the empirical analysis. Section 7 draws a comparison with the sticky-price model. Section 8 studies the sensitivity of the results to the information structure. Section 9 offers some concluding remarks.

2 The Model

The framework is a two-country open-economy monetary model that follows the international macroeconomic tradition initiated by Obstfeld and Rogoff (1995). The setup is similar to Corsetti, Dedola, and Leduc (2010). The world economy consists of two countries of unit mass, denominated H (Home) and F (Foreign), each populated by households, a continuum of monopolistically competitive producers, and a monetary authority. Each country specializes in the production of one type of tradable goods, produced in a number of varieties or brands, with measure equal to the population size. All goods produced are traded and consumed in both countries. Prices are set in the currency of the producer; therefore, the law of one price holds. Deviations of the real exchange rate from purchasing-power parity arise because households exhibit home bias in consumption preferences.

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All information is, in principle, available to every agent; however, firms can only pay limited attention to the information available, owing to finite information-processing capacity (Sims, 2003). Following Woodford (2002) and Melosi (2014), this idea is modeled by assuming that firms do not perfectly observe current and past realization of the variables in the model, but rather only observe private noisy signals about the state of nominal aggregate demand and technology.[3] Firms use these signals to draw inferences about other model variables. Households and the monetary authorities are assumed, for tractability, to observe the current and past realization of all the model variables. Below I present the structure of the Home economy in more detail. The Foreign economy is symmetric, and foreign variables will be denoted with an asterisk.

2.1 Preferences and Households

The utility function of the representative household in country H is

$$\mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \int_0^1 \frac{L_{ht}^{1+1/\psi}}{1 + 1/\psi} di \right] \right\} \tag{1}$$

The representative household has full information, $\mathbb{E}(.)$ denotes the statistical expectations operator, and $\beta < 1$ is the discount factor. Households receive utility from consumption C_t and disutility from working, where L_{ht} indicates hours of labor input in the production of domestic variety $h \in [0,1]$. Risk is pooled internally, to the extent that all domestic agents receive the same consumption level. The parameter $\psi > 0$ represents the Frisch elasticity of labor supply. Following Woodford (2003, Ch. 3), each of the home varieties (indexed by h over the unit interval) uses a specialized labor input in its production. As noted by Woodford, this type of differentiated labor markets generates more strategic complementarities in price-setting.[4]

Households consume both types of traded goods. The consumption of these goods is denoted by C_{Ht} and C_{Ft}. For each type of goods, one brand or variety is an imperfect substitute for all the other brands, and γ is the elasticity of substitution between brands. Mathematically, consumption baskets of Home and Foreign goods by Home agents are a CES aggregate of Home and Foreign brands, respectively:

$$C_{Ht} \equiv \left(\int_0^1 C_t(h)^{\frac{\gamma-1}{\gamma}} dh \right)^{\frac{\gamma}{\gamma-1}} \qquad C_{Ft} \equiv \left(\int_0^1 C_t(f)^{\frac{\gamma-1}{\gamma}} df \right)^{\frac{\gamma}{\gamma-1}} \qquad \gamma > 1$$

The overall consumption basket, C_t, is defined as

$$C_t \equiv \left[\alpha^{\frac{1}{\omega}} (C_{Ht})^{\frac{\omega-1}{\omega}} + (1-\alpha)^{\frac{1}{\omega}} (C_{Ft})^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \qquad \omega > 0$$

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[3]The implications of relaxing this assumption are explored in Section 8.

[4]Pricing decisions are strategic complements if, when other firms raise their prices, a given firm i wishes to raise its price as well. It is closely related to the concept of "real rigidity", in that it depends solely upon real factors: the structure of production costs and of demand. Strategic complementarities arise also in the presence of decreasing returns or input-output structures in production (Basu, 1995). For a discussion, see Ball and Romer (1990) and Woodford (2003, Ch. 3).

where α is the weight of the home consumption good and ω is the elasticity of substitution between home and foreign goods, which I alternatively refer to as the trade elasticity. The utility-based consumption price index (CPI) is

$$P_t = \left[\alpha P_{Ht}^{1-\omega} + (1-\alpha)P_{Ft}^{1-\omega}\right]^{\frac{1}{1-\omega}}$$

where P_{Ht} and P_{Ft} are the price sub-indices for the home- and foreign-produced goods, expressed in domestic currency

$$P_{Ht} = \left(\int_0^1 p_t(h)^{1-\gamma}dh\right)^{\frac{1}{1-\gamma}} \qquad P_{Ft} = \left(\int_0^1 p_t(f)^{1-\gamma}df\right)^{\frac{1}{1-\gamma}}$$

Foreign prices are similarly defined. The Foreign CPI is

$$P_t^* = \left[(1-\alpha)(P_{Ht}^*)^{1-\omega} + \alpha(P_{Ft}^*)^{1-\omega}\right]^{\frac{1}{1-\omega}}$$

Let $\mathcal{Q}_t$ denote the real exchange rate, that is, the relative price of consumption: $\mathcal{Q}_t \equiv \frac{\epsilon_t P_t^*}{P_t}$, where ϵ_t is the nominal exchange rate expressed in domestic currency per foreign currency. Even if the law of one price holds at the individual good level (i.e., $P_t(h) = \epsilon_t P_t(h)^*$, which implies $P_{Ht} = \epsilon_t P_{Ht}^*$), the presence of home bias in consumption—that is $\alpha > 1/2$—implies that the price of consumption may not be equalized across countries. Put differently, purchasing-power parity ($\mathcal{Q}_t = 1$) will generally not hold. The terms of trade are defined as the price of imports in terms of exports: $\mathcal{T}_t = \frac{P_{Ft}}{\epsilon_t P_{Ht}^*}$. If the law of one price holds, the real exchange rate will be proportional to the terms of trade

$$q_t = (2\alpha - 1)t_t \tag{2}$$

where, throughout the paper, lower-case letters denote percentage deviations from steady state.[5] Equation (2) implies that an improvement in the terms of trade always appreciates the real exchange rate. This is consistent with the empirical evidence (Obstfeld and Rogoff, 2000). Minimizing expenditure over brands and over goods, one can derive the domestic household demand for a generic good h, produced in country H, and the demand for a good f, produced in country F:

$$C_t(h) = \left(\frac{P_t(h)}{P_{Ht}}\right)^{-\gamma}\left(\frac{P_{Ht}}{P_t}\right)^{-\omega}\alpha C_t \qquad C_t(f) = \left(\frac{P_t(f)}{P_{Ft}}\right)^{-\gamma}\left(\frac{P_{Ft}}{P_t}\right)^{-\omega}(1-\alpha)C_t$$

Assuming that the law of one price holds, total demand for a generic home variety h or foreign variety f may be written as

$$Y_t^d(h) = \left(\frac{P_t(h)}{P_{Ht}}\right)^{-\gamma}\left(\frac{P_{Ht}}{P_t}\right)^{-\omega}\left[\alpha C_t + (1-\alpha)\mathcal{Q}_t^\omega C_t^*\right] \tag{3}$$

$$Y_t^d(f) = \left(\frac{P_t(f)^*}{P_{Ft}^*}\right)^{-\gamma}\left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\omega}\left[(1-\alpha)\mathcal{Q}_t^{-\omega}C_t + \alpha C_t^*\right] \tag{4}$$

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[5]This result assumes symmetric initial conditions.

2.2 Budget Constraint

The generic home household's budget constraint can be written as

$$P_t C_t + \int q_{H,t}(s_{t+1}) \mathcal{B}_{H,t}(s_{t+1}) ds_{t+1} \leq \int_0^1 W_{ht} L_{ht} dh + \mathcal{B}_{H,t} + P_t \int_0^1 \Pi_{ht} dh \quad (5)$$

$\mathcal{B}_{H,t}(s_{t+1})$ is the holding of state-contingent claims that pay off one unit of domestic currency if the realized state of the world at time $t+1$ is s_{t+1} and $q_{H,t}(s_{t+1})$ is the time-t price of such an asset. W_{ht} is the wage for the h-th type of labor input and Π_{ht} are the real profits of domestic firm h. Maximizing (1) subject to (5) gives the static first-order condition:

$$C_t^\sigma L_{ht}^{1/\psi} = W_{ht}/P_t \quad (6)$$

and the following Euler equation

$$1 = \beta(1 + R_{t+1}) \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \quad (7)$$

where R_{t+1} is the net risk-free rate of return between t and $t+1$.

2.3 Monetary Policy

Following Woodford (2002) and Carvalho and Nechio (2011), I leave the specification of monetary policy implicit, and assume that the growth rate of nominal aggregate demands $M_t = P_t C_t$ and $M_t^* = P_t^* C_t^*$ follows exogenous autoregressive processes

$$\Delta m_t = \rho_m \Delta m_{t-1} + u_t^m \quad (8)$$

$$\Delta m_t^* = \rho_{m^*} \Delta m_{t-1}^* + u_t^{m^*} \quad (9)$$

where $\Delta m_t \equiv \ln M_t - \ln M_{t-1}$ and the monetary shocks u_t^m and $u_t^{m^*}$ are $i.i.d.$, distributed as $\mathcal{N}(0, \sigma_m^2)$ and $\mathcal{N}(0, \sigma_{m^*}^2)$ and uncorrelated across countries.[6] I refer to these shocks as nominal demand shocks or monetary shocks, with the understanding that they capture structural shocks that move nominal aggregate demand. The variable M_t can be interpreted as a measure of money supply, such as M1 or M2, or more broadly as a measure of aggregate demand, such as nominal GDP. This specification is widely used in the monetary literature and has been shown to be a good approximation of the process that implements estimated Taylor rules of the types studied in Christiano, Eichenbaum, and Evans (1998).

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[6]This formulation of aggregate demand can also be justified by the presence of a cash-in-advance constraint.

2.4 Exchange Rate Determination

Asset markets are assumed to be internationally complete. Complete markets implies the following risk-sharing condition

$$\left(\frac{C_t}{C_{t+1}}\right)^\sigma \frac{P_t}{P_{t+1}} = \left(\frac{C_t^*}{C_{t+1}^*}\right)^\sigma \frac{\epsilon_t P_t^*}{\epsilon_{t+1} P_{t+1}^*}$$

This equation relates the cross-country differential in the growth rate of consumption to the depreciation of the exchange rate. Assuming symmetric initial conditions, this can be rewritten as

$$\frac{\epsilon_t P_t^*}{P_t} = \left(\frac{C_t}{C_t^*}\right)^\sigma \tag{10}$$

Equation (10) is an efficiency condition that equates the marginal rate of substitution between home and foreign consumption to their marginal rate of transformation, expressed as equilibrium prices, i.e., the real exchange rate. A key consequence is that home consumption can rise relative to foreign consumption only if the real exchange rate depreciates.[7] Equation (10), combined with the processes for nominal aggregate demand and optimal prices, determines real and nominal exchange rates under complete markets.

2.5 Price-setting Decisions

Firms do not perfectly observe the state of aggregate demand and their marginal cost, but at each date they receive private signals about economic conditions. Prices are set in the producer's currency and there are no barriers to trade, so the law of one price always holds. Firm h's expected real profits in period t, conditional on the history of signals observed by that firm at time t, are given by

$$\Pi_{ht} = \mathbb{E}_{ht}\left[\frac{P_t(h)}{P_t}Y_t^d(h) - \frac{W_{ht}}{P_t}L_{ht}\right] \tag{11}$$

where $\mathbb{E}_{ht}$ is the expectation operator conditional on firm h's information set, $\mathcal{I}_h^t$. The production function is given by

$$Y_t(h) = A_t L_{ht} \tag{12}$$

Total factor productivity, A, in the two countries follows the processes

$$\ln A_t = \rho_a \ln A_{t-1} + u_t^a \tag{13}$$

$$\ln A_t^* = \rho_{a^*} \ln A_{t-1}^* + u_t^{a^*} \tag{14}$$

The shocks u are mean zero and have variances σ_a^2 and $\sigma_{a^*}^2$, respectively.

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[7]This implication is known to be at odds with the the data, where real exchange rates and consumption differentials exhibit low or negative correlation (Backus and Smith, 1993). This counterfactual implication could be fixed by assuming incomplete asset markets or by making preference assumptions that break the tight link between marginal utilities and current consumption (e.g., introducing habit formation or non-separable utility). Previous work on this topic suggests that, if anything, these modifications would increase the volatility and persistence of the real exchange rate, thus strengthening my results. For tractability, I proceed with the assumption of complete markets.

Each firm in the home country receives the following signals:

$$
Z_{h,t} = \begin{bmatrix} z_{h,t}^m \\ z_{h,t}^{m^*} \\ z_{h,t}^a \\ z_{h,t}^{a^*} \end{bmatrix} = \begin{bmatrix} m_t \\ m_t^* \\ a_t \\ a_t^* \end{bmatrix} + \begin{bmatrix} \tilde{\sigma}_m & 0 & 0 & 0 \\ 0 & \tilde{\sigma}_{m^*} & 0 & 0 \\ 0 & 0 & \tilde{\sigma}_a & 0 \\ 0 & 0 & 0 & \tilde{\sigma}_{a^*} \end{bmatrix} \begin{bmatrix} v_{h,t}^m \\ v_{h,t}^{m^*} \\ v_{h,t}^a \\ v_{h,t}^{a^*} \end{bmatrix} \tag{15}
$$

where $v_{h,t}^m, v_{h,t}^{m^*}, v_{h,t}^a, v_{h,t}^{a^*} \sim \mathcal{N}(0,1)$, $a_t = \ln A_{h,t}$ and $a_t^* = \ln A_t^*$. $m_t = \ln M_t$ and $m_t^* = \ln M_t^*$ represent the nominal aggregate demands (or money supplies), and the signal noises are assumed to be independently and identically distributed across firms and over time. Foreign firms receive similar signals drawn from the same distributions. In every period t, firms observe the history of their signals Z_h^t (that is, their information set is $\mathcal{I}_{ht} = \{Z_{h,\tau}\}_{\tau=-\infty}^t$) and maximize (11) subject to (12) and (4). The first-order condition yields

$$
P_t(h) = \frac{\gamma}{\gamma - 1} \frac{\mathbb{E}_{ht}\left[\left(\frac{1}{P_{Ht}}\right)^{-\gamma} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} \frac{C_t^W}{P_t} \frac{W_{ht}}{P_t A_t} \right]}{\mathbb{E}_{ht}\left[\left(\frac{1}{P_{Ht}}\right)^{-\gamma} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} \frac{C_t^W}{P_t} \right]} \tag{16}
$$

where $C_t^W \equiv \alpha C_t + (1-\alpha)\mathcal{Q}_t^\omega C_t^*$. Equation (16) states that a firm optimally sets its price to a markup, $\frac{\gamma}{\gamma-1}$, over its *perceived* marginal cost. Following the tradition in this literature, I log-linearize the price-setting equation around the deterministic steady state so that the transition equations of average prices are linear. I assume that firms use the log-linearized model, rather than the original nonlinear model when addressing their signal-extraction problem. This assumption greatly simplifies the analysis, because it allows for the use of the Kalman filter to characterize the dynamics of firms' beliefs. Finally, I assume that at the beginning of time, firms are endowed with an infinite history of signals. This implies that the Kalman gain matrix is time-invariant and identical across firms.

2.6 Real Exchange Rate Dynamics

In this section I characterize the solution for the real exchange rate. To simplify the algebra and convey intuition, I henceforth assume log utility for consumption ($\sigma = 1$). Appendix A shows how the model can be solved also for a generic value of σ. As also shown in Appendix A, under the producer currency pricing (PCP) assumption, the log-linearized first-order condition for a generic h and f firm, combined with equation (2), reads

$$
p_t(h) = E_{ht}\left[(1-\xi)p_{Ht} + \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)} t_t + \xi(m_t - a_t) \right] \tag{17}
$$

$$
p_t^*(f) = E_{ft}\left[(1-\xi)p_{Ft}^* - \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)} t_t + \xi(m_t^* - a_t^*) \right] \tag{18}
$$

where $\xi = \frac{1+\psi}{\gamma+\psi}$. These equations show the interdependence of the optimal price with their foreign counterpart through the terms of trade. In particular, if home and foreign goods are substitutes ($\omega > 1$), other things equal, a rise in the price of foreign goods (that is, a rise in

t_t) causes expenditure switching away from foreign goods toward home goods. The increased demand for home goods increases firm's h marginal cost and makes it optimal to raise $p_t(h)$. If goods are instead complements ($\omega < 1$), a rise in t_t decreases demand both for foreign and home output, hence the optimal price for a home good $p_t(h)$ falls.

The parameter $1 - \xi$ is related to the degree of strategic complementarities in price-setting, i.e., it tells by how much the optimal price of an individual firm changes when all the other domestic competitors are changing their prices. Because $\gamma > 1$, then $0 < \xi < 1$. Integrating (17) over domestic agents and (18) over foreign agents and noting that the log-linear price indices read as $p_{Ht} = \int_0^1 p_t(h) dh$ and $p_{Ft}^* = \int_0^1 p_t^*(f) df$, I obtain

$$p_{Ht} = \bar{\mathbb{E}}_t^{(1)} \left[(1 - \xi) p_{Ht} + \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)} t_t + \xi(m_t - a_t) \right] \tag{19}$$

$$p_{Ft}^* = \bar{\mathbb{E}}_t^{(1)} \left[(1 - \xi) p_{Ft}^* - \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)} t_t + \xi(m_t^* - a_t^*) \right] \tag{20}$$

where $\bar{\mathbb{E}}_t^{(1)}(\cdot) = \int_0^1 \bar{\mathbb{E}}_{it}(\cdot) di$ for $i = h, f$ denotes a first-order average expectation. Note that $\int_0^1 \bar{\mathbb{E}}_{ht}(\cdot) dh = \int_0^1 \bar{\mathbb{E}}_{ft}(\cdot) df$ follows from the symmetry of the information structure. Equations (19) and (20) can be disentangled following the tradition of the "sum" versus "differences" approach in general equilibrium open-economy models (Aoki, 1981). Specifically, I can take the sum of (19) and (20) to obtain

$$p_{Ht} + p_{Ft}^* = \bar{\mathbb{E}}_t^{(1)} \left[(1 - \xi)(p_{Ht} + p_{Ft}^*) + \xi(m_t + m_t^*) - \xi(a_t + a_t^*) \right] \tag{21}$$

which yields the solution

$$p_{Ht} + p_{Ft}^* = \xi \sum_{k=1}^{\infty} (1 - \xi)^{k-1} \bar{\mathbb{E}}_t^{(k)} (m_t^W - a_t^W) \tag{22}$$

where for any variable x_t, I define $x_t^W \equiv x_t + x_t^*$ and $x_t^D \equiv x_t - x_t^*$. Additionally $\bar{\mathbb{E}}_t^{(k)}(\cdot) = \int_0^1 \bar{\mathbb{E}}_{it}^{(k-1)}(\cdot) di$ denotes the k-th-order average expectation. By taking the difference between (19) and (20) and substituting the solution for the terms of trade, I obtain

$$p_{Ht} - p_{Ft}^* = \bar{\mathbb{E}}_t^{(1)} \left[(1 - \varphi)(p_{Ht} - p_{Ft}^*) + \varphi(m_t - m_t^*) - \xi(a_t - a_t^*) \right] \tag{23}$$

where $\varphi \equiv \frac{(1+\psi)+4\alpha(1-\alpha)(\omega-1)}{\gamma+\psi}$. The solution to the above equation yields

$$p_{Ht} - p_{Ft}^* = \varphi \sum_{k=1}^{\infty} (1 - \varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} m_t^D - \xi \sum_{k=1}^{\infty} (1 - \varphi)^{k-1} \bar{\mathbb{E}}_t^{(k)} a_t^D \tag{24}$$

The solution for p_{Ht} and p_{Ft}^* can be found by taking sums and differences of equations (22) and (24). Proposition 1 follows.[8]

Proposition 1 *Under the assumption of log-utility and complete asset markets, the real exchange*

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[8] Detailed derivations are in Appendix A.

rate is given by

$$q_t = (2\alpha - 1)\left(m_t^D - \varphi\sum_{k=1}^{\infty}(1-\varphi)^{k-1}\bar{\mathbb{E}}_t^{(k)}m_t^D - \xi\sum_{k=1}^{\infty}(1-\varphi)^{k-1}\bar{\mathbb{E}}_t^{(k)}a_t^D\right) \tag{25}$$

where $1 - \varphi \equiv 1 - \frac{(1+\psi)+4\alpha(1-\alpha)(\omega-1)}{\gamma+\psi}$ *governs the degree of strategic complementarity.*

The intuition behind this equation is straightforward. Focus for a moment on the first two terms on the right-hand side of (25) and consider a relative shock to nominal demands, m_t^D. Under full-information rational expectations, we expect the shock to have no effect on the real exchange rate, because nominal prices should adjust one for one with the nominal demands. Indeed, with full information $\bar{\mathbb{E}}_t^{(k)}m_t = m_t$ and $\bar{\mathbb{E}}_t^{(k)}m_t^* = m_t^*$ for every k so that $m_t^D = \varphi\sum_{k=1}^{\infty}(1-\varphi)^{k-1}\bar{\mathbb{E}}_t^{(k)}m_t^D$, and the real exchange rate responds only to real shocks.

Under imperfect information instead, as long as we have home bias, the real exchange rate also responds to nominal shocks to the extent that higher-order expectations deviate from full-information rational expectations. Equation (25) shows also that the persistence of the response of the real exchange rate to relative monetary shocks depends on how quickly the weighted average of higher-order expectations $\varphi\sum_{k=1}^{\infty}(1-\varphi)^{k-1}\bar{\mathbb{E}}_t^{(k)}m_t^D$ adjusts. As shown in section 3, the speed of adjustment depends on the degree of strategic complementarities (φ for relative variables) and on the signal-to-noise ratios $\sigma_m/\tilde{\sigma}_m$ and $\sigma_{m^*}/\tilde{\sigma}_{m^*}$. Specifically, the signal-to-noise ratios determine how quickly the different order of expectations in the summation will adjust to shocks. The strategic-complementarity parameter determines the *weights* attached to the different orders. For instance, the average first-order expectation about m_t^D receives a weight φ, the second order receives a weight $\varphi(1-\varphi)$, the third $\varphi(1-\varphi)^2$, and so on.

The last term on the right-hand side of (25) indicates that the real exchange rate always responds to relative technology shocks or, in the presence of dispersed information, to the weighted-average of higher-order beliefs about the shock.

2.7 Strategic Complementarities in the Open Economy

As discussed above, the strategic-complementarity parameter $(1-\varphi)$ is an important determinant of the dynamics of the real exchange rate, as it affects the weights attached to different orders of expectations. In this section I explain how this parameter crucially depends on the elasticity of substitution between home and foreign goods. In the case of log utility we have

$$(1 - \varphi) = 1 - \frac{(1+\psi)+4\alpha(1-\alpha)(\omega-1)}{\gamma+\psi} = 1 - \xi\left[1+2\zeta\right] \tag{26}$$

where I define $\zeta = \frac{2\alpha(1-\alpha)(\omega-1)}{1+\psi}$. To build intuition let us focus on the case of a closed economy first, obtainable by setting the home-bias parameter α to one (which implies $\zeta = 0$). In this case, the optimal pricing equations (17) and (18) would read

$$p_t(h) = E_{ht}\left[(1-\xi)p_{Ht} + \xi(m_t - a_t)\right] \tag{27}$$

$$p_t^*(f) = E_{ft}\left[(1-\xi)p_{Ft}^* + \xi(m_t^* - a_t^*)\right] \tag{28}$$

Here the degree of strategic complementarity is governed by $1 - \xi = 1 - \frac{1+\psi}{\gamma+\psi} \in (0,1)$. Consider the experiment of increasing p_{Ht}, keeping everything else constant. A domestic firm h responds to a increase in the average price p_{Ht} by increasing its own price. This happens because the increase in p_{Ht} shifts demand away from competitors toward firm's h output, and with specialized labor markets firm's h marginal cost is increasing in its own output. The strength of the increase in $p_t(h)$, measured by $(1-\xi)$, depends on the size of the change in firm's h demand, as captured by the elasticity of substitution between domestic goods γ, and on the slope of the labor supply curve, governed by the Frisch elasticity ψ.

Now consider the same experiment as above in the case in which the economies are open. Rewriting the pricing equations (17) and (18) using the solution for the terms of trade yields

$$p_t(h) = E_{ht} \left\{ [1 - \xi(1+\zeta)]p_{Ht} + \xi\zeta(p^*_{Ft} + m^*_t - m_t) + \xi(m_t - a_t) \right\} \tag{29}$$

$$p^*_t(f) = E_{ft} \left\{ [1 - \xi(1+\zeta)]p^*_{Ft} - \xi\zeta(m^*_t - m_t - p_{Ht}) + \xi(m^*_t - a^*_t) \right\} \tag{30}$$

Now the response of $p_t(h)$ to an increase in the average domestic price, p_{Ht}, is determined by the strategic-complementarity parameter $[1 - \xi(1 + \zeta)]$, which will have the same sign as $(1 - \varphi)$ in (26). Note that this response might be smaller or larger than in the closed-economy case, depending on whether the value of ω is above or below unity. The intuition goes as follows. Under our maintained assumption of log utility in this Section, when $\omega > 1$, home and foreign goods are net substitutes. This diminishes strategic complementarity relative to the closed economy, because an increase in p_{Ht} now shifts demand *away* from all the other domestic goods, partly *toward* firm h's good and partly toward foreign goods. Thus firm h experiences a milder increase in marginal cost and changes its price by a smaller amount than if it were to operate in a closed economy. Conversely, when $\omega < 1$, home and foreign goods are net complements. An increase in p_{Ht} induces a larger increase in firm's h marginal cost relative to the closed-economy case, and firm h raises its price by a larger amount. These additional effects are captured in the strategic-complementarity parameter $1 - \varphi$ via ζ. Thus the substitutability between home and foreign goods has important implications for the degree of strategic complementarity, which in turns affects the dynamics of the real exchange rate through the channels described in Section 3.

3   Analytical Results

To gain intuition about the cyclical properties of the real exchange rate in response to monetary shocks, let us abstract from technological shocks and study the simple case in which money supplies follow a random walk. Precisely, for this section I assume that $a_t = a^*_t = a$ and

$$m_t = m_{t-1} + u^m_t \tag{31}$$

$$m^*_t = m_{t-1} + u^{m^*}_t \tag{32}$$

which is obtained as a special case from equation (8) setting $\rho_m = \rho_{m^*} = 0$. With random walks in nominal spending and linear updating implied by the signal-extraction problem, I can establish Proposition 2.

Proposition 2 *Assuming random-walk processes for nominal spending and complete asset markets (CM), the real exchange rate follows an AR(1) process*

$$q_t = \nu q_{t-1} + (2\alpha - 1)\nu(u_t - u_t^*)$$

where $\varphi \equiv \frac{(1+\psi)+4\alpha(1-\alpha)(\omega-1)}{\gamma+\psi}$, $1 - \nu = \varphi \times \kappa_1 + (1 - \varphi) \times \kappa_2 \in (0,1)$, and κ_1, κ_2 are the non-zero elements of the Kalman gains matrix. The autocorrelation and variance of the real exchange rate are

$$\rho_{\hat{Q}} = \nu \quad \sigma_{\hat{Q}}^2 = (2\alpha - 1)^2 \left(\frac{\nu}{1-\nu}\right)^2 (\sigma_u^2 + \sigma_{u^*}^2)$$

Proof. *In the Appendix.*

Proposition 2 shows that, under the above assumptions, the real exchange rate follows an AR(1) process. The Proposition highlights how its persistence, ν, depends on the relevant degree of strategic complementarity, φ, and the precision of the signals that determine the weights κ_1 and κ_2 in the Kalman gain matrix. Larger noise and more strategic complementarity increase the persistence of the exchange rate. This is illustrated in Figure 2, which depicts the iso-persistence of the real exchange rate as a function of φ and the inverse signal-to-noise ratios $\tilde{\sigma}_m^2/\sigma_m^2$, assumed to be identical for m_t and m_t^*.

A lower φ indicates a higher degree of strategic complementarities, which means that agents put a larger weight on their beliefs about others' actions (and beliefs about others' beliefs about others' actions) relative to their own belief about the current state of nominal demand. This implies that higher-order beliefs receive a higher weight than lower-order beliefs. With high-order beliefs moving more sluggishly than low-order beliefs[9], prices adjust more sluggishly, which in turn implies slower movements in the real exchange rate following a money shock. Additionally, when the relative precision of the signal falls ($\tilde{\sigma}_m^2/\sigma_m^2 \downarrow$), agents will weight their prior more than their signals, failing to change prices and only slowly updating their beliefs when monetary shocks hit the economy. While the shock immediately affects the nominal exchange rate, the slow movement in prices again triggers slow reversion of the real exchange rate to purchasing-power parity.

Finally, notice from Propositions 2 that the a higher ν not only affects the persistence of the exchange rate, but also its volatility. To understand this, consider the response of prices when a monetary shock hits the Home economy. The higher the value of ν, the smaller the adjustment of home prices at the impact of the shock, for the same reasons discussed above. The small impact response of prices drives the amplification of monetary shocks onto the real exchange rate.

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[9]See Woodford (2002) or Melosi (2014) for further explanation and graphical examples.

4 Model Solution

Models with dispersed information and strategic interactions are hard to solve because they feature the "infinite regress" problem in which agents are required to forecast the forecast of others, which results in an infinite dimensional state space (Townsend, 1983). A number of approaches have been developed to solve this class of models. A numerical approach consists of guessing and verifying the laws of motion for the vector of higher-order beliefs. Since this vector is infinite-dimensional, in practice it is truncated at a sufficiently high order.[10] Another approach—used, for instance, in Lorenzoni (2009) and which will be used in some extensions below—uses a truncation in the time dimension.

In some cases, one can exploit the fact that only a particular *weighted average* of higher-order expectations matters for the solution of the model (Woodford, 2002; Melosi, 2014). The advantage of this approach is that the state vector has a finite dimension and there is no need to truncate it. The model developed here meets the conditions for the applicability of this method. By looking at equations (22) and (24), it is clear that determining the dynamics of $\varphi \sum_{k=1}^{\infty}(1-\varphi)^{k-1}\bar{\mathbb{E}}_t^{(k)} x_t^D$ and $\xi \sum_{k=1}^{\infty}(1-\xi)^{k-1}\bar{\mathbb{E}}_t^{(k)} x_t^D$ for $x = a, m$ is sufficient to determine the endogenous prices p_{Ht} and p_{Ft}^*. In turn, one can use these two variables together with the nominal exchange rate to solve for the rest of the model. Hence, to solve the model I guess that the state of the system includes the exogenous state variables plus the two specific weighted averages of high-order expectations implied by equations (22) and (24). In particular, I define $F_{\xi,t} \equiv \xi \sum_{k=1}^{\infty}(1-\xi)X_t^{(k)}$ and $F_{\varphi,t} \equiv \varphi \sum_{k=1}^{\infty}(1-\varphi)X_t^{(k)}$ where $X_t = [m_t, m_{t-1}, m_t^*, m_{t-1}^*, a_t, a_t^*]'$ is the vector of exogenous state variables. $X_t^{(k)}$ is shorthand notations for $\bar{\mathbb{E}}_t^{(k)} X_t$. The transition equation for the model can be shown to be

$$\bar{X}_t = \bar{B}\bar{X}_{t-1} + \bar{b}u_t \tag{33}$$

where

$$\bar{X}_t = \begin{bmatrix} X_t \\ \hline F_{\xi,t} \\ \hline F_{\varphi,t} \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B_{6\times 6} & 0 & 0 \\ \Gamma_{6\times 6}^{\xi,x} & \Gamma_{6\times 6}^{\xi,\xi} & 0 \\ \Gamma_{6\times 6}^{\varphi,x} & 0 & \Gamma_{6\times 6}^{\varphi,\varphi} \end{bmatrix} \quad \bar{b} = \begin{bmatrix} b_{6\times 4} \\ \hline \Gamma_{6\times 4}^{\xi,u} \\ \hline \Gamma_{6\times 4}^{\varphi,u} \end{bmatrix} \quad u_t = \begin{bmatrix} u_t^m \\ u_t^{m^*} \\ u_t^a \\ u_t^{a^*} \end{bmatrix}$$

Equation (33) is the state transition equation of the system. Firms in the model use the observation equation (15) and its foreign counterpart to form expectations about the state vector. The zeros in the $\bar{B}$ matrix reflect the fact that in this model, "sums" variables evolve independently of "differences" variables. The matrices B and b are given by the exogenous processes for monetary policy, whereas matrices Γ are to be determined by solving the signal-extraction problem of the firms using the Kalman filter. One can show that these matrices are functions of the parameters of the model and the Kalman gain matrix associated with the firms' signal-extraction problem. The algorithm used in Woodford (2002) can be easily extended to solve this model.

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[10]For an example, see Nimark (2011).

5 Impulse Responses

In this section I study the properties of the model in the more general case in which monetary processes can be autocorrelated ($\rho_m \neq 0$) and the economies are also hit by technology shocks.[11]

5.1 Monetary Shocks

Figure 3 shows the impulse responses of key variables to a positive monetary shock in the home country for a value of $\rho_m = 0.5$ and different signal-to-noise ratios. Prices for goods produced in the home country increase, but—because price adjustment is incomplete with imperfect information—domestic output also rises. Foreign output falls because, according to the parameterization used, home and foreign goods are net substitute. Consumption rises in both countries, more so in Home given the presence of home bias. The nominal exchange rate (not shown) depreciates as a result of the monetary expansion. The difference between home and foreign goods' prices rises by less than the nominal exchange rate, resulting in a worsening (upward movement) of the terms of trade. The real exchange rate depreciates, as it is proportional to the terms of trade. Finally, domestic inflation rises as the prices of both home goods and foreign goods rise in domestic currency. Conversely, foreign inflation falls, as foreign goods' prices are unchanged, and home goods' prices fall in domestic currency.

The introduction of persistent monetary shocks results in hump-shaped responses for most key macro variables, including the real exchange rate. The hump in the response of the real exchange rate is consistent with the empirical literature (Steinsson, 2008). Interestingly, domestic producer-price inflation displays a hump for persistent monetary shocks. Hence, this model is consistent with the inertial behavior of inflation observed in the data(Christiano, Eichenbaum, and Evans, 2005).

An increase in private signal noise delivers more volatility and persistence in the exchange rate. The intuition for this result is the same as that highlighted in the previous section, whereby with noisy signals, firms put little weight on new information and adjust prices very slowly. Figure 4 shows that higher strategic complementarity also contributes to increased volatility and persistence in the exchange rate, as it increases the weight put on other firms' beliefs in price-setting decisions.

5.2 Technology Shocks

Figure 5 shows the impulse responses to a home technology shock with persistence $\rho_a = 0.95$ for different signal-to-noise ratios. A home technology shock raises domestic output and lowers the prices of home-produced goods. The shock is transmitted internationally via a depreciation of the real exchange rate. Consumption rises in both countries but more markedly in the home country. Varying the signal-to-noise ratio, we observe that more noise tends to dampen the effect of technology shocks, although it contributes to somewhat higher persistence. The intuition behind these result relies on the fact that in this model, output can rise in response to technology

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[11]In this section, the model is parameterized using the calibration and prior means described in Section 6.2, unless otherwise noted.

shocks only if prices fall, because nominal expenditure is fixed by the levels of money supplies.[12] When signals are more precise firms change prices quickly and output can rise substantially. When signals are noisier, firms fail to lower prices enough, and therefore output increases by a smaller amount.

An interesting feature of this model compared to a sticky-price model à la Calvo is that it potentially allows for slow responses to nominal shocks and quicker responses to supply shocks if technology shocks are observed with relatively high precision. A sticky-price model would instead imply a more similar speed of adjustments to different shocks, governed by a single parameter: the exogenous probability of resetting prices.

6   Empirical Analysis

This section contains the econometric analysis that evaluates whether the dispersed-information model can account for the empirical properties of the Euro Area/Dollar real exchange rate. The analysis will proceed as follows. First, I will estimate the model parameters using Bayesian techniques. The estimation will help me pin down values for the parameters of the model, in particular the signal-to-noise ratios, for which empirical micro evidence is scarce. I will then use the estimated model to test how well it captures the dynamics of the real exchange rate.

6.1   Data and Empirical Strategy

I estimate the parameters of the dispersed-information model using data on the US and Euro Area. The US data comes the FRED database, while the European data comes from the Area Wide Model database.[13] I use the time series of the growth rate of GDP and GDP deflators that I map to the variables $[\Pi_t^H, \Pi_t^F, \Delta Y_t^H, \Delta Y_t^F]$ in the model, where $\Pi_t^H = \frac{P_{Ht}}{P_{H,t-1}}$ and $\Pi_t^F = \frac{P_{Ft}^*}{P_{F,t-1}^*}$. For the US, I construct GDP growth by taking the log-difference of real GDP (*GDPC96*) divided by the civilian non institutional population over 16 (*CNP16OV*). The growth rate of the GDP deflator is the log-difference of *GDPDEF*. For the Euro Area, I take the log difference of the real GDP (*YER*) divided by population. Population data for the 17 countries in the Euro Area, consistent with the GDP series, is taken from the OECD database.[14] The sample period goes from 1971:I to 2011:IV. All series are demeaned to be consistent with the model. The US is considered to be the home country. Before estimation the model is stationarized (details are in Appendix B).

My empirical strategy is as follows. I estimate the parameters of the dispersed-information model using data on real GDP and GDP deflators for the US and the Euro Area. Using these four observables allows me to pin down the key parameters of the model, including the signal-to-noise ratios related to monetary and technology shocks, for which there is scarce micro evidence. Given the estimated parameters, I subsequently test whether the dispersed information model is

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[12]This corresponds to the case in which monetary authorities do not accommodate technology shocks.

[13]I am grateful to my advisor, Susanto Basu, for granting me access to the Euro Area data.

[14]Population data are available only at annual frequency. I use linear interpolation to obtain the quarterly frequency.

quantitatively able to generate the volatility and persistence observed in the Euro/Dollar real exchange rate. This empirical strategy is analogous in spirit to the common practice of calibrating a model to fit certain moments (in this case, the moments of real GDP growth and GDP-deflator inflation rates included in the likelihood function) and testing how well the model reproduces other moments in the data (here, the moments of the real exchange rate). This setup effectively allows me to conduct an out-of-sample test on the real exchange rate, as none of its moments were directly used in the estimation of the model parameters.

It is important to notice that the four series used in the estimation contain very little information about the real exchange rate. First, the real exchange rate in the data is constructed using CPIs rather than GDP deflators. The regressions reported in Table 2 show that the four series used in the estimation explain at most 17% of the variation in the real exchange rate. Second, most of the variation in the real exchange rate in the data comes from movements in the nominal exchange rate, which has not been used in the estimation.[15]

6.2 Fixing Parameters and Priors

I fix the values of the parameters that are not well identified in the estimation process. Specifically, I set home bias $\alpha = 0.9$ to match the average import-to-GDP ratio for the US over the sample. The parameter γ is set to 7 following Mankiw and Reis (2010), which implies a steady-state markup of 16.7%. Finally, I set σ to 4, a slightly lower value than Steinsson (2008). The calibration is summarized in Table 1. I estimate the rest of the parameters.[16]

The prior distributions for the estimated parameters are summarized in Table 3. The priors for the standard deviation of shocks and noise terms follow Melosi (2014). There is no clear evidence on the value of the trade elasticity ω, although macro studies usually point toward low values.[17] The prior mean is set to one. The parameter $\xi = \frac{1+\psi}{\gamma+\psi}$ is related to strategic complementarities, and its prior mean is set to 0.4, implying a prior mean for ψ of 3. The persistence parameters for technology shocks are centered at 0.86 whereas the persistence for monetary shocks is set at 0.5—although for these parameters, I let the data guide the estimation by leaving priors fairly loose.

The model is estimated using Bayesian techniques, as explained in Herbst and Schorfheide (2016). Specifically, I draw from the posterior distribution $p(\Theta|Y)$, where Θ is the parameter vector and Y the data, using a standard random walk Metropolis-Hasting algorithm. The variance-covariance matrix of the proposal distribution, Σ, is set to the variance-covariance matrix of the estimated parameters at the mode of the posterior distribution. I then draw 1,000,000 parameter vectors from the posterior distribution. With this procedure, I get an acceptance rate of about 25%.

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[15]Regressing the real exchange rate on the nominal exchange rate alone produces an R^2 of 91%.

[16]The discount factor β does not appear in any linearized model equation.

[17]See Corsetti, Dedola, and Leduc (2008b) for a discussion.

6.3 Posterior Distribution

In Table 4, I present the estimates for the benchmark economy. The table shows the posterior median, together with a 90% posterior confidence band. The posterior of ξ is relatively tight around 0.21, lower than the prior mean, suggesting that the data are informative about this parameter. The posterior median for the persistence of technology shocks in the two countries is 0.98, in line with many other studies. The persistence of money growth processes is 0.45 for the US and 0.76 for the Euro Area. These values are linked to the growth rate of nominal GDP over the sample period for the two countries.[18]

The median estimate for the trade elasticity ω is 0.49. While relatively low, this number is comparable to the estimates of other studies that use a likelihood approach on US and Euro Area data. Lubik and Schorfheide (2006) estimate a trade elasticity of 0.43, while Rabanal and Tuesta (2010)'s estimates are in the range of 0.16-0.94, depending on the model specification. In the present context, low trade elasticity contributes to generating strategic complementarity as discussed in Section 2.7.

The parameters that are most important for the present analysis are the signal-to-noise ratios. For monetary or nominal demand shocks, the median estimates are $\sigma_m/\tilde{\sigma}_m = 0.08$ and $\sigma_m^*/\tilde{\sigma}_m^* = 0.07$. For the technology shocks, $\sigma_a/\tilde{\sigma}_a = 0.57$ and $\sigma_a/\tilde{\sigma}_a = 0.78$. These results indicate that firms are more informed about technology shocks than they are about nominal demand shocks by a factor of seven. Melosi (2014), who estimates a closed-economy model similar to the one used here, also finds that firms pay more attention to technology than to nominal demand shocks, and shows that this is consistent with the predictions of a rational inattention model (Sims, 2003), in which firms have to optimally choose how much attention to allocate to the two types of shocks. In a posterior predictive check, Melosi shows that the estimated signal-to-noise ratios are consistent with micro evidence on the absolute sizes of price changes (Nakamura and Steinsson, 2008).

The presence of information frictions implies that firms do not generally set their price equal to the profit-maximizing price, which is defined as the price a particular firm would set if it had complete information. I further validate the estimates of the signal-to-noise ratios by asking how much firms lose, in terms of profits, from being imperfectly informed. Arguably, it would not be plausible to remain poorly informed about the state of the economy if that implied incurring large profit losses. In Appendix C, I show that the estimated signal-to-noise ratios imply profit losses well below 1% of steady state revenues—0.5% of steady state revenues for a US firm and 0.8% for a European firm. These profit losses are small, and comparable in size to empirical estimates of the information cost of price adjustment, which is 1.22% of a firm's revenues according to the findings of Zbaracki et al. (2004). The Appendix also shows that the losses in the dispersed-information model are one order of magnitude smaller than the losses that would arise in a sticky-price model à la Calvo that generates similar real effects from monetary shocks.

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[18]Note that the observables Π_t^H and ΔY_t^H sum to the log of nominal GDP growth in the data and to Δm_t in the model.

6.4 How Well Does the Model Explain the Real Exchange Rate?

Here I test how well the estimated model captures the dynamics of the real exchange rate observed in the data. The real exchange rate consists of the nominal exchange rate in U.S. dollar per Euros, converted to the real exchange rate index by multiplying it by the Euro area CPI (*HICP*) and dividing it by the U.S. CPI (*CPIAUCSL*). The "synthetic" US/Euro nominal exchange rate prior to the launch of the Euro also comes from the the Area Wide Model Database. As for the Bayesian estimation, the sample period runs from 1971:I to 2011:IV.

Following the empirical approach of Steinsson (2008) and Carvalho and Nechio (2011), I calculate measures of persistence of the real exchange rate based on the estimates of an AR(p) process of the form.

$$q_t = \mu + \alpha q_{t-1} + \sum_{j=1}^{p} \psi_j \Delta q_{t-j} + \epsilon_t \tag{34}$$

where I calculate median unbiased estimates of μ, α, and ψ's using the grid-bootstrap method described by Hansen (1999). I set $p = 5$.

The first three columns of Table 5 report several measures of persistence and volatility of the real exchange rate. In the top part of the table, I compute the half-life (HL), up-life (UP), and quarter-life (QL) following a unitary impulse response. The half-life is defined as the largest T such that the impulse response $IR(T - 1) > 0.5$ and $IR(T) < 0.5$. The up-life and quarter-life are defined similarly, but with thresholds 1 and 0.25, respectively. All these measures are useful in capturing the non monotonically decaying shape of the exchange rate impulse response. I also consider the more traditional measures of persistence, such as the sum of autoregressive coefficients (captured by α) and the autocorrelation of the HP-filtered exchange rate. All the statistics are reported in years. The second part of Table 5 reports measures of volatility and cross-correlation of the real exchange rate.

We can analyze the persistence of the real exchange rate by looking at its response to a unitary impulse, depicted with a black line in Figure 6. The response displays a typical hump-shaped behavior, peaking in the second quarter at about 1.3 and not falling below the initial impulse—the up-life—for 9 quarters. The half-life of the exchange rate, the most commonly used measure of persistence, is about 4.4 years, which is well in line with previous evidence. Finally, the quarter-life of the exchange rate is 6.7 years, which implies that the time the exchange rate spends below one half of the initial response but above one quarter of it is 2.3 years, suggesting a moderate acceleration in the rate of decay when short-run dynamics start to die out. These findings are well in line with empirical evidence from other countries (Steinsson, 2008), and point to the presence of a hump shape in the impulse response of the exchange rate also for the US and Euro Area as well. Moreover, Table 5 highlights that the real exchange rate is extremely volatile: 5.8 times as volatile as consumption and 4.8 times as volatile as output. Finally, the correlation between the real and nominal exchange rate is 0.99.

To assess the empirical success of the dispersed-information model, I use the following algorithm to compute statistics that are comparable with the data:

- Step 1: Draw a parameter vector from $p(\Theta|Y)$.

- Step 2: Simulate the dispersed-information model for n periods and discard the initial $n/2$ observations. n is chosen such that $n/2$ is equal to the length of the actual data.

- Step 3: Estimate equation (34) on the simulated data and compute the relevant statistics.

I repeat the procedure for 10,000 iterations and consider the 90% posterior band. The last three columns of Table 5 report the results from the model.

The table shows that the model is notably successful in matching the moments from the data, even though its parameters were not estimated by targeting these moments. The model predicts a median half-life of 5.17 years, which is close to the 4.38 years observed in the data. The model implies a ratio of up-life to half-life that is somewhat larger than in the data. Similar to what is found in the data, the rate of decay of the exchange rate moderately accelerates in later periods, as can be seen in the difference between the half-life and the quarter-life. Finally, the autocorrelation of the HP filtered exchange rate is also in line with the empirical results. Figure 6 displays these results visually by superimposing the impulse responses from the model and the data. While the model implies a slightly more pronounced hump in the early quarters after the impulse, the dynamics farther out from the initial impulse tend to be quite similar to the data. The simulated real exchange rates also exhibit the high volatility and the strong correlation with the nominal exchange rate observed in the data. Overall, the estimated dispersed-information model successfully replicates the observed real exchange rate dynamics. These results are noteworthy, considering that the model parameters were not pinned down to match the empirical moments of the real exchange rate and that they imply reasonably small profit losses from limited attention.

6.5   Monetary Shocks and Persistent Real Exchange Rates

The previous section showed how the dispersed-information model, driven by monetary shocks and technology shocks, is able to capture the large and persistent fluctuations in real exchange rates. These are statements about the unconditional moments of the real exchange rate. Two interesting questions that remain open are the following: does the estimated model deliver persistent responses of the real exchange rate following monetary shocks alone? And are these dynamics consistent with the observed behavior of the real exchange rate, conditional on monetary shocks?

To address the first question, I simulate the model at the median estimates under the assumption that all fluctuations are due to monetary shocks. The corresponding properties of the real exchange rate are reported in the last column of Table 5. The table shows that the implied volatility of the real exchange rate, relative to consumption and output, is similar to the the data. Additionally, the model still generates highly persistent real exchange rate dynamics. The half-life of the exchange rate falls only to 4.13 from 5.17 years when both monetary shocks are productivity shocks were present. A half-life of 4.13 years is very well in line with the observed half life of 4.38 years. These results suggest that monetary shocks in the dispersed information model are able to generate empirically relevant volatility and persistence in real exchange rate dynamics.

I proceed to examine how the real exchange rate responds to monetary shocks in the data. A vast literature attempts to identify the effects of monetary shocks on real exchange rates (e.g., Clarida and Gali, 1994; Rogers, 1999; Eichenbaum and Evans, 1995). The literature highlights the difficulty of the task. Following the empirical macro literature, I identity a monetary shock in the data by means of a structural VAR.

I estimate a two-variable VAR using the real exchange rate and the CPI differential between the US and the Euro Area. Specifically, the variables are collected in the vector $X_t = [\Delta \ln RER_t, \Delta(\ln CPI_t^{US} - \ln CPI_t^{EU})]$. To identify the monetary shock in the VAR, I use the restriction that monetary shocks have no long-run effects on the real exchange rate (e.g., Blanchard and Quah, 1989). This identification scheme is consistent with the dispersed-information model. In keeping with the Bayesian spirit of the paper, I follow Sims and Zha (1998) in specifying the prior distribution for the VAR parameters. I obtain 10,000 posterior draws using the Gibbs sampler.

Figure 7 reports the impulse response of the level of the real exchange rate to a monetary shock from the estimated VAR, along with the median impulse response to a home monetary shock in the dispersed-information model.

The impulse response from the VAR highlights the fact that monetary shocks have persistent effects on the real exchange rate. The real exchange rate peaks two quarters after the impulse, displaying hump shaped dynamics, like in the unconditional dynamics. The dispersed information model does really well in capturing these dynamics, as well as the size of the response. The response from the model peaks three quarters after the impulse and then decays at a slightly slower rate compared to the data, but well within the 70% posterior credible set. Again, it should be noted that the parameters in the model were estimated without any reference to real exchange rates.

6.6 Business-cycle Moments

To understand how the model performs along other dimensions of the international business cycle, this subsection presents results for several business-cycle statistics commonly analyzed in the literature. Table 6 reports the business-cycle moments obtained from the dispersed information model and compares them with the analogous statistics obtained from the data. For the data, the statistics are based on logged and HP-filtered quarterly data for the period 1971:I to 2011:IV. For the model economy, I simulate time series of 158 quarters from the model and HP-filter the simulated data. In the Table, I report the average statistics across 200 replications.

The table shows that the model produces reasonable results along most of the business-cycle dimensions considered. In terms of volatilities, as in the data, the model predicts that consumption is less volatile than GDP. The model also accurately predicts that nominal exchange rates are more volatile than real exchange rates, which are in turn more volatile than the foreign versus domestic price ratio. This is a considerable improvement relative to the sticky-price model of Chari, Kehoe, and McGrattan (2002), in which the price ratio is much more volatile than in the data. The model also predicts quite volatile net exports.

As to the autocorrelations, the model generates considerable persistence in most of the vari-

ables considered, delivering long-lasting dynamics not only in prices but also in quantities such as real GDP, consumption, and net exports, although not so much for employment. In this respect, the dispersed-information model is more successful than the sticky-price model developed by Chari, Kehoe, and McGrattan (2002), which does not generate quite as much persistence in output and consumption.

The model reproduces the positive correlation between home and foreign consumption, output, and employment observed in the data. In terms of the constitutive "pieces" of the real exchange rate, the model also predicts reasonably well the negative correlations between real exchange rate and price ratio, and between nominal exchange rate and price ratio, in addition to the already noted strong positive correlation between nominal and real exchange rates.

There are a few limitations to the model's predictions, which relate to some of the assumptions made in order to keep the model tractable enough to be estimated. The model predicts a strong positive correlation between real exchange rates and relative consumption, which is at odds with the data. This discrepancy is expected, given our assumption of complete asset markets and the results of Backus and Smith (1993). Given the low estimate for the trade elasticity, the findings of Corsetti, Dedola, and Leduc (2008b) suggest that assuming incomplete international asset markets is likely to significantly reduce the positive correlation in the model. As commonly found in international real business-cycle models and in contrast to the data, Home and Foreign GDPs exhibit lower cross-correlations than consumptions.[19] Chari, Kehoe, and McGrattan (2002) show that this issue can be addressed by assuming that monetary shocks are correlated across countries.

Finally, the model predicts a pro-cyclical trade balance, while in the data it is counter-cyclical. This is also expected because of the absence of investment in the model. Indeed, by simple national accounting, one can show that if consumption is less volatile than output, the trade balance must be pro-cyclical.[20] Introducing capital accumulation can ameliorate the predictions of the model along this dimension, provided that consumption and investment move in the right direction.[21]

7 Comparison with Sticky-price Model

A natural question that arises in evaluating the empirical success of the imperfect-information model is how well it performs relative to a more traditional sticky-price model à la Calvo (1983). In this section I address the question in two ways. First, I estimate a model with sticky prices à la Calvo and compare its fit to the data relative to the dispersed-information model. Second, I compare the sticky-price model's ability to reproduce the observed real exchange rate dynamics relative to the model with information frictions.

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[19]See, for instance Backus, Kehoe, and Kydland (1994) and Heathcote and Perri (2002).

[20]By national accounting in this model, $C = Y - NX$. Hence $Var(C) = Var(Y) + Var(NX) - 2Cov(Y, NX)$, which implies that if $Var(C) < Var(Y)$ then $Cov(Y, NX) > 0$.

[21]For a discussion of the matter, see Raffo (2010).

7.1 The Calvo Model

Households and monetary authorities are modeled in the same way as in the benchmark economy. Firms can perfectly observe the current and past realization of shocks, but can only reset their prices with a random probability $1 - \theta$. The derivations of the model are standard and can be found, for instance, in Corsetti, Dedola, and Leduc (2010). The dynamics of inflation can be described by the New Keynesian Phillips Curves:

$$\pi_t^H = \kappa \left[\frac{\sigma\psi + 1}{\gamma + \psi} y_{H,t} - \frac{2(1-\alpha)\alpha\psi(\sigma\omega - 1)}{\gamma + \psi} \tau_t - \frac{1 + \psi}{\gamma + \psi} a_t \right] + \beta\mathbb{E}_t \pi_{t+1}^H \tag{35}$$

$$\pi_t^F = \kappa \left[\frac{\sigma\psi + 1}{\gamma + \psi} y_{F,t} + \frac{2(1-\alpha)\alpha\psi(\sigma\omega - 1)}{\gamma + \psi} \tau_t - \frac{1 + \psi}{\gamma + \psi} a_t^* \right] + \beta\mathbb{E}_t \pi_{t+1}^F \tag{36}$$

where $\pi_t^H = p_{H,t} - p_{H,t-1}$, $\pi_t^F = p_{F,t}^* - p_{F,t-1}^*$ and $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}$. These two equations replace equations (22) and (24) of the dispersed-information model.

7.2 Bayesian Model Comparison

In this section I take a Bayesian approach to compare the dispersed-information model and the sticky-price model. I start by parameterizing the Calvo model. The parameters α,γ, and σ are calibrated to the same values used in the benchmark model. The discount factor β and the Calvo parameter θ cannot be identified separately in the estimation, as they both enter the slope of the Phillips Curve in a non linear fashion. I calibrate the discount factor β to 0.99. I estimate the parameter $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}$. I set the prior of κ such that the median implies a value of the Calvo parameter $\theta = 0.69$, and the 5th and 95th percentile imply values for θ of approximately 0.5 and 0.90. This range broadly covers the micro and macro estimates for the frequency of price adjustment. The prior for the remaining parameters shared across models is the same as in Section 6. I report my estimates in Table 7, along with the median estimates from the dispersed-information model.

The most remarkable difference in posterior estimates across the two models is the estimated standard deviation of productivity shocks. The Calvo model estimates are 3 times as large those of the benchmark model. Note that the latter estimates are consistent with a standard real business cycle calibration of these shocks, while the Calvo estimates are much larger (Kydland and Prescott, 1982). The median estimate for κ implies a value for the Calvo parameter θ of 0.67, and that prices change every three quarters.

The Bayesian approach used in this paper allows me to compare how the dispersed-information and the sticky-price frameworks fit the data overall by computing the posterior probability of each model. I refer to the dispersed-information and the Calvo model with $\mathcal{M}_{DI}$ and $\mathcal{M}_C$, respectively. I denote the parameter vector associated with each model as Θ_{DI} and Θ_C, respectively. The posterior probability of model $\mathcal{M}_i$ with $i \in \{DI, C\}$ is given by

$$\pi_{T,\mathcal{M}_i} = \frac{\pi_{0,\mathcal{M}_i} p(Y|\mathcal{M}_i)}{\sum_{s \in \{DI,C\}} \pi_{0,\mathcal{M}_s} p(Y|\mathcal{M}_s)}$$

where $\pi_{0,\mathcal{M}_s}$ is the prior probability of model $\mathcal{M}_s$ and Y denotes the dataset used in the estimation. $p(Y|\mathcal{M}_s) = \int \mathcal{L}(\Theta_s|Y,\mathcal{M}_s)p(\Theta_s|\mathcal{M}_s)d\Theta_s$ is the marginal data density (MDD) or marginal likelihood of model $\mathcal{M}_s$, where $\mathcal{L}(\cdot)$ is the likelihood function and $p(\Theta_s|\mathcal{M}_s)$ denotes the prior distribution for the parameter vector Θ_s. As is standard, the prior probabilities, $\pi_{0,\mathcal{M}_s}$, are assumed to be the same across models, that is, $\pi_{0,\mathcal{M}_s} = 1/2$ for all $s \in \{DI,C\}$. Therefore, the model that attains the largest posterior probability is the one with the highest MDD.

The last row of Table 7 reports the log of the MDD for the two models. The comparison reveals that the dispersed-information model has a larger posterior probability than the sticky-price model by 43.4 log points. This difference is sizable. It implies that the prior probability ratio in favor of the Calvo model would need to be larger than $7.05e^{18}$ in order for the Calvo model to attain a higher posterior probability than the dispersed-information model. The fact that the Calvo model has fewer parameters than the dispersed-information model is not worrisome, because the MDD penalizes models for the number of their parameters. These findings suggest that the model with information frictions is considerably better suited for explaining the joint dynamics of US and Euro Area key macro variables than the sticky-price model.

7.3   Real Exchange Rate Statistics

Now I compare the models' ability to reproduce the real exchange rate dynamics. I focus on the ability of the two models to generate the volatility and the persistence of the exchange rate. Table 9 presents statistics for the real exchange rate in the two models unconditionally or conditionally on monetary shocks. Results reported are the median estimates from 200 simulations, using the same methodology described in Section 6.4. All statistics are reported in years.

Comparing columns 2 and 4 reveals that the estimated Calvo model delivers both low volatility and low persistence conditional on monetary shocks. The half-life of the real exchange rate in the model is just above 2 years. These results provide additional support for Chari, Kehoe, and McGrattan (2002)'s claim that monetary shocks in sticky price model cannot explain the persistence of the real exchange rate found in the data. While the models considered differ in some assumptions, the result obtained by those authors in a calibration exercise are qualitatively similar to the results obtained here, where the model is instead estimated. Additionally, while the model is able to explain fairly well the volatility of the exchange rate relative to output or consumption, it explains only about half of the absolute volatility of the real exchange rate. Column 4 shows that the dispersed information model is more successful in all these dimensions as already discussed in Section 6.4.

In column 3, we observe that when technology shocks are added to the picture, the sticky price model still delivers too little volatility in the real exchange rate, but now it generates counterfactually high persistence. The half life of the exchange rate increases from 2.07 years with only monetary shocks to more than 10 years with both shocks. Hence, the model predicts a half-life that is twice as large as that observed in the data. Additionally, the median quarter-life is around 15 years, about 8 years longer than the quarter life observed in the data. In contrast, column 5 shows that the dispersed-information model delivers a half-life and quarter-life that are only marginally higher than with monetary shocks alone, keeping the real exchange rate

dynamics in line with the data. The dispersed-information model also explains more than three quarters of the volatility observed in the data, compared to the 56% explained by the Calvo model.

The difference in the performance of the two models comes from (i) the different size of the estimated technology shocks in the two models and (ii) the different response to technology shocks across model. To the first point, we have seen above that the Calvo model estimates for technology shocks is 3 to 4 times larger than the estimates from the dispersed-information model. Intuitively, this happens because the estimated Calvo models requires large technology shocks to account for the volatility and persistence of output and domestic price indices while the dispersed-information model relies more on monetary shocks to explain that feature of the data.

The second point can be understood by examining the impulse-response function of the real exchange rate to monetary and technology shocks in the two models. The left panels of Figure 8 compare the response of the real exchange rate to a home monetary shock across the two models. For ease of comparison, the sizes of the shocks are set to the estimated standard deviations for the dispersed-information model reported in Table 7. The panel shows how the dispersed-information model delivers substantially more persistence from these shocks and a much more pronounced hump shape. This different response explains the difference between columns 2 and 4 of Table 9.

The right panels show the response of the exchange rate to productivity shocks. A few results emerge. First, the impact response of the real exchange rate is larger in the model with information friction than in the Calvo model. This happens because the presence of sticky prices substantially dampens the effect of productivity in the Calvo model relative to the efficient response. On the other hand, productivity shocks in the dispersed-information model are observed relatively precisely by firms, making the response to these shocks look more like an efficient response. Second, for similar reasons productivity shocks in the Calvo model damp out more slowly relative to the dispersed-information model. One can intuitively see from the picture that the half-life of these shocks is significantly greater in the sticky-price model.

This last fact and the different sizes of estimated technology shocks in the two models are responsible for the considerable difference in unconditional persistence found in the two models in columns 3 and 5 of Table 9.

7.4 Discussion

The results of this section highlight a number of differences between traditional sticky-price models and models in which slow price adjustment is the endogenous response to information frictions. First, estimation of the two models suggests that the dispersed-information model fits the data on output and output deflators better than the sticky-price model. The former also delivers estimates for the size of productivity shocks that are consistent with real business-cycle calibrations of these shocks. The sticky-price model instead requires substantially larger productivity shocks to explain the data.

Second, comparing the two models' ability to reproduce the empirical properties of the real

exchange rates demonstrates the strength of the information-friction model relative to frameworks that model the nominal rigidity exogenously. In this particular case, assuming that prices can be reset with an exogenous probability irrespective of the shocks hitting the economy limits the model's ability to match the data. In the class of model with exogenous nominal rigidities considered here, there is a trade-off between obtaining amplification from nominal shocks and obtaining large effects from real shocks. This trade-off does not necessarily arise in models with dispersed information of the kind considered here, in which prices can respond differently to different kinds of shocks. Indeed the estimation results from the model with information frictions pushes exactly in this direction when seeking to fit the output and prices data, trying to obtain a slow response to monetary shock and a quick response to real shocks. It turns out that that this feature is important when it comes to predicting the real exchange rate.

In contrast, the Calvo model delivers too little persistence with only monetary shocks and too much persistence when both shocks are present. The result that monetary shocks cannot explain the persistence of the real exchange rate is qualitatively reminiscent of the results of Bergin and Feenstra (2001) and Chari, Kehoe, and McGrattan (2002). Differently from those, these findings are obtained in the context of an estimation exercise. The fact that real shocks cannot explain jointly the volatility and persistence of the exchange rate is also discussed in Iversen and Söderström (2014) in the context of a calibrated two-country model. There are modeling differences between the model considered here and theirs. Nonetheless, both models produce the result that with low elasticity of substitution between home and foreign goods, the presence of real shocks, while helping explain the volatility of the exchange rate relative to output and consumption, exaggerates the up-life, half-life, and quarter-life of the real exchange rate.

Taken together, these results suggest that the dispersed-information model outperforms the sticky-price model not only in explaining domestic variables, such as domestic output and prices, but it also better explains international price movements. I further validate this point by including the real exchange rate series among the observables and re-estimating both models. To accommodate the additional observable variable and avoid stochastic singularity, I add a measurement error to the real exchange rate equation. With these modifications, the dispersed-information model and the Calvo model deliver MDDs of 2518.45 and 2479.02, respectively. The difference of about 40 log points is sizable and points to the stronger ability of the model with information frictions to fit the data, consistently with other results in this section.

8   Sensitivity to Information Structure

In this section I investigate the role of the information structure in generating the persistence of the real exchange rate. The assumptions that firms observe signals about aggregate nominal demand and technology with finite precision is a simple way of capturing the idea that there is a cost in acquiring and processing information. In this context, the lower the cost of acquiring information, the higher the precision of the signals.

Nevertheless, it is worth noting that different signals may carry different information about

the variables that matter for firms' decisions. In particular, the literature started by Grossman (1976) and Hellwig (1980) stresses the idea that, under certain conditions, prices may aggregate disparate information that different economic agents have. When making optimal pricing decisions, an important variable for the firms in the model are the aggregate price levels in the two countries. This can be seen from the first-order conditions (19) and (20), which I repeat here for convenience:

$$p_t(h) = \mathbb{E}_{ht}\left[(1-\xi)p_{Ht} + \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}t_t + \xi(m_t - a_t)\right]$$

$$p_t^*(f) = \mathbb{E}_{ft}\left[(1-\xi)p_{Ft}^* - \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}t_t + \xi(m_t^* - a_t^*)\right]$$

These two equations make clear that a firm's optimal price depends on its expectation about the aggregate price level for domestically produced goods and a measure of relative prices.

Here I entertain the hypothesis that firms observe signals about these prices. Specifically, in addition to the four signals observed in the benchmark model, firms in both the Home and Foreign country now have access to the following two signals:

$$z_{i,t}^{p^H} = p_{H,t} + x_t + v_{i,t}^{p^H}$$
$$z_{i,t}^{p^F} = p_{F,t}^* + x_t + v_{i,t}^{p^F}$$

for $i = h, f$. The signals about aggregate prices contain an aggregate noise component, x_t, and an idiosyncratic noise component $v_{i,t}$. All the noise terms are iid, normally distributed with mean zero and variances σ_x^2, $\tilde{\sigma}_{p^H}^2$ and $\tilde{\sigma}_{p^F}^2$, respectively. To assess the robustness of my results, I re-estimate the model allowing for the presence of these two additional signals and compare the real exchange rate between my benchmark model and this augmented model. I use flat priors for the additional parameters so as to let the data entirely guide the estimation.

It is worth noting that these new signals are endogenous, and they depend on equilibrium prices. This feature breaks the finite state-space representation of the model solution described in Section 4. Hence, I adapt the solution method used in Lorenzoni (2009), allowing it to handle two countries. The details of the algorithm are provided in the Appendix. To solve the model, I write the state-space as the history of the time-t state variables used in the firm's inference problem: $[m_t, m_t^*, a_t, a_t^*, p_{H,t}, p_{F,t}^*]$ and I truncate it at $t - T$. I choose T sufficiently high, so that by increasing T the impulse response functions of the model do not change.

Table 8 presents the posterior mode of the new estimates *vis-à-vis* the benchmark estimates. Most of the parameters that are common across the two model have similar estimates in the two cases. The aggregate noise and the idiosyncratic noises have estimated standard deviations of 25, 46, and 24, respectively. These values are larger then the standard deviations of the signals about monetary shocks, indicating that the data favor the idea that signals about aggregate prices are fairly noisy. The value of the posterior at the mode changes only marginally, suggesting that this extended model is not significantly superior in fitting the data relative to the benchmark model.

How does this affect the main results? Standard signal-extraction theory suggests that an

agent should optimally put little weight on more imprecise signals. Hence we can already expect agents to put little weight on these new signals. Figure 9 shows the impulse response of the real exchange rate in the benchmark model and in the model with endogenous signals. There are only minor differences between the two, indicating that the dynamics of the model are not quantitatively affected by the presence of endogenous signals on prices once the model is re-estimated. Finally, Table 10 confirms these results by comparing the up-life, half-life, and quarter-life of the exchange rate in the two versions of the model and find no significant differences.

9 Conclusions

Existing New-Keynesian models with sticky prices struggle to deliver the persistence in the real exchange rate observed in the data under plausible nominal rigidities. In this paper, I argue that the persistence of the real exchange rate, together with its other empirical features, can be explained by a model with strategic complementarity and dispersed information among price-setting firms. In this environment, firms' beliefs about economic conditions and about other firms' expectations become endogenous state variables that result in increased persistence in real exchange rates. Once taken to the data, the model is shown to successfully explain the volatility and persistence of the real exchange rate. The model also generates persistent real exchange rate dynamics following monetary shocks, which is consistent with the empirical evidence documented by a structural VAR. Taken together, my findings suggest that dispersed information is a quantitatively important channel for real exchange rate dynamics that should be taken into account in future research on this topic.

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Tables

Table 1: Calibrated Parameters

Name	Description	Value
γ	Elasticity of substitution between domestic goods	7
α	Home bias	0.9
σ^{-1}	Intertemporal elasticity of substitution	1/4

Table 2: Real Exchange Rate Predictability

	ΔRER_t	ΔRER_t	RER_t	RER_t
$\Delta RGDP_t^{US}$	-0.471	-0.456	-2.616	-2.988*
	(-1.00)	(-1.00)	(-1.73)	(-2.00)
$\Delta RGDP_t^{EU}$	0.832	0.656	-1.579	-2.198
	(1.39)	(1.14)	(-0.82)	(-1.16)
$\Delta DEFL_t^{US}$	-1.455	-3.428*	15.28***	6.523
	(-1.40)	(-2.53)	(4.60)	(1.47)
$\Delta DEFL_t^{EU}$	0.533	2.251**	-10.32***	-9.872***
	(0.78)	(2.68)	(-4.69)	(-3.59)
ΔCPI_t^{US}		2.737***		7.591**
		(3.41)		(2.88)
ΔCPI_t^{EU}		-2.735**		-0.216
		(-3.13)		(-0.08)
N	163	163	163	163
R^2	0.025	0.121	0.173	0.216
Adj. R^2	0.000	0.087	0.152	0.187

$Notes$: t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3: Priors

Name	Description	Shape	Median	0.05	0.95
ξ	Strategic complementarity	$\mathcal{B}$	0.40	0.24	0.57
ω	Trade elasticity	$\mathcal{N}$	1.00	0.5	1.5
ρ_a	Persistence of technology shock (H)	$\mathcal{B}$	0.86	0.70	0.96
ρ_a^*	Persistence of technology shock (F)	$\mathcal{B}$	0.86	0.70	0.96
ρ_m	Persistence of monetary shock (H)	$\mathcal{B}$	0.50	0.25	0.75
ρ_m^*	Persistence of monetary shock (F)	$\mathcal{B}$	0.50	0.25	0.75
$100\sigma_a$	Std of technology shock (H)	$\mathcal{IG}$	0.68	0.53	0.92
$100\sigma_a^*$	Std of technology shock (F)	$\mathcal{IG}$	0.68	0.53	0.92
$100\sigma_m$	Std of monetary shock (H)	$\mathcal{IG}$	1.80	0.60	6.03
$100\sigma_m^*$	Std of monetary shock (F)	$\mathcal{IG}$	1.80	0.60	6.03
$\sigma_a/\tilde{\sigma}_a$	Signal-to-noise — technology shock (H)	$\mathcal{NA}$	0.73	0.31	2.33
$\sigma_a^*/\tilde{\sigma}_a^*$	Signal-to-noise — technology shock (F)	$\mathcal{NA}$	0.73	0.31	2.33
$\sigma_m/\tilde{\sigma}_m$	Signal-to-noise — monetary shock (H)	$\mathcal{NA}$	0.11	0.07	0.15
$\sigma_m^*/\tilde{\sigma}_m^*$	Signal-to-noise — monetary shock (F)	$\mathcal{NA}$	0.11	0.07	0.15

Notes : The letters $\mathcal{B}, \mathcal{N}, \mathcal{IG}$ denote the beta, normal, and inverse gamma distributions. $\mathcal{NA}$ is used for implied priors, which do not belong to any family of theoretical distributions.

Table 4: Posterior Estimates

Name	Description	Median	0.05	0.95
ξ	Strategic complementarity	0.21	0.16	0.29
ω	Trade elasticity	0.50	0.39	0.62
ρ_a	Persistence of technology shock (H)	0.98	0.97	0.99
ρ_a^*	Persistence of technology shock (F)	0.98	0.97	0.99
ρ_m	Persistence of monetary shock (H)	0.41	0.29	0.52
ρ_m^*	Persistence of monetary shock (F)	0.74	0.67	0.83
$100\sigma_a$	Std of technology shock (H)	0.97	0.76	1.19
$100\sigma_a^*$	Std of technology shock (F)	0.71	0.58	0.85
$100\sigma_m$	Std of monetary shock (H)	0.89	0.80	0.98
$100\sigma_m^*$	Std of monetary shock (F)	0.77	0.70	0.85
$\sigma_a/\tilde{\sigma}_a$	Signal-to-noise — technology shock (H)	0.51	0.42	0.75
$\sigma_a^*/\tilde{\sigma}_a^*$	Signal-to-noise — technology shock (F)	0.97	0.61	1.01
$\sigma_m/\tilde{\sigma}_m$	Signal-to-noise — monetary shock (H)	0.11	0.05	0.12
$\sigma_m^*/\tilde{\sigma}_m^*$	Signal-to-noise — monetary shock (F)	0.10	0.04	0.11

Notes : The table reports the median, the 5th, and 95th percentile of the estimates for the parameters of the dispersed-information model.

Table 5: Estimation Results

	Data			Model			Model
	Median	0.05	0.95	Median	0.05	0.95	Only M shocks
α	0.96	0.903	0.995	0.97	0.95	0.98	0.96
Half-life (HL)	4.38	2.05	38.54	5.17	3.68	7.43	4.13
Quarter-life (QL)	6.73	2.95	68.10	7.18	5.06	10.63	5.62
UL/UH	0.45	0.15	0.65	0.61	0.52	0.69	0.45
QL-HL	2.35	0.62	13.17	1.98	1.19	3.26	1.49
$\rho(q_{hp})$	0.84	0.75	0.87	0.89	0.87	0.92	0.89
$\frac{\sigma(q_{hp})}{\sigma(c_{hp})}$	5.83	-	-	4.24	3.71	4.76	4.13
$\frac{\sigma(q_{hp})}{\sigma(y_{hp})}$	4.83	-	-	3.65	3.11	4.20	3.64
$\rho(q_{hp}, \varepsilon_{hp})$	0.99	-	-	0.95	0.93	0.97	0.96

$Notes$: Half-life (HL): the largest T such that $IR(T-1) \geq 0.5$ and $IR(T) < 0.5$. Quarter-life (QL): the largest T such that $IR(T-1) \geq 0.25$ and $IR(T) < 0.25$. Up-life (UL): the largest time T such that $IR(T-1) \geq 1$ and $IR(T) < 1$. ρ and σ correspond to first-order autocorrelation/cross-correlation and standard deviation, respectively. Only M shocks refer to the model driven only by monetary shocks.

Table 6: Business Cycle Statistics

Statistic	Data	Model
Standard deviations relative to GDP		
Consumption	0.82	0.86
Employment	0.89	1.17
Nominal Exchange Rate	4.94	4.54
Real Exchange Rate	4.73	3.55
Price Ratio	0.74	1.58
Net Exports	0.38	0.67
Autocorrelations		
GDP	0.87	0.83
Consumption	0.88	0.84
Employment	0.94	0.75
Nominal Exchange Rate	0.84	0.87
Real Exchange Rate	0.83	0.87
Price Ratio	0.89	0.92
Net Exports	0.86	0.85
Cross-correlations		
Home and Foreign GDP	0.52	0.10
Home and Foreign Consumption	0.36	0.57
Home and Foreign Employment	0.46	0.09
Net Exports and GDP	-0.53	0.52
RER and GDP	0.09	0.57
RER and Net Exports	0.18	0.92
RER and Relative Consumption	-0.14	1.00
Real and Nominal Exchange Rate	0.99	0.95
Nominal Exchange Rate and Price Ratio	-0.36	-0.73
RER and Price Ratio	-0.22	-0.50

Notes : With the exception of net exports, standard deviations and correlations in the table are based on logged and HP-filtered US and Euro Area data for the period 1971:I-2011:IV. Net exports are measured as the HP-filtered ratio of real net exports to real GDP. Thus, the standard deviation of net exports is simply the standard deviation of this ratio.

Table 7: Posterior Estimates

Name	Description	DI Model	Calvo Model
κ	$(1-\beta\theta)(1-\theta)/\theta$	—	0.16
ξ	Strategic complementarity	0.21	0.15
ω	Trade elasticity	0.49	0.58
ρ_a	Persistence of technology shock (H)	0.98	0.97
ρ_a^*	Persistence of technology shock (F)	0.98	0.96
ρ_m	Persistence of monetary shock (H)	0.45	0.28
ρ_m^*	Persistence of monetary shock (F)	0.76	0.66
$100\sigma_a$	Std of technology shock (H)	0.86	2.14
$100\sigma_a^*$	Std of technology shock (F)	0.81	2.62
$100\sigma_m$	Std of monetary shock (H)	0.90	0.87
$100\sigma_m^*$	Std of monetary shock (F)	0.77	0.75
$\sigma_a/\tilde{\sigma}_a$	Signal-to-noise — technology shock (H)	0.57	—
$\sigma_a^*/\tilde{\sigma}_a^*$	Signal-to-noise — technology shock (F)	0.78	—
$\sigma_m/\tilde{\sigma}_m$	Signal-to-noise — monetary shock (H)	0.08	—
$\sigma_m^*/\tilde{\sigma}_m^*$	Signal-to-noise — monetary shock (F)	0.07	—
MDD	Log Marginal Data Density	2461.9	2418.5

$Notes$: The table reports the median estimates for the parameters of the dispersed-information (DI) model and the Calvo model.

Table 8: Posterior Estimates Comparison

Name	Description	Benchmark	Endo Signals
ω	Trade elasticity	0.50	0.45
ρ_a	Persistence of technology shock (H)	0.99	0.99
ρ_a^*	Persistence of technology shock (F)	0.99	0.99
ρ_m	Persistence of monetary shock (H)	0.42	0.41
ρ_m^*	Persistence of monetary shock (F)	0.74	0.72
$100\sigma_a$	Std of technology shock (H)	1.47	1.38
$100\sigma_a^*$	Std of technology shock (F)	1.06	1.11
$100\sigma_m$	Std of monetary shock (H)	0.87	0.90
$100\sigma_m^*$	Std of monetary shock (F)	0.75	0.77
$100\tilde{\sigma}_a$	Std noise — technology shock (H)	2.45	2.38
$100\tilde{\sigma}_a^*$	Std noise — technology shock (F)	0.88	1.84
$100\tilde{\sigma}_m$	Std noise — monetary shock (H)	9.55	11.93
$100\tilde{\sigma}_m^*$	Std noise — monetary shock (F)	6.87	7.71
$100\sigma_x$	Std of aggregate noise	-	25.96
$100\sigma_{v_H}$	Std of idiosyncratic noise (H)	-	46.49
$100\sigma_{v_F}$	Std of idiosyncratic noise (F)	-	23.73
$p(\theta|Y)$	Log Posterior at the Mode	2505.2	2510.7

$Notes$: The table reports the mode the parameters of the dispersed-information model (Benchmark) and the model with endogenous signals (Endo Signals).

Table 9: Real Exchange Rate Statistics Comparison

	Data	Calvo	Calvo Only M	DI	DI Only M
Half-life (HL)	4.38	10.25	2.07	5.17	4.13
Up-life (UL)	1.99	5.15	0.97	3.05	2.56
Quarter-life (QL)	6.73	14.96	3.06	7.18	5.62
$\rho(q_{hp})$	0.84	0.86	0.81	0.89	0.89
$100\sigma(q_{hp})$	72.8	40.5	32.6	57.2	55.3
$100\sigma(y_{hp})$	15.7	12.8	10.0	15.5	15.4
$100\sigma(c_{hp})$	13.0	10.5	8.2	13.5	13.4

Notes : DI refers the dispersed-information model. Only M refers to the model driven only by monetary shocks.

Table 10: Exchange Rate Persistence

	Data	Baseline	Endogenous Signals
α	0.96	0.97	0.97
Half-life (HL)	4.38	5.17	5.82
Up-life	1.99	3.06	3.26
Quarter-life (QL)	6.73	7.18	8.34
$\rho(q_{hp})$	0.84	0.89	0.88
$\frac{\sigma(q_{hp})}{\sigma(c_{hp})}$	5.83	4.24	4.02
$\frac{\sigma(q_{hp})}{\sigma(y_{hp})}$	4.83	3.65	3.54
Log Posterior	-	2505.2	2510.7

Notes : Half-life (HL): the largest T such that $IR(T-1) \geq 0.5$ and $IR(T) < 0.5$. Quarter-life (QL): the largest T such that $IR(T-1) \geq 0.25$ and $IR(T) < 0.25$. Up-life (UL): the largest time T such that $IR(T-1) \geq 1$ and $IR(T) < 1$. ρ and σ correspond to first-oder autocorrelation and standard deviation, respectively.
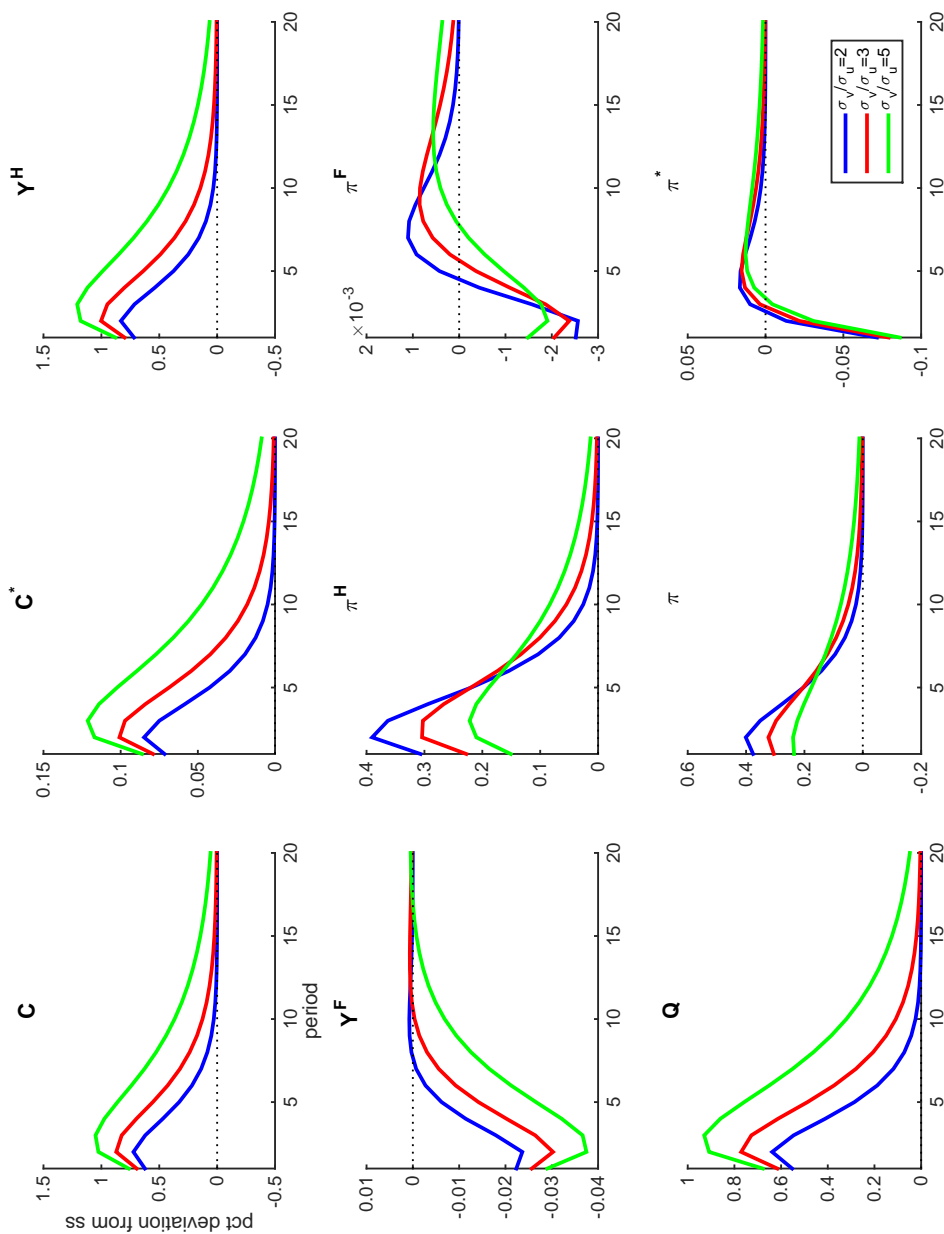
Figures

Figure 1: Dispersion in Beliefs
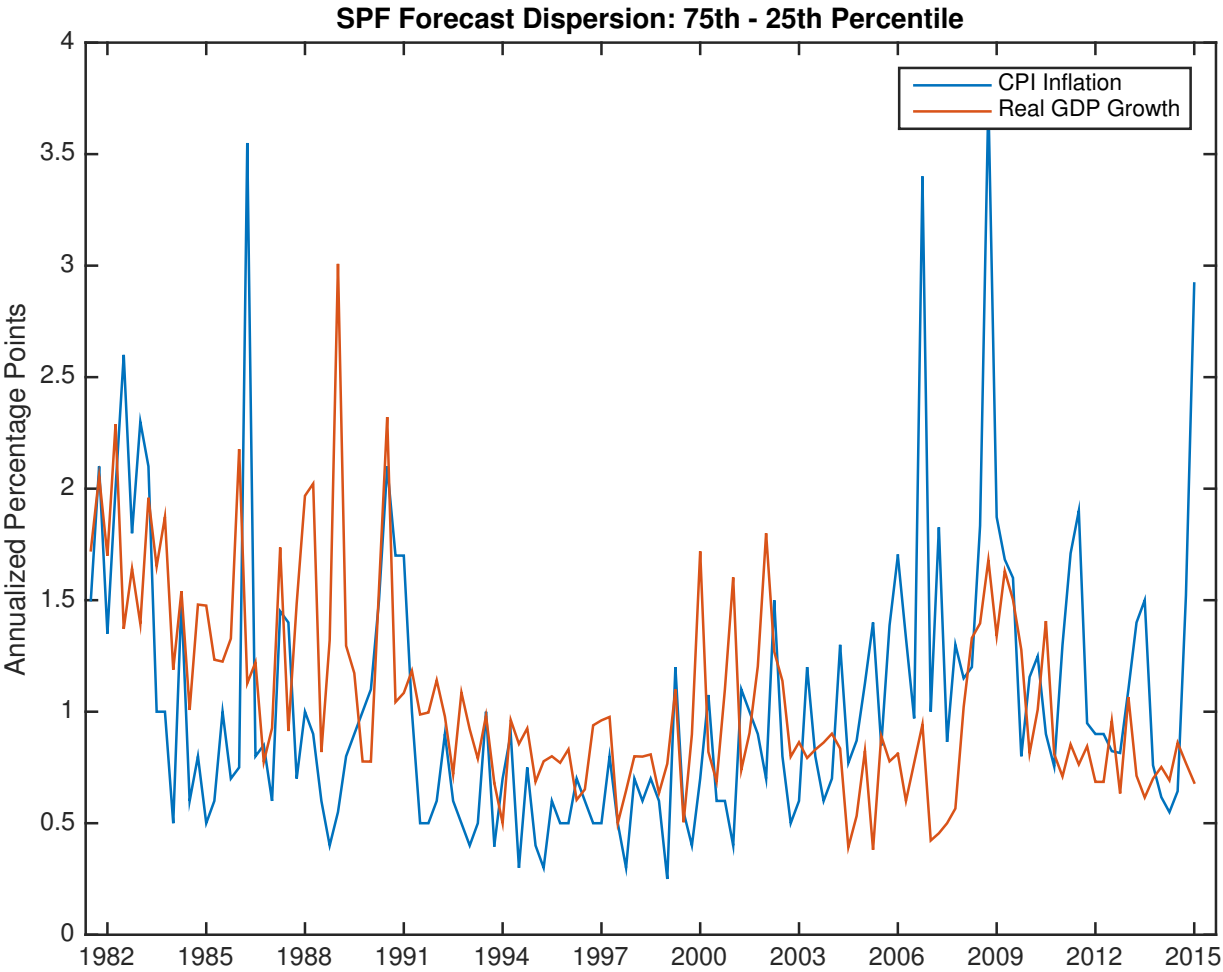
Figure 2: Iso-persistence Curves of the Real Exchange Rate
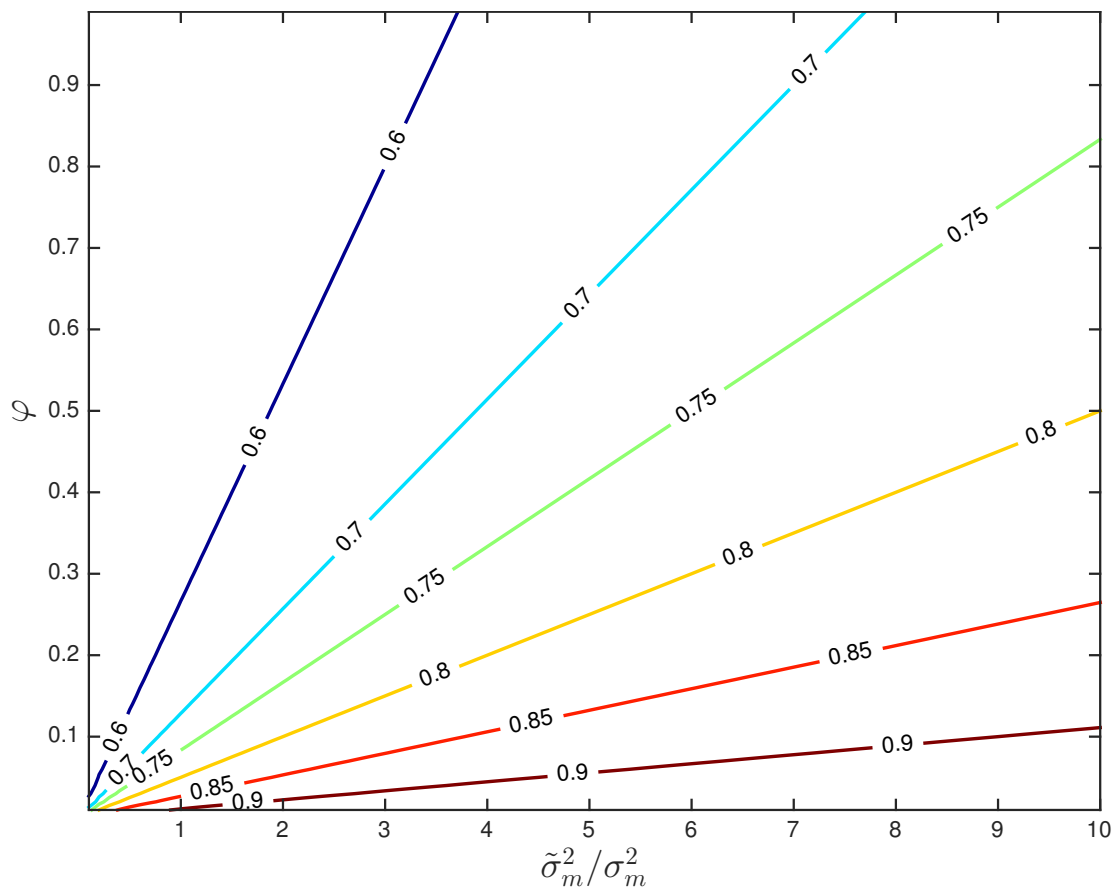
Notes: The Figure depicts the iso-persistence curves of the real exchange rate for different values of the inverse signal-to-noise ratio $\tilde{\sigma}_m^2/\sigma_m^2$ and of the strategic-complementarity parameter φ when money supplies follow a random walk. Lower φ indicates more strategic complementarity.

Figure 3: Impulse Responses to a Home Monetary Shock — $\rho = 0.5$

Notes: The figure depicts the impulse responses of key variables following a Home monetary shock for different values of noise in the signals (σ_v) relative to the standard deviation of the shock (σ_u).

Figure 4: Imperfect Information and Strategic Complementarities
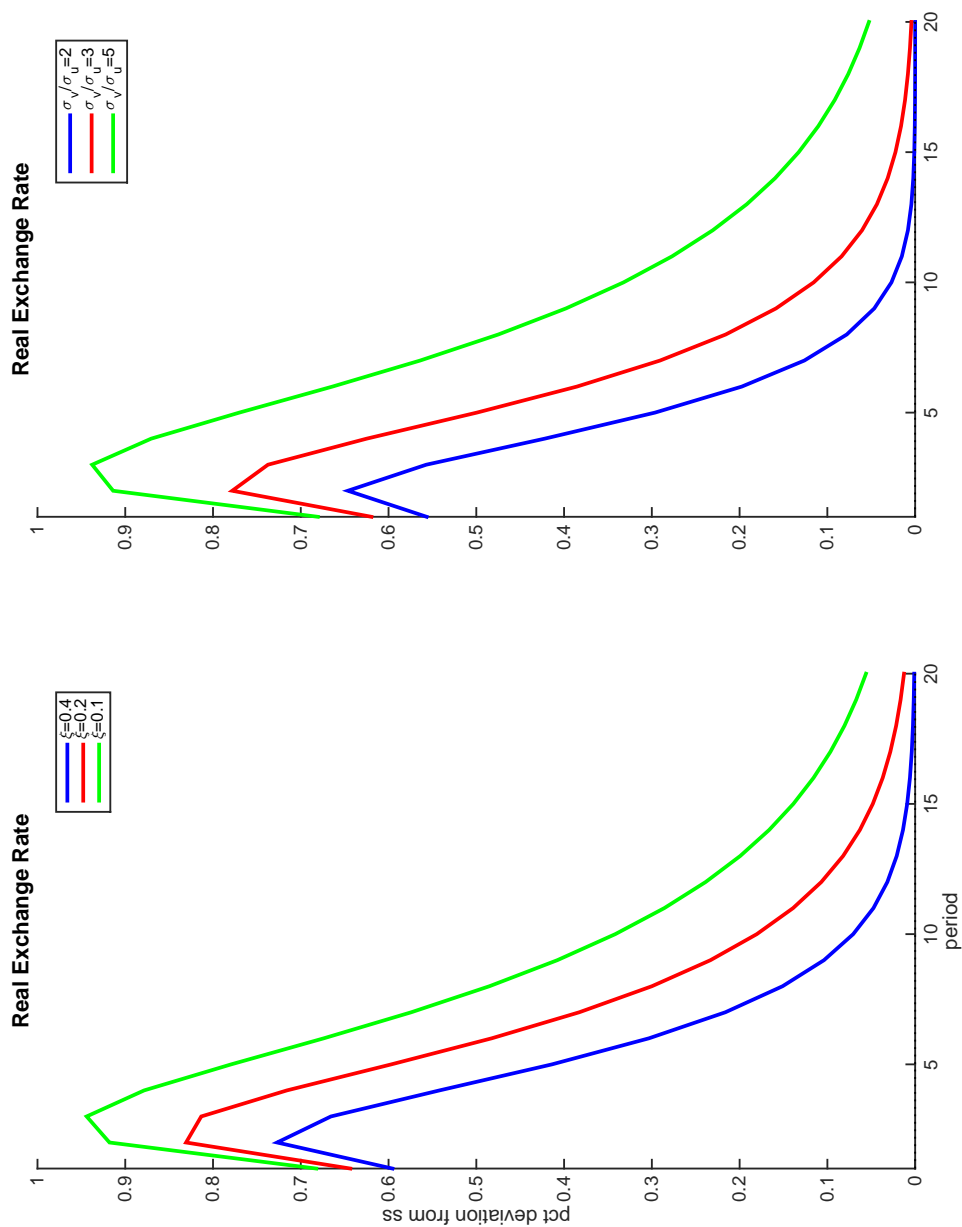
Notes: The figure depicts the impulse responses of the real exchange rate to a Home monetary shock for different values of noise in the signals (σ_v) relative to the standard deviation of the shock (σ_u) and for different values of the strategic-complementarity parameter ($1 - \xi$).

Figure 5: Impulse Responses to a Home Technology Shock
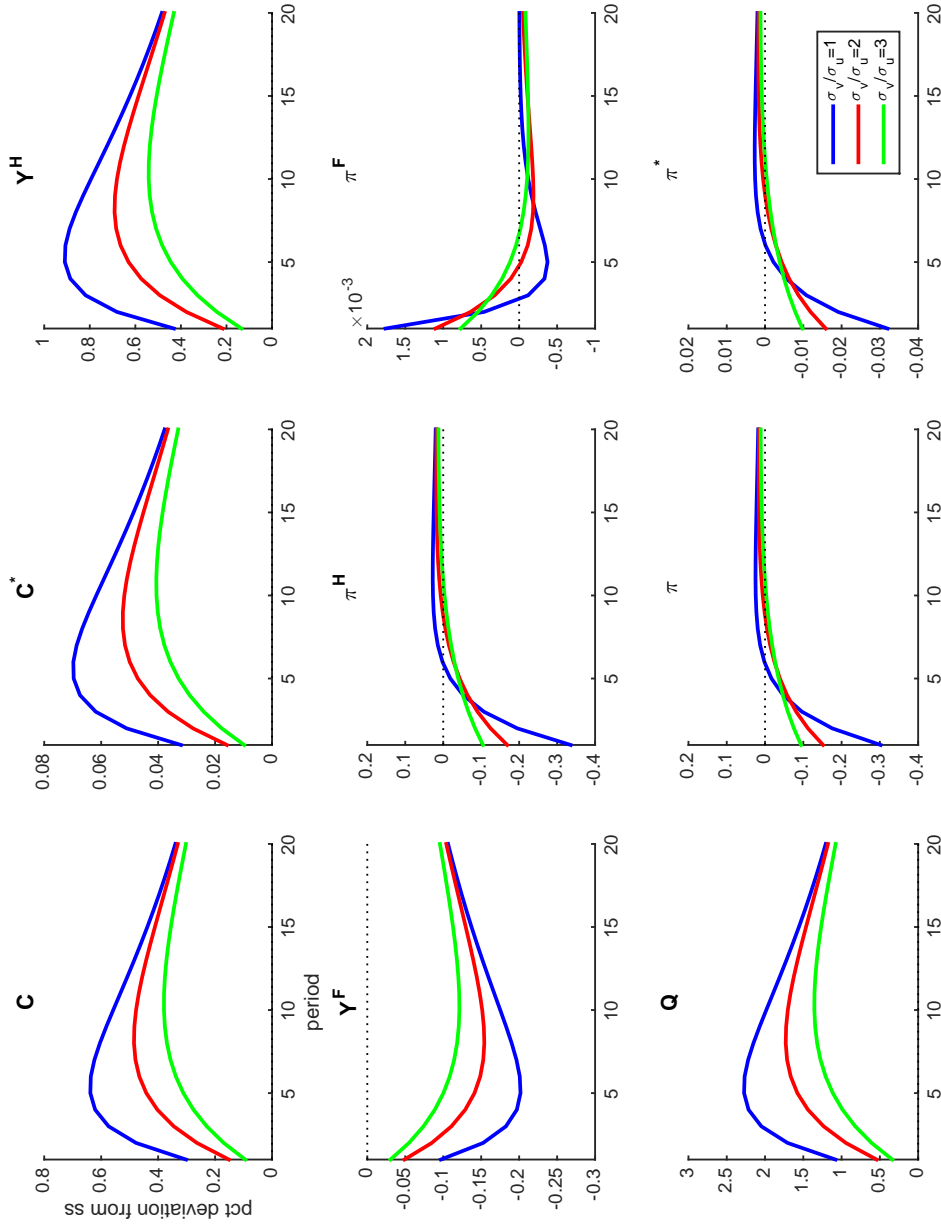
Notes: The figure depicts the impulse responses of key variables following a Home technology shock for different values of noise in the signals (σ_v) relative to the standard deviation of the shock (σ_u).

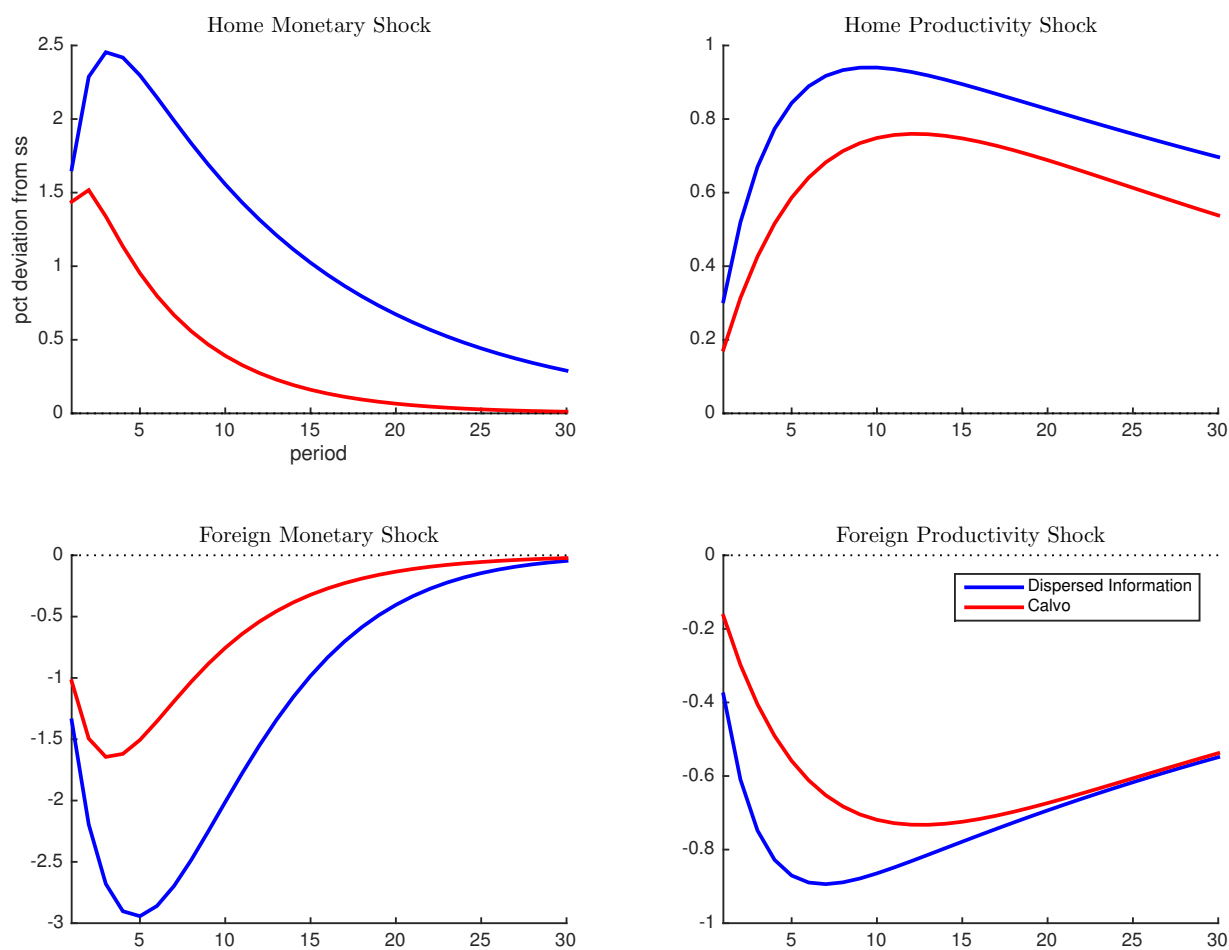Figure 6: Response of the Real Exchange Rate in the Data and in the Model
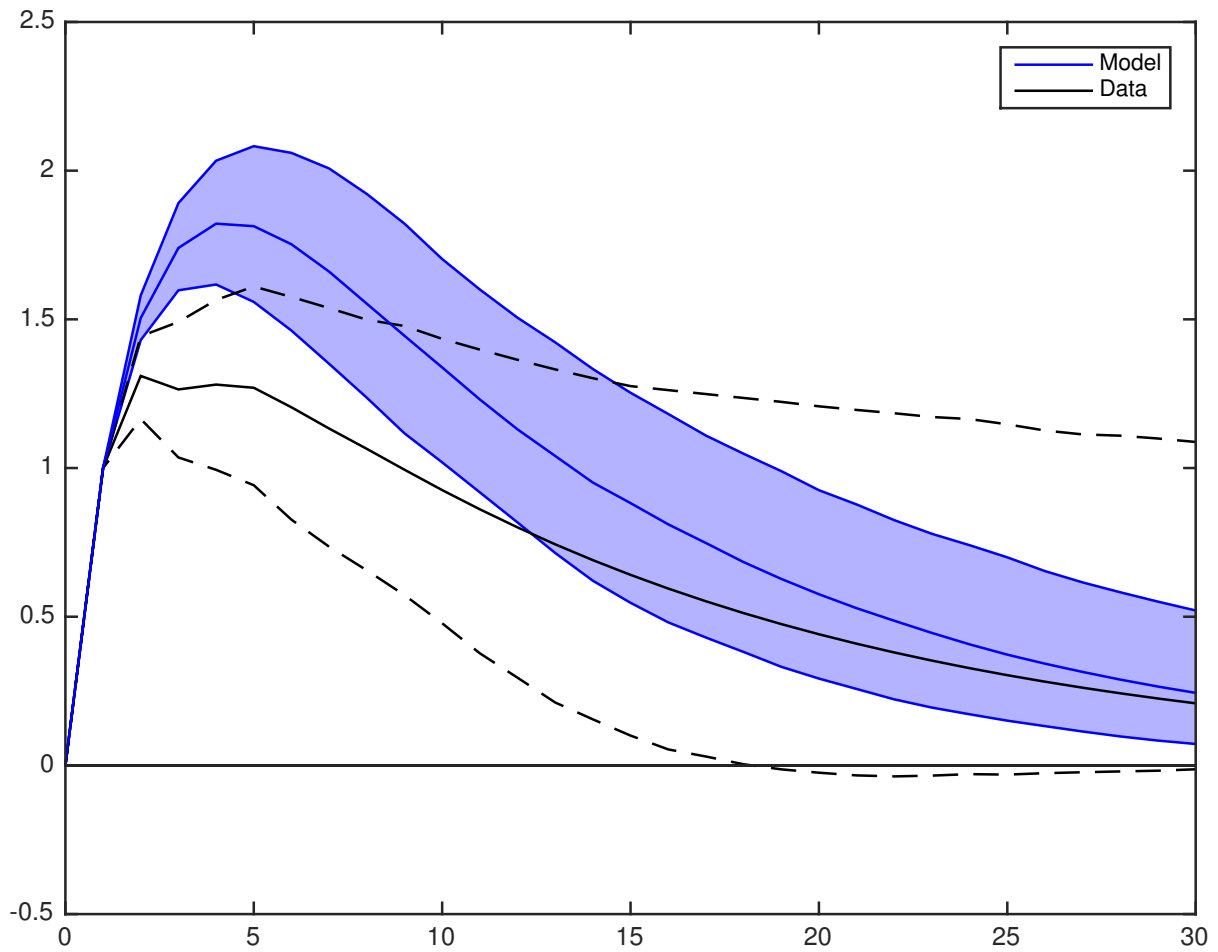
Notes: The black lines depict the median response and the associated 90% confidence band of the exchange rate from the data. The blue lines represent analogous objects from the simulated dispersed-information model, as explained in Section 6.4.

Figure 7: Response to a Monetary Shock in the VAR and in the Model
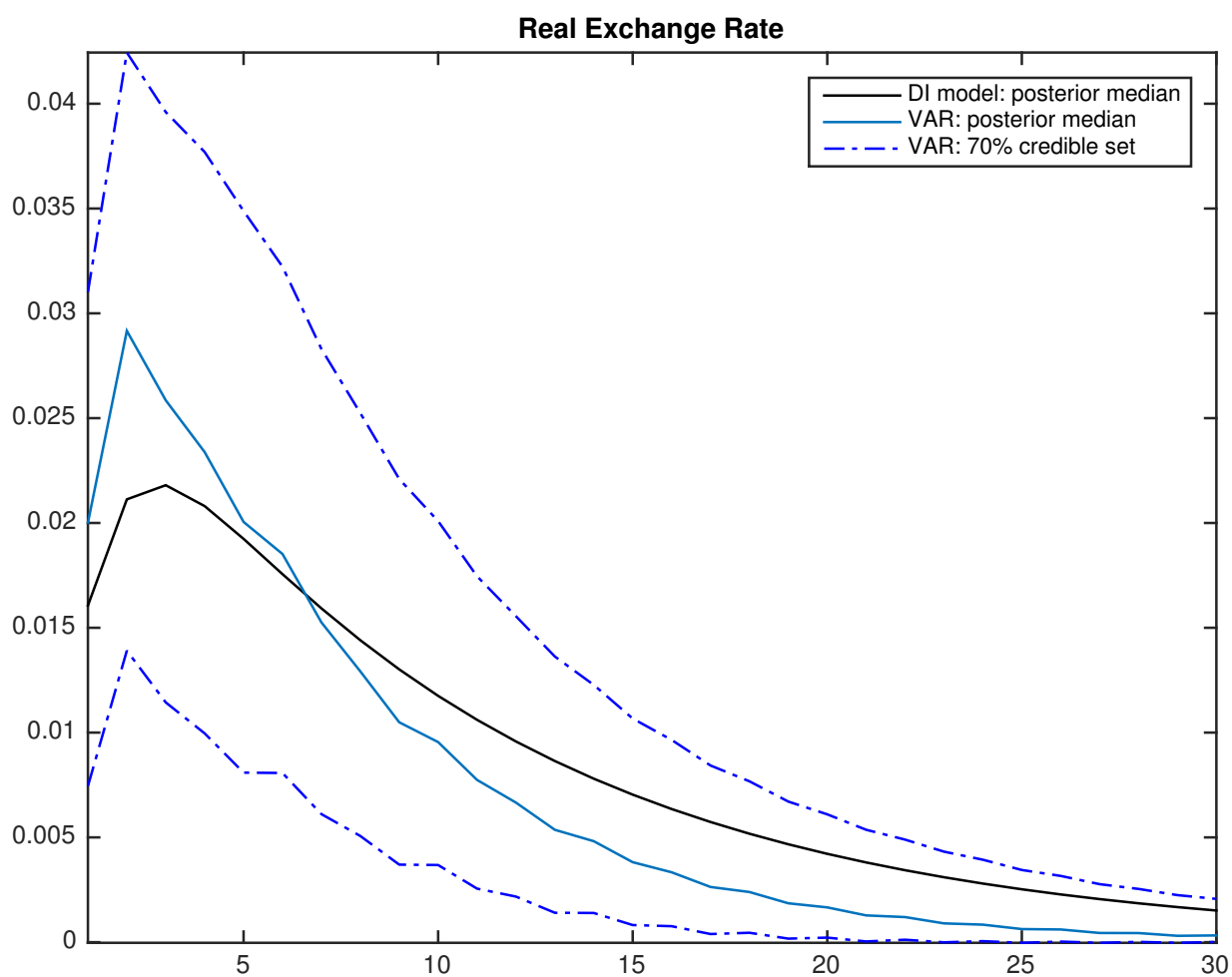
Real Exchange Rate

Notes: the black lines depict the median response of the exchange rate to a one standard deviation Home monetary shocks in the model. The blue lines represent the median response and the 70% credible set of the real exchange rate to a one standard deviation monetary shock in the VAR.

Figure 8: Impulse Responses of the Real Exchange Rate

Notes: The figure depicts the response of the real exchange rate to the four structural shocks. For all the panels, the shock sizes have been normalized to the median estimate of the standard deviation from the dispersed-information model.

Figure 9: Impulse Response Comparison with Endogenous Signals
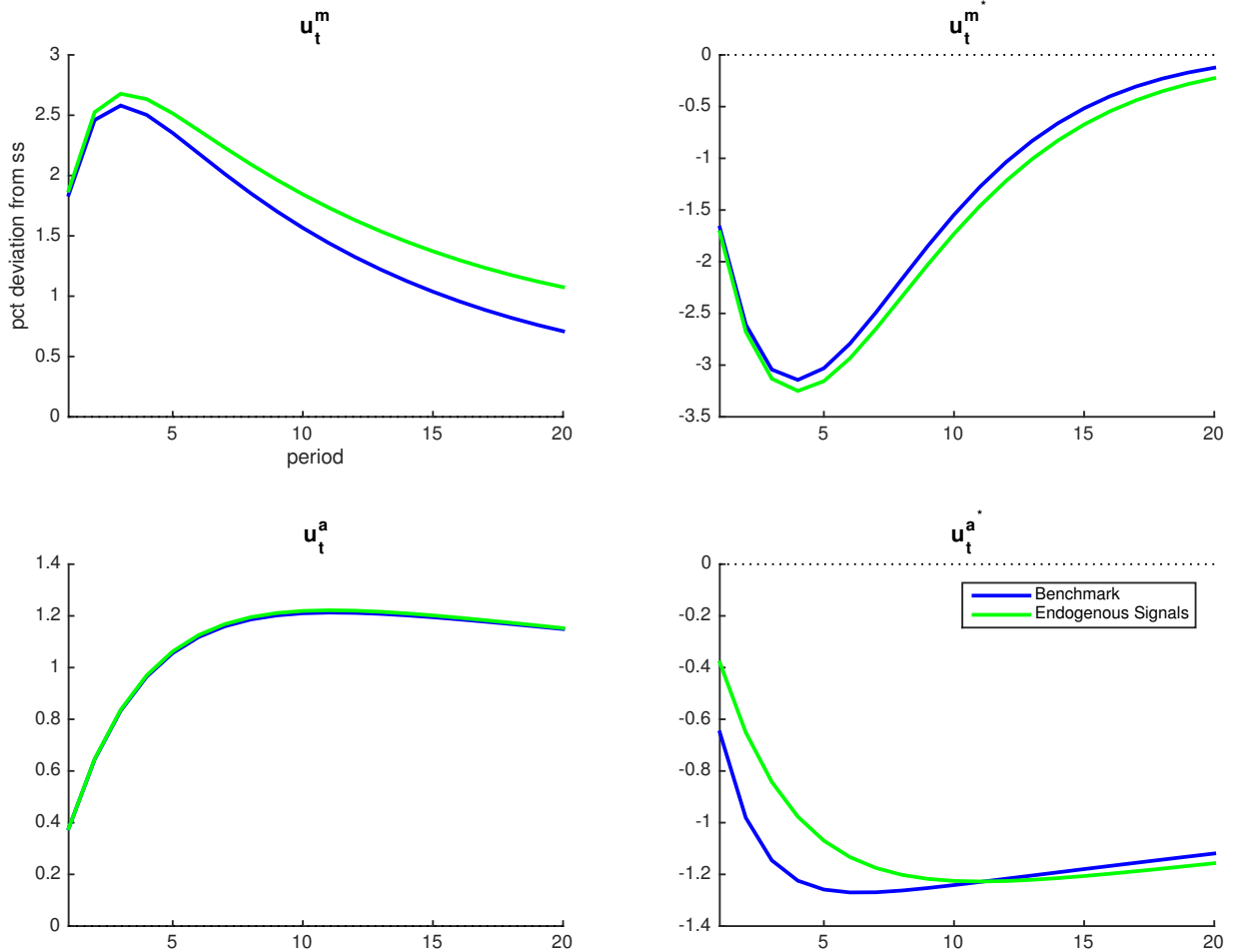
Notes: The figure depicts the response of the real exchange rate to the four structural shocks for the benchmark dispersed-information model and for the model with endogenous signals. The impulse responses are simulated using the posterior mode of the parameter estimates.

Appendices

Appendix A: Algebraic Results

Solution for p_{Ht} and p_{Ft}

Log linearizing the FOC, one obtains

$$p_t(h) = \mathbb{E}_{ht}(w_{ht} - a_t) \tag{37}$$

Add and subtract p_t inside the expectation

$$p_t(h) = \mathbb{E}_{ht}(w_{it} - p_t + p_t - a_t) \tag{38}$$

Now substitute $w_{ht} - p_t$ from the log-linear version of (6) to obtain

$$p_t(h) = \mathbb{E}_{ht}(\sigma c_t + \frac{1}{\psi} l_{it} + p_t - a_t) \tag{39}$$

Substitute the production function for l_{it}

$$p_t(h) = \mathbb{E}_{ht}(\sigma c_t + \frac{1}{\psi}(y_{it} - a_{it}) + p_t - a_t) \tag{40}$$

Now substitute the log-linearized demand for y_{it}

$$p_t(h) = \mathbb{E}_{ht}[\sigma c_t + p_t - (1 + \frac{1}{\psi}) a_t + \frac{1}{\psi}(-\gamma(p_t(h) - p_{Ht}) - \omega(p_{Ht} - p_t) + \alpha c_t + (1 - \alpha)(\omega q_t + c_t^*)]$$

Add and subtract p_{Ht} and rearrange to obtain

$$p_t(h) = \mathbb{E}_{ht}\left[-\left(\frac{1+\psi}{\gamma+\psi}\right) a_t + p_{Ht} - \left(\frac{\psi+\omega}{\gamma+\psi}\right)(p_{Ht} - p_t) + \alpha\left(\frac{\alpha+\psi\sigma}{\gamma+\psi}\right) c_t\right] +$$
$$\mathbb{E}_{ht}\left[(1-\alpha)\left(\frac{1}{\gamma+\psi}\right)(\omega q_t + c_t^*)\right]$$

Now recall that $p_{Ht} - p_t = -(1-\alpha)\tau_t$, so

$$p_t(h) = \mathbb{E}_{it}\left[-\left(\frac{1+\psi}{\gamma+\psi}\right) a_t + p_{Ht} + (1-\alpha)\left(\frac{\psi+\omega}{\gamma+\psi}\right)\tau_t + \alpha\left(\frac{\alpha+\psi\sigma}{\gamma+\psi}\right) c_t\right] +$$
$$\mathbb{E}_{ht}\left[(1-\alpha)\left(\frac{1}{\gamma+\psi}\right)(\omega q_t + c_t^*)\right]$$

Recall that $q_t = (2\alpha - 1)\tau_t$, hence

$$p_t(h) = \mathbb{E}_{ht}\left[p_{Ht} - (\gamma+\psi)^{-1}[(1+\psi) a_t + (1-\alpha)(\psi+\omega+\omega(2\alpha-1))\tau_t + (\alpha+\psi\sigma) c_t + (1-\alpha)c_t^*]\right]$$

$$p_t(h) = \mathbb{E}_{ht}\left\{p_{Ht} + (\gamma+\psi)^{-1}[(1-\alpha)(\psi+2\alpha\omega)\tau_t + (\alpha+\psi\sigma) c_t + (1-\alpha)c_t^* - (1+\psi) a_t]\right\} \tag{41}$$

A similar equation can be derived for $p_t(f)$. Rewrite this as

$$p_t(h) = \mathbb{E}_{it}\left\{p_{Ht} + (\gamma + \psi)^{-1}[(1-\alpha)(\psi + 2\alpha\omega)\tau_t + (1+\psi\sigma)c_t - (1-\alpha)(c_t - c_t^*) - (1+\psi)a_{it}]\right\}$$

using the fact that $c_t - c_t^* = (2\alpha - 1)\sigma^{-1}\tau_t$, I can write

$$p_t(h) = \mathbb{E}_{it}\left\{p_{Ht} + (\gamma + \psi)^{-1}[(1-\alpha)\left(\psi + 2\alpha\omega - (2\alpha - 1)\sigma^{-1}\right)\tau_t + (1+\psi\sigma)c_t - (1+\psi)a_{it}]\right\}$$

use the money process and the link between relative prices to write

$$\begin{aligned}p_t(h) = &\mathbb{E}_{ht}\left\{p_{Ht} + (\gamma + \psi)^{-1}[(1-\alpha)\left(\psi + 2\alpha\omega - (2\alpha - 1)\sigma^{-1}\right)\tau_t\right\} + \\ &\mathbb{E}_{ht}\left\{(1+\psi\sigma)(m_t - (1-\alpha)\tau_t - p_{Ht}) - (1+\psi)a_t]\right\}\end{aligned}$$

and finally

$$\begin{aligned}p_t(h) = &\mathbb{E}_{ht}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma + \psi}\right)p_{Ht}\right\} + \\ &\mathbb{E}_{ht}\left\{\left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha - 1)\sigma^{-1} - 1 - \psi\sigma)}{\gamma + \psi}\right]\tau_t + \left(\frac{1+\psi\sigma}{\gamma + \psi}\right)m_t - \left(\frac{1+\psi}{\gamma + \psi}\right)a_t\right\}\end{aligned}$$

Similarly, for the foreign country I have

$$\begin{aligned}p_t^*(f) = &\mathbb{E}_{ft}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma + \psi}\right)p_{Ft}^*\right\} - \\ &\mathbb{E}_{ft}\left\{\left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha - 1)\sigma^{-1} - 1 - \psi\sigma)}{\gamma + \psi}\right]\tau_t + \left(\frac{1+\psi\sigma}{\gamma + \psi}\right)m_t^* - \left(\frac{1+\psi}{\gamma + \psi}\right)a_t^*\right\}\end{aligned}$$

Notice that the last two equations collapse to (17) and (18) when $\sigma = 1$. By averaging these two equation over firms one obtains

$$\begin{aligned}p_{Ht} = &\bar{\mathbb{E}}_t^{(1)}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma + \psi}\right)p_{Ht}\right\} + \\ &\bar{\mathbb{E}}_t^{(1)}\left\{\left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha - 1)\sigma^{-1} - 1 - \psi\sigma)}{\gamma + \psi}\right]\tau_t + \left(\frac{1+\psi\sigma}{\gamma + \psi}\right)m_t - \left(\frac{1+\psi}{\gamma + \psi}\right)a_t\right\}\end{aligned}$$

and

$$\begin{aligned}p_{Ft}^* = &\bar{\mathbb{E}}_t^{(1)}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma + \psi}\right)p_{Ft}^*\right\} - \\ &\bar{\mathbb{E}}_t^{(1)}\left\{\left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha - 1)\sigma^{-1} - 1 - \psi\sigma)}{\gamma + \psi}\right]\tau_t + \left(\frac{1+\psi\sigma}{\gamma + \psi}\right)m_t^* - \left(\frac{1+\psi}{\gamma + \psi}\right)a_t^*\right\}\end{aligned}$$

When taking the sum of these two equations the terms of trade cancel out

$$p_{Ht} + p_{Ft}^* = \bar{\mathbb{E}}_t^{(1)}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma + \psi}\right)(p_{Ht} + p_{Ft}^*) + \left(\frac{1+\psi\sigma}{\gamma + \psi}\right)m_t^W - \left(\frac{1+\psi}{\gamma + \psi}\right)a_t^W\right\}$$

Recursively substituting $p_{Ht} + p^*_{Ft}$ on the right-hand side yields

$$p_{Ht} + p^*_{Ft} = \tilde{\xi} \sum_{k=1}^{\infty} (1 - \tilde{\xi})^{k-1} E_t^{(k)} \left(m_t^W - \frac{1+\psi}{1+\psi\sigma} a_t^W \right) \tag{42}$$

where

$$\tilde{\xi} = \frac{1+\psi\sigma}{\gamma+\psi} \tag{43}$$

Taking instead the difference of the average prices equations yields

$$p_{Ht} - p^*_{Ft} = \mathbb{E}_t^{(1)} \left\{ \left(1 - \frac{1+\psi\sigma}{\gamma+\psi} \right) (p_{Ht} - p^*_{Ft}) + \right\}$$

$$\mathbb{E}_t^{(1)} \left\{ 2 \left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha-1)\sigma^{-1} - 1 - \psi\sigma)}{\gamma+\psi} \right] \tau_t + \left(\frac{1+\psi\sigma}{\gamma+\psi} \right) m_t^D - \left(\frac{1+\psi}{\gamma+\psi} \right) a_{it}^D \right\}$$

Now I need to solve for τ_t in terms of $p_{Ht} - p^*_{Ft}$ and m_t^D

$$\varepsilon_t = q_t + p_t - p_t^* = (2\alpha - 1)\tau_t + m_t^D - c_t^D \tag{44}$$

$$= (2\alpha - 1)\tau_t - (2\alpha - 1)\sigma^{-1}\tau_t + m_t^D = (2\alpha - 1)(1 - \sigma^{-1})\tau_t + m_t^D \tag{45}$$

So

$$\tau_t = p^*_{Ft} - p_{Ht} + \varepsilon_t = p^*_{Ft} - p_{Ht} + (2\alpha - 1)(1 - \sigma^{-1})\tau_t + m_t^D \tag{46}$$

$$(1 - (2\alpha - 1)(1 - \sigma^{-1}))\tau_t = -(p_{Ht} - p^*_{Ft}) + m_t^D \tag{47}$$

or

$$\tau_t = \frac{1}{(1 - (2\alpha - 1)(1 - \sigma^{-1}))}(-(p_{Ht} - p^*_{Ft}) + m_t^D) \tag{48}$$

Substituting this above

$$p_t(h) - p_t(f)^* = \mathbb{E}_{it} \left\{ \left(1 - \frac{1+\psi\sigma}{\gamma+\psi} - 2 \left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha-1)\sigma^{-1} - 1 - \psi\sigma)}{(\gamma+\psi)(1 - (2\alpha-1)(1-\sigma^{-1}))} \right] \right) (p_{Ht} - p^*_{Ft}) \right\} +$$

$$\mathbb{E}_{it} \left\{ \left(\frac{1+\psi\sigma}{\gamma+\psi} + 2 \left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha-1)\sigma^{-1} - 1 - \psi\sigma)}{(\gamma+\psi)(1 - (2\alpha-1)(1-\sigma^{-1}))} \right] \right) m_t^D - \left(\frac{1+\psi}{\gamma+\psi} \right) a_t^D \right\}$$

Hence the solution for the price difference can be expressed as

$$p_{Ht} - p^*_{Ft} = \tilde{\varphi} \sum_{k=1}^{\infty} (1 - \tilde{\varphi})^{k-1} E_t^{(k)} \left(m_t^D - \frac{1+\psi}{(\gamma+\psi)\tilde{\varphi}} a_t^D \right) \tag{49}$$

where

$$\tilde{\varphi} = \left(\frac{1+\psi\sigma}{\gamma+\psi} + 2 \left[\frac{(1-\alpha)(\psi + 2\alpha\omega - (2\alpha-1)\sigma^{-1} - 1 - \psi\sigma)}{(\gamma+\psi)(1 - (2\alpha-1)(1-\sigma^{-1}))} \right] \right) \tag{50}$$

It can be easily verified that if $\sigma = 1$, then $\tilde{\xi} = \xi$ and $\tilde{\varphi} = \varphi$ and one goes back to the equations (22) and (24) in the main text. The solution for the real exchange rate follows by using the relationship $q_t = (2\alpha - 1)t_t$.

Proof of Proposition 2

The random-walk hypothesis implies that $m_t^D = m_{t-1}^D + u_t$, where $u_t \equiv u_t^m - u_t^{m*}$. The proof follows the guess-and-verify approach used by Woodford (2002). The guess is that:

$$p_{Ht} - p_{Ft}^* = \nu(p_{Ht} - p_{Ft}^*) + (1 - \nu)m_t^D \tag{51}$$

Denote with i subscript a generic firm in either Home or Foreign. Equation (15) shows that firms in each country receive two signals about the money supplies: one for Home ($z_{i,t}^m$) and one for Foreign ($z_{i,t}^{m*}$). Given the properties of the signals, it is as if firm i received one signal about the difference in money supplies: $s_{i,t} = m_t^D + \eta_{i,t}$ with $\eta_{i,t} = v_{i,t}^m - v_{i,t}^{m*}$. Writing compactly the process for the money supplies, the guess for the price difference, and this signal in a state-space representation, we have:

$$\begin{bmatrix} m_t^D \\ p_{Ht} - p_{Ft}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1-\nu & \nu \end{bmatrix} \begin{bmatrix} m_{t-1}^D \\ p_{H,t-1} - p_{F,t-1}^* \end{bmatrix} + \begin{bmatrix} 1 \\ 1-\nu \end{bmatrix} u_t \implies x_t = Mx_{t-1} + du_t \tag{52}$$

$$s_{i,t} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} m_t^D \\ p_{Ht} - p_{Ft}^* \end{bmatrix} + \eta_{i,t} \implies s_{i,t} = ex_t + \eta_{i,t} \tag{53}$$

Here I have defined the new vector x_t and matrices M, d and e to write the problem as a state-space system. The Kalman filter implies:

$$\mathbb{E}_{it}(x_t) = \mathbb{E}_{i,t-1}(x_{t-1}) + \kappa[s_{i,t} - eM\mathbb{E}_{i,t-1}(x_{t-1})] \tag{54}$$

with $\kappa = [\kappa_1, \kappa_2]'$ being a 2×1 vector of Kalman gains. Given the symmetry of signals across countries, integrating the last expression over the continuum of Home or Foreign firms yields:

$$\bar{\mathbb{E}}_t^{(1)}(x_t) = \kappa eMx_{t-1} + (M - \kappa eM)\bar{\mathbb{E}}_{t-1}^{(1)}(x_{t-1}) + \kappa edu_t \tag{55}$$

Now note that equation (23), absent technology shocks, may be written as:

$$p_{Ht} - p_{Ft}^* = (1-\varphi)\bar{\mathbb{E}}_t^{(1)}(p_{Ht} - p_{Ft}^*) + \varphi\bar{\mathbb{E}}_t^{(1)}(m_t^D) \tag{56}$$

On the right-hand side of this expression, the average expectations of m_t^D and $p_{Ht} - p_{Ft}^*$ can be replaced using equation (55) after performing the matrix algebra. This yields:

$$p_{Ht} - p_{Ft}^* = \nu(p_{H,t-1} - p_{F,t-1}^*) + [\varphi\kappa_1 + (1-\varphi)\kappa_2]m_t^D + [(1-\nu) - \varphi\kappa_1 - (1-\varphi)\kappa_2]\bar{\mathbb{E}}_{t-1}^{(1)}(m_{t-1}^D)$$

This verifies the original guess in equation (51) and shows that $(1-\nu) = \varphi\kappa_1 + (1-\varphi)\kappa_2$. Now recall that with log utility the real exchange rate is given by $q_t = (2\alpha - 1)(p_{Ft}^* + \varepsilon_t - p_{Ht}) = (2\alpha - 1)(m_t^D - p_{Ht} + p_{Ft}^*)$. Using the solution for the price difference yields:

$$q_t = (2\alpha - 1)[m_{t-1} + u_t - (1-\nu)m_{t-1} - \nu(p_{H,t-1} - p_{F,t-1}^*) - (1-\nu)u_t]$$
$$= \nu q_{t-1} + (2\alpha - 1)\nu u_t$$

The expressions for the autocorrelation and the standard deviation of the real exchange rate immediately follow. ∎

Appendix B: Stationarizing the Model

In rewriting the model in a stationary representation, I can exploit the following facts:

- The level of the money supply is nonstationary, but money growth is stationary.

- Price levels and, more generally, higher-order beliefs about money supplies are non-stationary but deviations of these beliefs from the true levels of the money supplies are stationary.

The exogenous state variables are $X_t = [m_t, m_{t-1}, m_t^*, m_{t-1}^*, a_t, a_t^*]'$. The state-transition equation is given by:

$$\bar{X}_t = \bar{B}\bar{X}_{t-1} + \bar{b}u_t \tag{57}$$

where

$$\bar{X}_t = \begin{bmatrix} X_t \\ \hline F_{\xi,t} \\ \hline F_{\varphi,t} \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B_{6\times6} & 0 & 0 \\ \Gamma_{6\times6}^{\xi,x} & \Gamma_{6\times6}^{\xi,\xi} & 0 \\ \Gamma_{6\times6}^{\varphi,x} & 0 & \Gamma_{6\times6}^{\varphi,\varphi} \end{bmatrix} \quad \bar{b} = \begin{bmatrix} b_{6\times4} \\ \Gamma_{6\times4}^{\xi,u} \\ \Gamma_{6\times4}^{\varphi,u} \end{bmatrix} \quad u_t = \begin{bmatrix} u_t^m \\ u_t^{m^*} \\ u_t^a \\ u_t^{a^*} \end{bmatrix}$$

where $F_{\xi,t} \equiv \xi \sum_{k=1}^{\infty}(1-\xi)X_t^{(k)}$ and $F_{\varphi,t} \equiv \varphi \sum_{k=1}^{\infty}(1-\varphi)X_t^{(k)}$ are the weighted averages of higher-order beliefs that matter for the solution of the model. The matrices B and b are given exogenously, and the matrices Γ can be found as the solution to the fixed-point problem. Now define for any exogenous variable, x_t, the deviation of the variable itself from its weighted average of HOEs: $x_t^{-\xi} \equiv x_t - \xi \sum_{k=1}^{\infty}(1-\xi)x_t^{(k)}$ and the similar object for φ, so that in vectors this is $X_t^{-\xi} = X_t - \xi \sum_{k=1}^{\infty}(1-\xi)X_t^{(k)}$. Because the weighted average of HOEs converges in the long run to the respective variables, the dynamics of $X_t^{-\xi}$ and $X_t^{-\varphi}$ will be stationary. Furthermore, notice that the Kalman filter iteration implies that $\Gamma^{\xi,x} + \Gamma^{\xi,\xi} = \Gamma^{\varphi,x} + \Gamma^{\varphi,\varphi} = B$. Using this fact and equation (57), one can show that

$$X_t^{-\xi} = \Gamma^{\xi,\xi}X_{t-1}^{-\xi} + [b - \Gamma^{\xi,u}]u_t \tag{58}$$
$$X_t^{-\varphi} = \Gamma^{\varphi,\varphi}X_{t-1}^{-\varphi} + [b - \Gamma^{\varphi,u}]u_t \tag{59}$$

Finally, we stationarize the exogenous part of the system by rewriting it in terms of money growth rates

$$\underbrace{\begin{bmatrix} \Delta m_t \\ \Delta m_t^* \\ a_t \\ a_t^* \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} \rho_m & 0 & 0 & 0 \\ 0 & \rho_{m^*} & 0 & 0 \\ 0 & 0 & \rho_a & 0 \\ 0 & 0 & 0 & \rho_{a^*} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \Delta m_{t-1} \\ \Delta m_{t-1}^* \\ a_{t-1} \\ a_{t-1}^* \end{bmatrix}}_{Y_{t-1}} + \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{a} \begin{bmatrix} u_t^m \\ u_t^{m^*} \\ u_t^a \\ u_t^{a^*} \end{bmatrix} \tag{60}$$

So finally, our stationary state can be written as

$$
\underbrace{\begin{bmatrix} Y_t \\ X_t^{-\xi} \\ X_t^{-\varphi} \end{bmatrix}}_{\bar{Y}_t} = \underbrace{\begin{bmatrix} A & 0 & 0 \\ 0 & \Gamma^{\xi,\xi} & 0 \\ 0 & 0 & \Gamma^{\varphi,\varphi} \end{bmatrix}}_{\Phi} \begin{bmatrix} Y_{t-1} \\ X_{t-1}^{-\xi} \\ X_{t-1}^{-\varphi} \end{bmatrix} + \underbrace{\begin{bmatrix} a \\ b - \Gamma^{\xi,u} \\ b - \Gamma^{\varphi,u} \end{bmatrix}}_{C} u_t
\tag{61}
$$

Define the stationary variables: $mp_t \equiv m_t - p_{Ht}$ and $mp_t^* = m_t^* - p_{Ft}^*$. The solution for these variables can be written as

$$
mp_t = e_h \bar{Y}_t \tag{62}
$$

$$
mp_t^* = e_f \bar{Y}_t \tag{63}
$$

The other model equations can be written as

$$
\tau_t = [1 - (2\alpha - 1)(1 - \sigma^{-1})]^{-}1(mp_t - mp_t^*) \tag{64}
$$

$$
c_t = mp_t - (1 - \alpha)\tau_t \tag{65}
$$

$$
c_t^* = mp_t^* + (1 - \alpha)\tau_t \tag{66}
$$

$$
y_{Ht} = \omega(1 - \alpha)\tau_t + \alpha c_t + (1 - \alpha)(\omega q_t + c_t^*) \tag{67}
$$

$$
yf = -\omega(1 - \alpha)\tau_t + (1 - \alpha)(c_t - \omega q_t) + \alpha c_t^* \tag{68}
$$

$$
q_t = \sigma(c_t - c_t^*) \tag{69}
$$

$$
\pi_t^H = \Delta m_t - mp_t + mp_{t-1} \tag{70}
$$

$$
\pi_t^F = \Delta m_t^* - mp_t^* + mp_{t-1}^* \tag{71}
$$

$$
\pi = \alpha \pi_t^H + (1 - \alpha)(\Delta \varepsilon_t + \pi_t^F) \tag{72}
$$

$$
\pi^* = (1 - \alpha)(\pi_t^H - \Delta \varepsilon_t) + \alpha \pi_t^F \tag{73}
$$

$$
\Delta \varepsilon_t = \Delta m_t - \Delta m_t^* + (2\alpha - 1)(1 - \sigma^{-1})(\tau_t - \tau_{t-1}) \tag{74}
$$

$$
\Delta m_t = \rho_m \Delta m_{t-1} + u_t^m \tag{75}
$$

$$
\Delta m_t = \rho_m^* \Delta m_{t-1} + u_t^{m^*} \tag{76}
$$

$$
a_t = \rho_a a_{t-1} + u_t^a \tag{77}
$$

$$
a_t^* = \rho_a^* a_{t-1}^* + u_t^{a^*} \tag{78}
$$

These equations can be written compactly as

$$
Z_t = \Xi \bar{Y}_t \tag{79}
$$

Equations (61) and (79) form the stationary state-space representation of the model.

Simulating Firms' Prices

Consider the nonstationary system as defined above

$$
\bar{X}_t = \bar{B} \bar{X}_{t-1} + \bar{b} u_t \tag{80}
$$

Recall that the prices solutions are equations (42) and (49), which can be solved to get the individual prices as a function of the state

$$p_{Ht} = \frac{(m_t^\xi + m_t^{*\xi} + m_t^\varphi - m_t^{*\varphi}) - (\chi_w(a_t^\xi + a_t^{*\xi}) + \chi_d(a_t^\varphi - a_t^{*\varphi}))}{2} \tag{81}$$

$$p_{Ft}^* = \frac{[m_t^\xi + m_t^{*\xi} - (m_t^\varphi - m_t^{*\varphi})] - [\chi_w(a_t^\xi + a_t^{*\xi}) - \chi_d(a_t^\varphi - a_t^{*\varphi})]}{2} \tag{82}$$

where I defined $\chi_w = \frac{1+\psi}{(\gamma+\psi)\bar{\xi}}$ and $\chi_d = \frac{1+\psi}{(\gamma+\psi)\bar{\varphi}}$ and

$$\tau_t = \frac{1}{(1 - (2\alpha - 1)(1 - \sigma^{-1}))}[m_t - m_t^* - (m_t^\varphi - m_t^{*\varphi}) + \chi_d(a_t^\varphi - a_t^{*\varphi})] \tag{83}$$

Define two vectors v_h and v_f and v_τ such that $p_{Ht} = v_1\bar{X}_t$ and $\tau_t = v_2\bar{X}_t$. So

$$v_h = 1/2[0,0,0,0,0,0,1,0,1,0-\chi_w,-\chi_w,1,0,-1,0,-\chi_d,+\chi_d] \tag{84}$$

$$v_\tau = \frac{1}{(1-(2\alpha-1)(1-\sigma^{-1}))}[1,0,-1,0,0,0,0,0,0,0,0,0,-1,0,1,0,\chi_d,-\chi_d] \tag{85}$$

$$v_f = 1/2[0,0,0,0,0,0,1,0,1,0-\chi_w,-\chi_w,-1,0,1,0,\chi_d,-\chi_d] \tag{86}$$

Recall that the home price was

$$p_t(h) = \mathbb{E}_{it}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)p_{Ht}\right\} + \tag{87}$$

$$\mathbb{E}_{it}\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]\tau_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right)m_t - \left(\frac{1+\psi}{\gamma+\psi}\right)a_{it}\right\} \tag{88}$$

or

$$p_t(h) = \mathbb{E}_{it}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)v_1\bar{X}_t\right\} + \tag{89}$$

$$\mathbb{E}_{it}\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]v_2\bar{X}_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right)e_1\bar{X}_t - \left(\frac{1+\psi}{\gamma+\psi}\right)e_5\bar{X}_t\right\} \tag{90}$$

For foreign prices

$$p_t^*(f) = \mathbb{E}_{it}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)p_{Ft}^*\right\} - $$

$$\mathbb{E}_{it}\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]\tau_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right)m_t^* - \left(\frac{1+\psi}{\gamma+\psi}\right)a_{it}^*\right\}$$

or

$$p_t^*(f) = \mathbb{E}_{it}\left\{\left(1 - \frac{1+\psi\sigma}{\gamma+\psi}\right)v_f\bar{X}_t\right\} - $$

$$\mathbb{E}_{it}\left\{\left[\frac{(1-\alpha)(\psi+2\alpha\omega-(2\alpha-1)\sigma^{-1}-1-\psi\sigma)}{\gamma+\psi}\right]v_\tau\bar{X}_t + \left(\frac{1+\psi\sigma}{\gamma+\psi}\right)e_3\bar{X}_t - \left(\frac{1+\psi}{\gamma+\psi}\right)e_6\bar{X}_t\right\}$$

So you can write it as

$$p_t(h) = P_h \bar{X}_{t|t}^{(1)}(h) \qquad\qquad p_t^*(f) = P_f \bar{X}_{t|t}^{(1)}(f) \qquad\qquad (91)$$

where P are the appropriate matrices. The state space for the firm h is given by

$$\bar{X}_t = \bar{B}\bar{X}_{t-1} + \bar{b}u_t \qquad\qquad (92)$$

$$z_{it} = D\bar{X}_t + v_{it} \qquad\qquad (93)$$

where

$$D = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0_{1\times12} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0_{1\times12} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0_{1\times12} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0_{1\times12} \end{pmatrix} \qquad\qquad (94)$$

Beliefs about the state evolve according to

$$\bar{X}_{t|t}^{(1)}(i) = \bar{B}\bar{X}_{t-1|t-1}^{(1)}(i) + K[z_{it} - D\bar{B}\bar{X}_{t-1|t-1}^{(1)}(i)] \qquad\qquad (95)$$

where $K = \Sigma D'(D\Sigma D' + \Sigma_v)^{-1}$ and

$$\Sigma = \bar{B}\Sigma\bar{B} - \bar{B}\Sigma D'(D\Sigma D' + \Sigma_v)^{-1}D\Sigma\bar{B}' + \bar{b}\Sigma_u\bar{b}' \qquad\qquad (96)$$

These are the same matrices K, Σ that one finds with the model solution.

Appendix C: Calculating Profit Losses

Profit Losses in the Dispersed-information Model

Modeling imperfect information with noisy signals is a simple way of formalizing the idea that a cost is associated with gathering and processing the information that is relevant for firms' optimal pricing decisions. In the context of the present model, that information consists of aggregate economic conditions and the prices set by domestic and foreign competitors. One way to evaluate the plausibility of the estimated signal-to-noise ratios is to consider the individual profit loss that a firm incurs when they observe signals only with finite precision. Indeed, one may argue that if paying limited attention to macroeconomic conditions leads to high profit losses, a firm should pay more attention to those conditions. On the other hand, if profit losses are small, then a firm's cost of acquiring more information would outweigh the gain in profits that derive from obtaining more information.

I here explore this reasoning in the context of my model estimates. Recall that the price set by firm h in the home country in the model with dispersed information is given by equation (19). Instead, the price that a firm would set under full information, expressed in log-deviations

from the steady state, is

$$p_t^\diamond(h) = (1-\xi)p_{Ht} + \frac{2\alpha(1-\alpha)(\omega-1)}{(\gamma+\psi)}t_t + \xi(m_t - a_t) \tag{97}$$

Firm's h expected per-period profit loss due to imperfect information is then given by

$$\mathbb{E}\left[\Pi_{h,t}(P_t^\diamond(h), \cdot) - \Pi_{h,t}(P_t(h), \cdot)\right]$$

The Appendix shows that after taking a log-quadratic approximation to the profit function, this expression simplifies to

$$-\frac{\Pi_{11}}{2}\mathbb{E}\left[(p_t^\diamond(h) - p_t(h))^2\right]$$

where Π_{11} is the curvature of the profit function with respect to the firm's own price. As shown in the Appendix, the expression $(p_t^\diamond(h) - p_t(h))^2$ can be computed analytically once the model is solved, using the matrices from the firm's signal-extraction problem. Using the posterior mode, I find that the expected profit loss from imperfect tracking of economic conditions is 0.5% of steady-state revenue for a US firm and 0.8% for a European firm.[22] These numbers suggest that the profit losses from imperfect information are small and plausible. Empirical evidence on the cost of price adjustment indicates that the cost of price adjustment in US industrial manufacturing is 1.22% of a firm's steady-state revenues (Zbaracki et al., 2004). The figure implied by the estimated model is well below the empirical value, suggesting that it is rational for firms to settle on an equilibrium with imperfect information, as the cost of being fully informed would outweigh the profit gain.

Profit Losses In the Calvo Model

The presence of nominal rigidities in the Calvo model implies that generally, firms do not set their prices to the level that would maximize their profits. Firms are thus subject to profit losses that can be compared to the losses in the dispersed-information model.

The profit-maximizing price, $P_t^\diamond(h)$, or the price that a firm would set under flexible prices, taking as given the level of demand and the level of aggregate prices, is given in equation (97), because the structure of the economy is the same as in the dispersed information model. Instead, the price that a firm in the Home country sets if subject to the Calvo friction, $P_t^C(h)$, is the optimal reset price if it has a chance to update its price, or its old price otherwise. That is, $P_t^C(h) = (1-\theta)P_t^R(h) + \theta P_{t-1}^C(h)$. The optimal reset price, in log-linear terms, is given by

$$p_t^R(h) - p_{H,t} = (1-\theta\beta)mc_t(h) + \theta\beta E_t\{(p_{t+1}^R(h) - p_{H,t+1}) + \pi_{t+1}^H\} \tag{98}$$

where $mc_t(h) = \left[\frac{\sigma\psi+1}{\gamma+\psi}y_{H,t} - \frac{2(1-\alpha)\alpha\psi(\sigma\omega-1)}{\gamma+\psi}\tau_t - \frac{1+\psi}{\gamma+\psi}a_t\right]$. In this case there is no closed-form solution for the expected profit-loss expression, but the model can be simulated to compute the expectation.

---

[22]Virtually all the expected profit loss comes from the imperfect tracking of monetary shocks. Specifically, the expected profit loss due to imperfect tracking of monetary shocks for a US firm is 0.33% of steady-state revenues. The profit loss due to imperfect tracking of technology shocks is only 0.006% of steady-state revenues.

To make the profit losses comparable across models, I use the following calibration. For the parameters that are common across models, I calibrate the Calvo model using the median estimates from the dispersed-information model. This set of parameters includes the volatility and persistence of shocks. This choice keeps the models comparable, as the profit losses, calculated with a quadratic approximation, are affected by the size of the shocks. As evident from the Phillips Curves equations, the sticky-price model additionally requires to calibrate the discount factor, β, and for the probability of non-price adjustment, θ. I set β to 0.99, the standard value in the literature. For θ, I search for the value that allows me to match the impulse response of the real exchange rate from the two models following a home monetary shock. I find that value to be $\theta = 0.86$, which implies a median price duration of 7 quarters. Empirical estimates of the median price duration range between 4 months and 8-10 months (Bils and Klenow, 2004; Nakamura and Steinsson, 2008). In this sense, the sticky-price model requires "too much price stickiness" to explain the real exchange rate persistence, even in the presence of strategic complementarities that flatten the Phillips Curve for a given degree of nominal rigidities. This confirms the intuition of Chari, Kehoe, and McGrattan (2002) mentioned in the introduction.

Using this calibration, I find that the Calvo friction delivers an expected per-quarter profit loss of 5.11% and 7.77% of steady-state revenue for a US firm and European firm, respectively. These losses are one order of magnitude larger than in the dispersed-information model and quite substantial. In particular, they are greater than the estimated cost of price adjustment in Zbaracki et al. (2004).

Why are the differences so large? Recall the expression for the profit-maximizing price in (97). This equation makes clear the role of strategic complementarities. The stronger the strategic complementarities, the larger $(1 - \xi)$, and the more the optimal price of a particular firm depends on the aggregate price level. In the dispersed-information model, large strategic complementarities imply that firms place large weights on higher-order beliefs relative to lower-order beliefs. Because higher-order beliefs adjust more sluggishly, all prices in equilibrium adjust sluggishly and they tend to be close together. At the same time, high strategic complementarity implies that the profit-maximizing price is also close to the average price in the economy. As a result, the difference between a particular firm's equilibrium price and its profit-maximizing price tends to be small, implying small profit losses.

In the Calvo model, while strategic complementarity still requires the profit-maximizing price to be close to the average price, an individual firm's price may be arbitrarily far away from the average if the firm did not have the chance to reset the price for a long time. Thus, expected profit losses increase rapidly with the probability of non-price adjustment. In this case, with the value of θ required to match the persistence in the real exchange rate, the losses are substantial.

Appendix D: Solving the Model with Endogenous Signals

This algorithm is an adaptation of Lorenzoni (2009)'s solution method.

Law of Motion For The State

Define the vectors $z_t = [m_t, m_t, a_t, a_t^*, p_{Ht}, p_{Ft}^*]'$ and $\mathbf{z_t} = [z_t, z_{t-1}, z_{t-2}, ...]$. We are looking for a linear equilibrium of the form:

$$\mathbf{z_t} = \mathbf{A}\mathbf{z_{t-1}} + \mathbf{B}\mathbf{u_t} \tag{99}$$

where $\mathbf{u_t} = [e_t^m, e_t^{m^*}, e_t^a, e_t^{a^*}]$. The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{bmatrix} A_m \\ A_{m^*} \\ A_a \\ A_{a^*} \\ A_{p_H} \\ A_{p_F^*} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} B_m \\ B_{m^*} \\ B_a \\ B_{a^*} \\ B_{p_H} \\ B_{p_F^*} \end{bmatrix}$$

where $A_m, A_{m^*}, A_a, A_{a^*}, B_m, B_{m^*}, B_a, B_{a^*}$ are known exogenous vectors and $A_{p_H}, A_{p_F^*}, B_{p_H}, B_{p_F^*}$ are to be determined. The pricing equations can be written as

$$p_{Ht} = \Psi_H E_t[\mathbf{z_t}] \tag{100}$$

$$p_{Ft} = \Psi_F E_t[\mathbf{z_t}] \tag{101}$$

for known selector vectors Ψ_H, Ψ_F.

Individual Inference

We can write the vector of signals for a Home firm as

$$\mathbf{s_t^H} = \mathbf{F}\mathbf{z_t} + \mathbf{G}\mathbf{v_t} \tag{102}$$

Bayesian updating requires

$$E_{h,t}[\mathbf{z_t}] = E_{h,t-1}[\mathbf{z_t}] + \mathbf{C}(\mathbf{s_t^H} - E_{h,t-1}[\mathbf{s_t^H}]) \tag{103}$$

Define $\mathbf{\Omega} = Var_{h,t-1}[\mathbf{z_t}]$. The Kalman gain $\mathbf{C}$ and the matrix $\mathbf{\Omega}$ must satisfy

$$\mathbf{C} = \mathbf{\Omega}\mathbf{F}'(\mathbf{F}\mathbf{\Omega}\mathbf{F}' + \mathbf{G}\mathbf{V}\mathbf{G}')^{-1} \tag{104}$$

$$\mathbf{\Omega} = \mathbf{A}(\mathbf{\Omega} - \mathbf{C}\mathbf{F}\mathbf{\Omega})\mathbf{A}' + \mathbf{B}\mathbf{\Sigma}\mathbf{B}' \tag{105}$$

Fixed Point

The average first-order beliefs can be expressed as a function of the state as

$$\mathbf{z_{t|t}} = \mathbf{\Xi^H z_t} \tag{106}$$

Using the updating equation and aggregating across home firms

$$\mathbf{z_{t|t}} = (I - \mathbf{CF})\mathbf{A z_{t-1|t-1}} + \mathbf{CF z_t} \tag{107}$$

So the matrix $\mathbf{\Xi^H}$ must satisfy

$$\mathbf{\Xi^H z_t} = (I - \mathbf{CF})\mathbf{A\Xi^H z_{t-1}} + \mathbf{CF z_t} \tag{108}$$

A similar matrix $\mathbf{\Xi^F}$ is defined for foreign firms' first-order beliefs. These matrices allow me to rewrite equations (95) and (96) as

$$p_{H,t} = \mathbf{\Xi^H z_t} \tag{109}$$

$$p_{F,t} = \mathbf{\Xi^F z_t} \tag{110}$$

The equilibrium is characterized by the vectors $A_{p_H}, A_{p_F^*}, B_{p_H}, B_{p_F^*}$ and matrices $\mathbf{\Xi^H}, \mathbf{\Xi^F}$ which are consistent with the law of motion of the state equation and with the signal-extraction problem of the firms. The equilibrium can be computed iterating over the relevant equations until convergence is achieved. The convergence criterion is given by the quadratic distance of the impulse-response functions of p_H and p_F^* to the exogenous shocks in $\mathbf{u_t}$, with weights given by the variances of the shocks.