# Econometric Methods for Modelling Systems with a Mixture of I(1) and I(0) Variables[#]

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## Summary

This paper considers structural models when both I(1) and I(0) variables are present. The structural shocks associated with either set of variables could be permanent or transitory. We therefore classify the shocks as (P1,P0) and (T1,T0), where P/T distinguishes permanent and transitory, while 1/0 means they are attached to either I(1) or I(0) variables. We first analyse what happens when there are P0 shocks. This is done using a sequence of examples and shows a variety of outcomes that differ from standard results in the cointegration literature. Then conditions are derived upon the nature of the SVAR in the event that T0 (and no P0) shocks are present. Following this a general method that allows for either P0 or T0 shocks is described and related to the literature that treats I(0) variables as cointegrating with themselves. Finally, we turn to an examination of a well-known empirical SVAR where there are P0 shocks. This SVAR is re-formulated so that the extra shock coming from the introduction of an I(0) variable does not affect relative prices in the long-run i.e. it is T0, and it is found that this has major implications for whether there is a price puzzle. It is also shown how to handle long-run parametric restrictions in the presence of P0 shocks when some shocks are identified using sign restrictions.

*Key Words*: Mixed models, transitory shocks, permanent shocks, long-run restrictions, sign restrictions, instrumental variables

*JEL Classification*: C32, C36, C51

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## 1. Introduction

It seems likely that macroeconometric modelling will involve a mixture of variables that are I(1) and I(0). However most textbooks and applied work deal with the case when all series are I(1), while reviews such as Juselius (2006) make the assumption that all series are either I(1) or I(2). So there appears to be no systematic examination of the estimation issues raised by a mixture of I(1) and I(0) variables.

When there is no cointegration structural models are generally formulated in terms of changes in the I(1) variables. With cointegration present some of the changes in I(1) variables are replaced by error correction (EC) terms when setting up a structural VAR (SVAR). When there are only I(1) variables present in a system, and there is cointegration, structural shocks have been traditionally classified as permanent and transitory. We will refer to such shocks as P1 and T1 respectively. Now, when I(0) variables are present in the system, they will be in levels, and some assumption needs to be made about the nature of the extra shocks arising from the introduction of these variables into the system. They could either be transitory or could have permanent effects on all or some of the I(1) variables. We will refer to these as T0 and P0 shocks respectively.

Section 2.1 presents some simple examples to show what the effects are of having P0 shocks present in the structural system. It starts with no P0 shocks, being just a standard cointegrated two I(1) variable model. Then an I(0) variable is added with a P0 shock and it is shown that some of the familiar results from cointegration analysis no longer apply. A third example incorporates three I(1) variables and a single I(0) variable with a P0 shock, while a fourth example returns to the first example and adds on two I(0) variables with P0 shocks. Just as with the second example, the third and fourth examples also show that traditional results from the cointegration literature no longer need apply.

Section 2.2 then looks at the case where the introduced shocks from I(0) variables are transitory i.e. T0. It is shown that this requires a particular type of model design. Specifically one needs to force some of the structural equations to have changes rather than the levels of I(0) variables, which extends the result found in Pagan and Pesaran (2008). A good deal of empirical work seems to have this situation in mind but does not recognize that the system needs to be designed to ensure that the shocks are transitory. Thus studies that have either the growth rate of output or the change in the nominal exchange rate in the SVAR, along with the inflation rate and the level of interest rates,

need to design the SVAR to ensure that monetary policy shocks do not have long-run effects on output or relative prices such as the real exchange rate. Canova, Gambetti and Pappa (2007), del Negro and Schorfheide (2004), Smets (1997), and the FAVAR SVAR of Bernanke *et al*. (2005) are examples of papers that have growth rates in the real variables in the SVAR but which do not ensure that monetary policy has zero long-run effects on real variables. We don't think that the researchers working with these systems intended such outcomes but they did not formulate SVAR specifications which ensured that the shocks were transitory.

Section 2.3 turns to an examination of a device that has been suggested as a way of handling mixtures of I(1) and I(0) variables, namely treating the I(0) variables as "cointegrating with themselves". That strategy requires the introduction of "pseudo" cointegrating vectors as well as true ones. We first use this device in section 2.2, when the extra shocks arising from the I(0) variables are assumed to have purely transitory effects, i.e. are T0. Then, using a simple example in section 2.3, we show that this method provides the correct computation of the permanent component of the I(1) variable, even when the shock coming from the introduction of the I(0) variable has permanent effects i.e. when it is P0. Section 2.4 then introduces a general method for computing the permanent component of the I(1) variables when the shocks coming from I(0) variables are either transitory (T0) or permanent (P0). This method allows us to relate the transitory component in the I(1) series to the error correction terms and lagged I(0) variables. It also allows us to establish that the pseudo-cointegration approach to calculating the permanent component in the I(1) variables applies generally.

It is not possible to study the many papers that feature structural models with P0 shocks. Consequently, in section 3 we illustrate some of the outcomes by looking at an influential study by Peersman (2005) which had this feature. Peersman sets up a SVAR involving three I(1) variables and one I(0) variable, with no co-integration between the I(1) variables. He works with three permanent shocks among the I(1) variables and one coming from the I(0) variable. This P0 shock is that stemming from the introduction of an I(0) nominal interest rate into the system. He regards the interest rate shock as having no effect upon output in the long-run, but it is allowed to affect both the general price level and the price of oil in the long-run. As there is nothing imposed on the model to say that these two prices change by the same amount in the long-run then the real price of oil must be affected by interest rate shocks at that horizon. This is the same mechanism that makes the real exchange rate respond in the long-run to monetary policy shocks in the application by Smets (1997) mentioned earlier. Examining the implications of this result in Peersman's case we find that the absence of price and output "puzzles" in his estimated model stems from the fact that a

monetary policy shock has a permanent effect on the real oil price. When this shock is taken to be transitory the puzzles re-appear.

Now the most common case where P0 shocks arise may be when sign rather than parametric restrictions are applied to identify impulse responses, since these determine only the signs of the responses for a finite number of periods, and nothing is said about the long-run outcomes. Consequently, when SVARs are adopted which include growth rates of output, the monetary policy shocks found from sign restrictions will almost always have a long-run impact on the *level* of output. Accordingly, when Peersman (2005) moved to sign restrictions to identify his shocks, the resulting impulse responses to monetary shocks show long-run effects on real variables. This would be true of many other studies with I(1) variables using sign restrictions to identify shocks. Therefore, in section 4 we look at how one can impose a zero long-run restriction on a P0 shock within a sign restrictions framework. Once again we use Peersman's set up to illustrate the approach. Section 5 then concludes.

## 2. The nature of shocks in structural models with I(0) and I(1) variables

### 2.1. Definitions of shocks with mixtures of variables

When all variables are I(1) and there is cointegration between them, shocks can be separated into whether they are permanent or transitory. These terms describe the long-run effects on the variables of the structural shocks i.e. for the second structural shock $\varepsilon_{2t}$ and, for the level of the first variable $y_{1t}$, the *long-run* effect will be $\lim\limits_{j\to\infty}\left(\dfrac{\partial y_{1t+j}}{\partial \varepsilon_{2t}}\right)$.[1] Specifically, when a shock is applied that lasts only for a single period it is called transitory if it has a zero effect on all the variables at infinity. A permanent shock is required to have a non-zero long-run effect on *at least one* of the variables. This allows for the possibility that a permanent shock may have a zero long-run effect upon *some* of the I(1) variables.

We will use a set of examples to illustrate some of the issues that arise when the shocks coming from the I(0) variables are permanent. In our first example there are only two I(1) variables and there is cointegration. Then the long-run response matrix $C = \lim\limits_{j\to\infty}\left(\dfrac{\partial y_{t+j}}{\partial \varepsilon_t'}\right)$ will have rows corresponding to the variables $y_t$ and columns representing the structural shocks. Thus the (1,2)

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[1] If $y_{1t}$ is I(1) then it would enter the SVAR as $\Delta y_{1t}$.

element of $C$ is $C_{12} = \lim\limits_{j \to \infty} \left( \dfrac{\partial y_{1t+j}}{\partial \varepsilon_{2t}} \right)$. Now in this system a permanent-transitory decomposition for

the structural shocks exists so the $C$ matrix takes the form $\quad C = \begin{bmatrix} * & 0 \\ * & 0 \end{bmatrix}$, where the * indicate non-

zero elements. This system has one permanent shock and one transitory shock. Standard results from the cointegration literature (Lütkepohl, 2006, p. 369) are; (i) the number of independent permanent components among the I(1) series (sometimes called stochastic trends) is equal to the rank of $C$; (ii) the number of permanent shocks is also equal to the rank of $C$; and (iii) the number of transitory shocks is equal to the number of cointegrating vectors. All of these results hold in this example.

It is useful to think of the first example above as being the structure for a market model in which $y_{1t}$ is the log of quantity and $y_{2t}$ the log of price. The supply shock $\varepsilon_{1t}$ has permanent effects while the demand shock $\varepsilon_{2t}$ is transitory. Now suppose that an I(0) variable is added to the system and that the new shock has permanent effects on the second of the I(1) variables. This involves the addition of a third variable to the first example. In terms of the "market" model it might be termed "sunspots" and these are assumed to have a permanent effect on prices. Consequently, the supply and sunspots shocks have permanent effects but the demand shock is transitory. In this set-up the long-run response matrix for the three variables will be[2]

$$ C = \begin{bmatrix} * & 0 & 0 \\ * & 0 & * \\ 0 & 0 & 0 \end{bmatrix}. $$

Because the Beveridge-Nelson decomposition has $\Delta y_t^p = C\varepsilon_t$, it means that $\Delta y_{1t}^p = C_{11}\varepsilon_{1t}$ and $\Delta y_{2t}^p = C_{21}\varepsilon_{1t} + C_{23}\varepsilon_{3t}$. Hence there are two permanent components (or, as sometimes said, two stochastic trends), which agrees with the rank of $C$. However there is no linear combination of $y_{1t}^p$ and $y_{2t}^p$ that is I(0), as one cannot eliminate the influence of the third shock. Hence there is no cointegration between the two I(1) variables. *Accordingly, the presence of a P0 shock has removed the cointegration that previously held between the I(1) variables.* This is because the DGP of $y_t$ has changed i.e. in terms of the market model the permanent sunspots shocks result in prices and quantities not being cointegrated, but when sunspots are absent there is cointegration between

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[2] Clearly we can differentiate between all the structural shocks here just using the long-run responses. If however the (1,3) element in $C$ had been non-zero then we would need some short-run restriction to separate the first and third shocks.

those variables. It is also interesting to note that if $C = \begin{bmatrix} * & 0 & * \\ * & 0 & * \\ 0 & 0 & 0 \end{bmatrix}$ and $c_{11} = c_{21};\ c_{13} = c_{23}$, then

the two stochastic trends would be identical and so there is only one independent one i.e. cointegration would be re-established. Of course in this case one needs some short-run information to differentiate between the two permanent shocks.

The next example adds a further I(1) variable to the second example and assumes that there are two cointegrating relations among the three of them. This will generate a $C$ matrix of the form

$$C = \begin{bmatrix} * & 0 & 0 & 0 \\ * & 0 & 0 & * \\ * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

In this case the number of permanent components is two (which equals the rank of $C$) and the number of permanent shocks is two. Examining cointegration we find that there is one cointegrating vector as there will be a linear combination of $y_{1t}^P$ and $y_{3t}^P$ that is zero. This occurs because each depends only on the first structural shock. One cannot find a combination of $y_{2t}^P$ with either permanent component in the other two variables that would be zero. Notice however that there are two transitory shocks and *so the standard relation between the number of cointegrating vectors and the number of transitory shocks is now broken*. This is because the DGP of $y_t$ has changed.

Finally, return to the first example and add two extra I(0) variables that have P0 shocks. This results in

$$C = \begin{bmatrix} * & 0 & 0 & * \\ * & 0 & * & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Inspection shows that the rank of $C$ is two and, writing out the permanent components for the first and second variables, shows why they are independent. But now there are *three* permanent shocks – the first, third and fourth. So this *illustrates the fact that the number of independent permanent components can be less than the number of permanent shocks when there are P0 shocks*.

We now investigate the implications of including both I(0) and I(1) variables together in systems when the shocks associated with the former are, firstly, transitory (T0) (covered in section 2.2) and then permanent (P0) (covered in sections 2.3 and 2.4).

### 2.2. Shocks associated with I(0) Variables are Transitory

This section shows how to treat I(0) variables in structural models that contain cointegrating relationships among the $n_1$ I(1) variables when the extra shocks coming from the I(0) variables are taken to be transitory. We will often refer to these extra shocks as those "associated with the I(0) variables". By this we mean that the addition of $q$ I(0) variables to the structural system necessitates the addition of $q$ structural equations and $q$ shocks so that the latter will be those "associated with the I(0) variables".

For simplicity, consider a structural VAR(2) model of $n$ variables of the form

$$A_0 x_t = A_1 x_{t-1} + A_2 x_{t-2} + \varepsilon_t \tag{1}$$

where $A_i$ are $n \times n$ matrices of unknown coefficients, $A_0$ is non-singular and $\varepsilon_t$ is an $n \times 1$ vector of structural shocks with mean zero and covariance matrix $D_n$. We assume that there are $n_1$ variables which are I(1) and $q$ which are I(0) giving $n = n_1 + q$, while among the I(1) variables there are $r$ cointegrating relations. We refer to the latter as 'true' i.e. the actual cointegrating relations, as distinct from the $q$ 'pseudo' cointegrating relations coming from the treatment of each of the I(0) variables as 'cointegrating with itself'. This is probably the standard way of handling I(0) variables in SVECMs that is currently in the literature. We will refer to these structures as pseudo-SVECMs.

Because there are $r$ cointegrating relations among the I(1) variables, there are $m = n_1 - r$ independent I(1) processes driving the $n_1$ I(1) variables and the $m$ shocks driving these are the permanent shocks in a pseudo-SVECM. Without loss of generality, let

$$x_t = \begin{pmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{pmatrix},$$

where $x_{1t}$ is the $m \times 1$ vector of I(1) variables whose structural shocks are known to have permanent effects, $x_{2t}$ is the $r \times 1$ vector of I(1) variables whose structural shocks are known to have

transitory effects, and $x_{3t}$ is the $q \times 1$ vector of I(0) variables whose structural shocks will be assumed to have transitory effects. Then there are $r + q$ transitory shocks in the SVECM. Let

$$\tilde{\beta} = \begin{pmatrix} \beta_1 & 0 \\ \beta_2 & 0 \\ 0 & I_q \end{pmatrix},$$

where $\tilde{\beta}$ is an $n \times (r + q)$ matrix. The matrices $\beta_1$ and $\beta_2$ are $m \times r$ and $r \times r$, respectively. The first column of block matrices in $\tilde{\beta}$ are the coefficients in the 'true' cointegrating relations among the I(1) variables, while the second column gives the 'pseudo' cointegrating relations. The latter involve a coefficient of one on a given stationary variable, and a coefficient of zero on all the remaining variables, and so are represented by the identity matrix. Analogously the loadings vector $\tilde{\alpha}^*$ can be partitioned as

$$\tilde{\alpha}^* = \begin{pmatrix} \alpha_1^* & \delta_1^* \\ \alpha_2^* & \delta_2^* \\ \alpha_3^* & \delta_3^* \end{pmatrix},$$

where $\tilde{\alpha}^*$ is an $n \times (r + q)$ matrix. The sub-matrices $\alpha_1^*$, $\alpha_2^*$ and $\alpha_3^*$ are of dimension $m \times r$, $r \times r$ and $q \times r$, respectively. Similarly, the sub-matrices $\delta_1^*$, $\delta_2^*$ and $\delta_3^*$ are of dimension $m \times q$, $r \times q$ and $q \times q$, respectively. The first column of block matrices in $\tilde{\alpha}^*$ shows the loadings on the 'true' cointegrating relations for each group of structural equations while the second shows the loadings on the I(0) variables.

The VAR model of (1) can now be written as the pseudo-SVECM

$$A_0 \Delta x_t = -\tilde{\alpha}^* \tilde{\beta}' x_{t-1} + A_2 \Delta x_{t-1} + \varepsilon_t. \tag{2}$$

The $r \times 1$ vector of 'true' error correction terms, $\xi_t$, can be written as

$$\xi_t = \beta_1' x_{1t} + \beta_2' x_{2t}. \tag{3}$$

Following the development in Pagan and Pesaran, we proceed to express the SVECM model of (2) as a structural vector autoregressive (SVAR) model of order two in the variables $\Delta x_{1t}$, $\xi_t$ and $x_{3t}$. From (3), we have

$$\Delta \xi_t = \beta_1' \Delta x_{1t} + \beta_2' \Delta x_{2t} \,,$$

from which it follows that

$$\Delta x_{2t} = (\beta_2')^{-1} (\Delta \xi_t - \beta_1' \Delta x_{1t}) \,, \tag{4}$$

provided the $r \times r$ matrix $\beta_2'$ is non-singular.[3]

The first $m$ equations in (2) are

$$A_{11}^0 \Delta x_{1t} + A_{12}^0 \Delta x_{2t} + A_{13}^0 \Delta x_{3t} = -\alpha_1^* \xi_{t-1} - \delta_1^* x_{3t-1} + A_{11}^2 \Delta x_{1t-1} + A_{12}^2 \Delta x_{2t-1} + A_{13}^2 \Delta x_{3t-1} + \varepsilon_{1t} \,, \tag{5}$$

where the $A$ matrices are partitioned conformably with $\Delta x_t$. These equations contain the structural shocks with permanent effects. Pagan and Pesaran proved that $\alpha_1^* = 0$ in (5), so that the structural equations with the permanent shocks do not contain the lagged 'true' error correction (EC) terms. Here we show additionally that $\delta_1^* = 0$ when the structural shocks associated with the I(0) variables are transitory, i.e. for the SVAR(2) in $\Delta x_{1t}$, $\xi_t$ and $x_{3t}$. Using (4) to eliminate the terms in $\Delta x_{2t}$ in (5), one obtains

$$(A_{11}^0 - A_{12}^0 (\beta_2')^{-1} \beta_1') \Delta x_{1t} + A_{12}^0 (\beta_2')^{-1} \Delta \xi_t + A_{13}^0 \Delta x_{3t}$$
$$= -\alpha_1^* \xi_{t-1} - \delta_1^* x_{3t-1} + (A_{11}^2 - A_{12}^2 (\beta_2')^{-1} \beta_1') \Delta x_{1t-1} + A_{12}^2 (\beta_2')^{-1} \Delta \xi_{t-1} + A_{13}^2 \Delta x_{3t-1} + \varepsilon_{1t}. \tag{6}$$

Defining $w_t = \begin{pmatrix} \Delta x_{1t} & \xi_t & x_{3t} \end{pmatrix}'$ the SVAR(2) can be expressed as

$$B_0 w_t = B_1 w_{t-1} + B_2 w_{t-2} + \varepsilon_t. \tag{7}$$

Partitioning (7) into the form conformable with the partition used in (5), the first $m$ equations will be

$$B_{11}^0 \Delta x_{1t} + B_{12}^0 \xi_t + B_{13}^0 x_{3t}$$
$$= B_{11}^1 \Delta x_{1t-1} + B_{12}^1 \xi_{t-1} + B_{13}^1 x_{3t-1} + B_{11}^2 \Delta x_{1t-2} + B_{12}^2 \xi_{t-2} + B_{13}^2 x_{3t-2} + \varepsilon_{1t}$$

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[3]It may be necessary to take care in setting the system up to ensure that $\beta_2'$ is non-singular. To take an example, suppose there are three I(1) variables in the system with cointegrating vector $\beta' = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$. Then, if we select the first and second equations as the two whose structural shocks have permanent effects, $\beta_2' = 0$. So we would need to choose either the first and third or the second and third as the two variables whose associated equations have permanent shocks. In the former case $\beta_2' = -1$ and, in the latter, $\beta_2' = 1$ and so are non-singular.

which can be written as,

$$B_{11}^0 \Delta x_{1t} + B_{12}^0 \Delta \xi_t + B_{13}^0 \Delta x_{3t}$$
$$= B_{11}^1 \Delta x_{1t-1} + (B_{12}^1 + B_{12}^2 - B_{12}^0)\xi_{t-1} + (B_{13}^1 + B_{13}^2 - B_{13}^0)x_{3t-1} + B_{11}^2 \Delta x_{1t-2} - B_{12}^2 \Delta \xi_{t-1} - B_{13}^2 \Delta x_{3t-1} + \varepsilon_{1t}. \tag{8}$$

Comparing (8) with (6), we get

$$\alpha_1^* = -(B_{12}^1 + B_{12}^2 - B_{12}^0), \tag{9}$$

$$\delta_1^* = -(B_{13}^1 + B_{13}^2 - B_{13}^0). \tag{10}$$

Now (7) can be written in lag operator form as

$$B(L)w_t = \varepsilon_t,$$

where $B(L) = B_0 - B_1 L - B_2 L^2$ and $L$ is the lag operator. It then follows that the moving average representation will be

$$w_t = B(L)^{-1}\varepsilon_t = C(L)\varepsilon_t, \tag{11}$$

where $C(L) = C_0 + C_1 L + C_2 L^2 + C_3 L^3 + \dots$. Hence $C(1) = B(1)^{-1}$ implies that

$$C(1)B(1) = I_n. \tag{12}$$

By assumption shocks to the error correction terms $\xi_t$ are transitory, so it must be the case that $C_{12}(1) = 0$, where $C(1)$ is partitioned analogously to the partitioned matrices in (8). When shocks to the I(0) variables are transitory, it is the case that $C_{13}(1) = 0$. These both place restrictions on the $B$ matrices. To determine what they are multiply the first row of $C(1)$ with the second column of $B(1)$ to obtain the equation

$$C_{11}(1)B_{12}(1) + C_{12}(1)B_{22}(1) + C_{13}(1)B_{32}(1) = 0_{12}, \tag{13}$$

where $0_{12}$ is an $m \times r$ null matrix. Under the restrictions, (13) becomes $C_{11}(1)B_{12}(1) = 0_{12}$, from which it follows that $B_{12}(1) = 0_{12}$, since $C_{11}(1)$ has full rank $m$. But $B_{12}(1) = (B_{12}^0 - B_{12}^1 - B_{12}^2)$ so that $B_{12}(1) = 0_{12}$ means $\alpha_1^* = 0_{12}$ from (9). This is the Pagan and Pesaran result.

Similarly, multiplying the first row of $C(1)$ with the third column of $B(1)$ gives

$$C_{11}(1)B_{13}(1) + C_{12}(1)B_{23}(1) + C_{13}(1)B_{33}(1) = 0_{13}, \tag{14}$$

where $0_{13}$ is an $m \times q$ null matrix. Using the same reasoning, it follows that $B_{13}(1) = 0_{13}$, and noting that $B_{13}(1) = (B_{13}^0 - B_{13}^1 - B_{13}^2)$, (10) implies that $\delta_1^* = 0_{13}$. Thus, when the structural shocks coming from the I(0) variables are transitory, the $m$ structural equations with the permanent shocks do not contain the levels of the I(0) variables, only their differences. This case is an extension of the Pagan and Pesaran result for the SVAR involving $\Delta x_{1t}$ and $\Delta \xi_t$ - that the levels of the EC terms were replaced by their differences in the structural equations for $\Delta x_{1t}$.

*2.3. A Simple Example for Constructing the Permanent Component of an I(1) Variable when Shocks are P0 or T0.*

In order to see the effects of a P0 shock, it is useful to consider the following simple system:

$$\Delta y_t = \delta z_t + \varepsilon_{1t} \tag{15}$$

$$z_t = \gamma z_{t-1} + \varepsilon_{2t}, \tag{16}$$

where $y_t$ is I(1), $z_t$ is I(0), the shocks $\varepsilon_{1t}$ and $\varepsilon_{2t}$ are white noise and $\varepsilon_{2t}$ has a permanent effect on $y_t$. Hence, using the Beveridge-Nelson decomposition to find the permanent component of $y_t$ we have

$$y_t^p = y_t + E_t \sum_{j=1}^{\infty} \Delta y_{t+j} = y_t + E_t \sum_{j=1}^{\infty} (\delta z_{t+j} + \varepsilon_{1t+j}) = y_t + \frac{\delta\gamma}{1-\gamma} z_t,$$

so that, from (15) and (16),

$$\Delta y_t^p = \Delta y_t + \frac{\delta\gamma}{1-\gamma} \Delta z_t = \delta z_t + \varepsilon_{1t} + \frac{\delta\gamma}{1-\gamma} \Delta z_t$$

$$= \delta\gamma z_{t-1} + \delta\varepsilon_{2t} + \varepsilon_{1t} + \frac{\delta\gamma}{1-\gamma}[(\gamma-1)z_{t-1} + \varepsilon_{2t}]$$

$$= \varepsilon_{1t} + \frac{\delta}{1-\gamma}\varepsilon_{2t}.$$

This shows that $\Delta y_t^p$ is white noise as it is the sum of two white noise terms.

Turning to the case where $\varepsilon_{2t}$ has transitory effects requires the re-specification of (15) to

$$\Delta y_t = \delta \Delta z_t + \varepsilon_{1t}. \tag{17}$$

Then, by the Beveridge-Nelson decomposition,

$$y_t^p = y_t + E_t \sum_{j=1}^{\infty} (\delta \Delta z_{t+j} + \varepsilon_{1t+j}),$$

and, using the fact that $z_t^p = z_t + E_t \sum_{j=1}^{\infty} \Delta z_{t+j}$ means $z_t^p = 0$ ( $z_t$ is I(0) with zero mean), as well as

$\varepsilon_{1t}$ being white noise, we get

$$y_t^p = y_t + \delta(z_t^p - z_t) = y_t - \delta z_t,$$

so that

$$\Delta y_t^p = \Delta y_t - \delta \Delta z_t = \varepsilon_{1t}.$$

This shows that $\varepsilon_{2t}$ has only a transitory effect on any $y_t$. Of course, this is the result that we established more generally in the previous sub-section viz. that the I(0) variables have to be entered as differences if extra shocks in the system coming from the introduction of the I(0) variables are to have transitory effects.

It would generally be the case that the SVAR specified by empirical researchers would involve $\Delta y_t$ and $z_t$, and so it is clear that such a SVAR would not incorporate any structural equations that had $\Delta z_t$ on the RHS. One needs to modify existing SVAR programs to get that effect i.e. to make the extra shocks from the I(0) variables have transitory effects. That would entail having the first difference of the I(0) variables and not their levels, on the RHS of the structural equations for $\Delta y_t$. Notice that if one had used an SVAR with $\Delta y_t$ and $\Delta z_t$, this would change the specification of the second equation in (16) to an AR(1) in $\Delta z_t$. However this is a very different specification.

In an earlier section we noted that often it had been suggested that I(0) variables can be handled by using the idea of pseudo co-integrating vectors. Would this approach give a correct estimate of the permanent component of $y_t$ in the first system, (15) and (16), and in the second one, (17) and (16)? In both systems there is no cointegration so that $\beta$ will be the pseudo co-integrating vector $\beta' = \begin{pmatrix} 0 & 1 \end{pmatrix}$. The pseudo-SVECM for both systems has equations in $\Delta y_t$ and $\Delta z_t$. For the first system, the pseudo-SVECM comprises (15) along with

$$\Delta z_t = (\gamma - 1)z_{t-1} + \varepsilon_{2t}, \tag{18}$$

and the corresponding pseudo-VECM form is

$$\Delta y_t = \delta\gamma z_{t-1} + e_{1t} \tag{19}$$

$$\Delta z_t = (\gamma - 1)z_{t-1} + e_{2t}, \tag{20}$$

where the pseudo-VECM residuals are $e_{1t} = \varepsilon_{1t} + \delta\varepsilon_{2t}$ and $e_{2t} = \varepsilon_{2t}$. In cointegration analysis, Johansen's formula (1995) can be used to obtain the change in the permanent component of the series $y_t$ as

$$\begin{pmatrix} \Delta y_t^p \\ \Delta z_t^p \end{pmatrix} = \beta_\perp \left[ \alpha_\perp' A(1)\beta_\perp \right]^{-1} \alpha_\perp' e_t, \tag{21}$$

where $\beta'\beta_\perp = 0$, $\alpha'\alpha_\perp = 0$, and $A(1)$ is the (2x2) matrix whose elements are the sum of the coefficients on $\Delta y_t$, $\Delta z_t$ and their lags in each equation block. In the system (19) and (20),

$$\beta' = \begin{pmatrix} 0 & 1 \end{pmatrix}, \ \beta_\perp = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \alpha' = \begin{pmatrix} \delta\gamma & \gamma - 1 \end{pmatrix}, \ \alpha_\perp' = \begin{pmatrix} 1 & \dfrac{-\delta\gamma}{\gamma - 1} \end{pmatrix} \text{ and } A(1) = I_2, \text{ so that}$$

$$\left[ \alpha_\perp' A(1)\beta_\perp \right]^{-1} = 1 \text{ and}$$

$$\Delta y_t^p = \alpha_\perp' e_t = e_{1t} - \frac{\delta\gamma}{\gamma - 1}e_{2t} = \varepsilon_{1t} + \frac{\delta}{1-\gamma}\varepsilon_{2t}, \tag{22}$$

which is the expression we obtained earlier by direct methods for the system (15) and (16).

Now consider the second system. Here the pseudo-SVECM comprises (17) and (18) which has a pseudo-VECM form comprising (20) and

$$\Delta y_t = \delta(\gamma - 1)z_{t-1} + e_{1t}, \tag{23}$$

where $e_{1t} = \varepsilon_{1t} + \delta\varepsilon_{2t}$ and $e_{2t} = \varepsilon_{2t}$ as before. Now $\alpha' = \begin{pmatrix} \delta(\gamma - 1) & \gamma - 1 \end{pmatrix}$ and $\alpha_\perp' = \begin{pmatrix} 1 & -\delta \end{pmatrix}$, so that $\Delta y_t^p = \alpha_\perp' e_t = e_{1t} - \delta e_{2t} = \varepsilon_{1t}$, which is what we obtained earlier by direct methods for the second system where the second shock was transitory.

This analysis illustrates that the pseudo-cointegration approach will work, provided the SVECM is set-up appropriately, and what is appropriate will depend on whether the shocks arising from the I(0)

variables are assumed to have permanent or transitory effects. When they are allowed to have permanent effects, the structural equations for the I(1) variables must be specified in the levels of the I(0) variables. When they are restricted to have transitory effects, these equations must be specified to include changes of the I(0) variables. Care might need to be taken if one uses the pseudo-cointegration approach in conjunction with standard cointegration software since it would be rare for the SVECM to be formulated as involving both changes and levels of the I(0) variables i.e. typically the software would handle the case when the extra shocks are permanent but not when they are transitory.

We now derive a formula for the computation of the permanent component in an I(1) variable when there is a general VAR system i.e. in which shocks can be transitory or permanent. This formula also allows us to see the mapping from the changes in I(1) variables and the levels of I(0) variables to the transitory component of an I(1) variable. It can also be used to allow us to establish for the general case that the cointegration approach gives the correct estimate of the permanent component. This latter feature is demonstrated in Appendix 2.

*2.4. General formula for computation of permanent components of I(1) series when the Extra Shocks are either P0 or T0.*

We will consider the following VAR system

$$\Delta y_t = A_1 \Delta y_{t-1} + G z_{t-1} + e_{1t} \tag{24}$$

$$z_t = F z_{t-1} + \Phi \Delta y_{t-1} + e_{2t}, \tag{25}$$

where $z_t$ has both any true EC terms as well as the I(0) variables in it. To rationalize (24) and (25) think of the case where all variables $y_t$ are I(1) and there is co-integration. Then we would have $\Delta y_t = A_1 \Delta y_{t-1} + \alpha \beta' y_{t-1}$, and this can be written as $\beta' \Delta y_t = \Delta ec_t = \beta' A_1 \Delta y_{t-1} + \beta' \alpha ec_{t-1}$, thereby giving an equation that has the form $ec_t = (I + \beta' \alpha) ec_{t-1} + \beta' A_1 \Delta y_{t-1}$. So this has the structure of (25) with $ec_t$ being included in $z_t$. Although we are working with a first order system, higher order systems can be handled simply by reducing them to a first order form in the standard way.

Now the permanent component of $y_t$ is $y_t^p = y_t + E_t \sum_{j=1}^{\infty} \Delta y_{t+j}$, so we need to look at the second term which we denote as $E_t K_t$. This will be

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$$E_t K_t = E_t \sum_{j=1}^{\infty} \Delta y_{t+j} = E_t \sum_{j=1}^{\infty} (A_1 \Delta y_{t-1+j} + G z_{t-1+j} + e_{1t+j}). \tag{26}$$

Appendix 1 shows that

$$E_t K_t = (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi]\Delta y_t + (I - R\Phi)^{-1} R z_t \tag{27}$$

where

$$R = (I - A_1)^{-1} G (I - F)^{-1}. \tag{28}$$

Using (27) for $E_t K_t$ we can then compute $y_t^p$ from

$$y_t^p = y_t + E_t K_t. \tag{29}$$

The transitory component in $y_t$ is $-E_t K_t$ and the formula (27) shows how it is related to the error correction terms and the lagged I(0) variables i.e. for the case $G \neq 0$. When $G = 0$, the formula shows that the transitory component is $-(I - A_1)^{-1} A_1 \Delta y_t$, and so is not dependent on $z_t$. Further, we show in appendix 2 that this formula gives the change in the permanent component of the I(1) variables as

$$\Delta y_t^p = (I - R\Phi)^{-1}(I - A_1)^{-1}[e_{1t} + G(I - F)^{-1} e_{2t}]. \tag{30}$$

Application of (30) to the two simple systems in section 2.3 gives identical results to those established previously. This is demonstrated in appendix 1.

### 3. An illustration of the treatment of P0 shocks in Peersman's (2005) SVAR

In an influential paper, Peersman (2005) estimated an SVAR model of four variables to investigate the role played by the underlying structural shocks in the early millennium slowdown experienced in the United States and Europe. The VAR consisted of the oil price $(o_t)$, output $(y_t)$, consumer prices $(p_t)$ (all in log levels) and the short-term nominal interest rate $(s_t)$. The oil price, output and consumer prices are treated as I(1) variables and the short-term interest rate as an I(0) variable. There was no evidence for a cointegrating relation among the I(1) variables. In view of these properties of the data, Peersman followed common practice and specified an SVAR in the first difference of the I(1) variables and in the level of the stationary variable.

To exactly identify the SVAR, Peersman imposed two long-run and four contemporaneous restrictions. Under these restrictions, the structural shock to oil prices was interpreted as an oil price shock, to output as a supply shock, to consumer prices as a demand shock and to the interest rate as a monetary policy shock. The two long-run restrictions are that the demand and monetary policy shocks have a zero long-run effect on output, and these distinguish those shocks from the oil price and supply shocks. In order to distinguish the monetary policy shock from the demand shock, Peersman imposed the restriction that the monetary policy shock has a zero contemporaneous effect on output. Finally, he assumed that the change in oil prices does not depend on the contemporaneous change in output, consumer prices and the interest rate. These serve to differentiate the supply shock from the oil price shock and also imply that supply, demand and monetary policy shocks have a zero contemporaneous effect on oil prices. Hence the monetary policy shock is, in our terminology, a P0 shock, as it arises from the introduction of the I(0) interest rate variable and it is permitted to have a long-run effect on some of the I(1) variables, specifically, consumer and oil prices.

The SVAR was specified with three lags and each equation included a constant and a time trend. Peersman estimated the SVAR by maximum likelihood methods for the sample 1980Q1 – 2002Q2. Figure 1(a) of his paper (2005, p.189) shows the impulse responses of the variables to the structural shocks out to a 28 quarter horizon. An inspection of this figure reveals several features. First, in response to a monetary policy shock which raises the short-term interest rate, both consumer prices and oil prices fall over all horizons i.e. there are no "price puzzles". While output increases by only a small amount initially, it then falls over the next four quarters, after which it starts to gradually recover to its level prior to the monetary policy shock. Third, the monetary policy shock has a long-run effect on relative prices since oil prices fall proportionately more than consumer prices (2% compared with 0.3%) at the 28 quarter horizon. Fourth, the demand shock has a long-run effect on relative prices. Oil prices increase by 3% in response to a positive demand shock at the 28 quarter horizon while consumer prices increase by only 0.3%. While there are no "output" and "price" puzzles in the results, the monetary policy and demand shocks have a long-run effect on relative prices.[4] Because it is standard in most economic models for demand and monetary policy shocks to have only transitory effects on relative prices and output so that in the long-run relative prices and

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[4] We estimated Peersman's SVAR by IV and replicated his results. Note that IV estimation of SVARs was introduced by Shapiro and Watson (1988). Peersman's data was obtained from the data archive of the *Journal of Applied Econometrics*. We will later find some price and output puzzles in various SVARs we estimate. There are of course suggestions that these puzzles may not be so e.g. it has been argued that a rise in interest rates could increase the price level owing to increased working capital costs. However, mostly such results are regarded as abnormal, and so classified as puzzles. We will just follow the conventional approach here and classify rises in output and prices in response to monetary policy shocks as "puzzles".

output are unaffected by these shocks, we would expect that the SVAR should also be designed to have such properties. We now turn to how this is to be done.

## 3.1. Design of the SVAR

To arrive at a SVAR with the long-run properties just mentioned, we begin by replacing the price of oil with the *relative* price of oil, defined as $\varsigma_t = o_t - p_t$. This is also an I(1) variable, as Peersman found no co-integration between the I(1) variables. The resulting SVAR is:[5]

$$\Delta\varsigma_t = a_{11}^1\Delta\varsigma_{t-1} + a_{12}^0\Delta y_t + a_{12}^1\Delta y_{t-1} + a_{13}^0\Delta p_t + a_{13}^1\Delta p_{t-1} + a_{14}^0 s_t + a_{14}^1 s_{t-1} + \varepsilon_{1t} \tag{31}$$

$$\Delta y_t = a_{21}^0\Delta\varsigma_t + a_{21}^1\Delta\varsigma_{t-1} + a_{22}^1\Delta y_{t-1} + a_{23}^0\Delta p_t + a_{23}^1\Delta p_{t-1} + a_{24}^0 s_t + a_{24}^1 s_{t-1} + \varepsilon_{2t} \tag{32}$$

$$\Delta p_t = a_{31}^0\Delta\varsigma_t + a_{31}^1\Delta\varsigma_{t-1} + a_{32}^0\Delta y_t + a_{32}^1\Delta y_{t-1} + a_{33}^1\Delta p_{t-1} + a_{34}^0 s_t + a_{34}^1 s_{t-1} + \varepsilon_{3t} \tag{33}$$

$$s_t = a_{41}^0\Delta\varsigma_t + a_{41}^1\Delta\varsigma_{t-1} + a_{42}^0\Delta y_t + a_{42}^1\Delta y_{t-1} + a_{43}^0\Delta p_t + a_{43}^1\Delta p_{t-1} + a_{44}^1 s_{t-1} + \varepsilon_{4t} \tag{34}$$

The four long-run restrictions we impose are that demand and monetary policy shocks have a zero long-run effect on relative prices and output. With respect to relative prices, the restrictions are, respectively,

$$a_{13}^0 + a_{13}^1 = 0, \quad a_{14}^0 + a_{14}^1 = 0, \tag{35}$$

and, with respect to output, they would be

$$a_{23}^0 + a_{23}^1 = 0, \quad a_{24}^0 + a_{24}^1 = 0. \tag{36}$$

These enable demand and monetary policy shocks to be differentiated from relative oil price and supply shocks.[6]

We require two contemporaneous restrictions, one to separate demand from monetary policy shocks and the other to separate relative oil price from supply shocks. They are, respectively, that the demand and supply shocks have a zero contemporaneous effect on the relative price of oil. These are the equivalent of two of Peersman's short-run restrictions, though now with respect to the relative oil price.

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[5] For ease of exposition, our development assumes an SVAR of order one which does not include deterministic terms. It can be easily generalised to the SVAR we actually estimate which, following Peersman, has three lags and a constant and time trend in each equation.

[6] To implement these restrictions, we note, for example, that $a_{24}^0 s_t + a_{24}^1 s_{t-1}$ can be expressed as

$a_{24}^0 \Delta s_t + (a_{24}^0 + a_{24}^1) s_{t-1}$.

These restrictions can be imposed parametrically on (31)-(34). Let the (4×4) matrix of contemporaneous interactions among the variables be denoted by $A_0$, where the elements along the principal diagonal are unity, so the first structural equation is for the change in the relative oil price, the second for the change in output, the third for the change in consumer prices and the fourth for the interest rate. The relationship between the structural shocks and the reduced form errors $(e_t)$ is given by $\varepsilon_t = A_0 e_t$. Let the element in the $i$th row and the $j$th column of $A_0^{-1}$ be denoted as $a_0^{ij}$. Then the restriction that the demand shock has a zero contemporaneous effect on relative oil prices is expressed as $a_0^{13} = 0$. Because $e_t = A_0^{-1} \varepsilon_t$ the reduced form errors are linear combinations of the structural shocks, so that the restriction $a_0^{13} = 0$ means that the demand shock does not appear in the reduced form (VAR) error for relative oil prices. Consequently the residuals from the VAR equation for relative oil prices $(e_{1t})$ can be used as an instrument in the estimation of the consumer price equation. Similarly, the restriction that the supply shock has a zero contemporaneous effect on relative oil prices is $a_0^{12} = 0$, showing that the VAR relative oil price residuals can also be used as an instrument in the estimation of the output equation. The two contemporaneous restrictions together with the four long-run restrictions shown in (35) and (36) produce the correct number of restrictions to identify the SVAR parameters.

*3.2. Estimation*

Imposing the two long-run restrictions in (36) on (32), the equation for the change in output becomes

$$\Delta y_t = a_{21}^0 \Delta \varsigma_t + a_{21}^1 \Delta \varsigma_{t-1} + a_{22}^1 \Delta y_{t-1} + a_{23}^0 \Delta^2 p_t + a_{24}^0 \Delta s_t + \varepsilon_{2t}. \tag{37}$$

We estimate this equation using, as instruments, $\hat{e}_{1t}$, $\Delta p_{t-1}$, $s_{t-1}$, as well as $\Delta \varsigma_{t-1}$ and $\Delta y_{t-1}$. The next equation to estimate is the equation for the relative price of oil that is obtained by imposing the restrictions in (35) on (31). The resulting equation is

$$\Delta \varsigma_t = a_{11}^1 \Delta \varsigma_{t-1} + a_{12}^0 \Delta y_t + a_{12}^1 \Delta y_{t-1} + a_{13}^0 \Delta^2 p_t + a_{14}^0 \Delta s_t + \varepsilon_{1t}. \tag{38}$$

It is estimated using, as instruments, the residuals $\hat{\varepsilon}_{2t}$ from (37), along with $\Delta p_{t-1}$, $s_{t-1}$, $\Delta \varsigma_{t-1}$ and $\Delta y_{t-1}$. The next equation estimated is (33), the equation for consumer prices. Here the instruments are $\hat{\varepsilon}_{1t}$, $\hat{\varepsilon}_{2t}$, $\hat{e}_{1t}$, as well as $\Delta \varsigma_{t-1}$, $\Delta y_{t-1}$, $\Delta p_{t-1}$ and $s_{t-1}$. Finally, the last equation estimated is (34),

the interest rate equation. For this, we use the estimated residuals $\hat{\varepsilon}_{1t}$ , $\hat{\varepsilon}_{2t}$ and $\hat{\varepsilon}_{3t}$ as well as $\Delta \varsigma_{t-1}$ , $\Delta y_{t-1}$ , $\Delta p_{t-1}$ and $s_{t-1}$ , as the instruments.

*3.3 Results*

Figure 1 shows the impulse responses of the U.S. variables to the structural shocks out to a horizon of 28 quarters.[7] Note that the response of the oil price itself to a shock is simply the sum of the relative oil price and consumer price response to that shock. The identifying restrictions are apparent in the responses: the demand and monetary policy shocks have a zero long-run effect on relative oil prices and output, and the supply and demand shocks have a zero contemporaneous effect on relative oil prices.[8]

With this specification, however, there are "price" and "output" puzzles. In response to a monetary policy shock which raises the interest rate, consumer prices steadily rise and by 28 quarters have increased by 0.3%. The oil price initially falls by about 3%, so there is no "relative oil price" puzzle, and by 28 quarters it has increased by the same proportionate amount as consumer prices, leaving long-run relative oil prices unchanged. Output initially rises by around 0.3% following the monetary policy shock so there is an "output puzzle".

We estimated several SVARs under other combinations of two zero contemporaneous restrictions while maintaining the four long-run restrictions. In all the SVAR's, at least one puzzle was apparent in the responses. When there was a consumer price puzzle, there was no oil price puzzle and vice-versa, and it was only in specifications which restricted the contemporaneous response of output to the monetary policy shock to zero that the output puzzle disappeared.[9] It appears that, once demand and monetary policy shocks are restricted to have only transitory effects on relative prices, "puzzles" emerge. Once these shocks are allowed to have permanent effects on relative prices, the "puzzles" disappear. Our experience in other applications is that this is a common phenomenon and it should force empirical researchers to justify why they allow nominal shocks to have long-run effects on real variables and relative prices.

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[7] The responses at 28 quarters are sufficient to show the long-run as they are indistinguishable from those at much longer horizons (we generated responses out to 200 quarters and saw no discernible differences). The impulse responses are shown together with their one standard error bands based on 1000 bootstrapped draws. In the bootstrap, the forecast values and re-sampled residuals from the reduced-form VAR model estimated with actual data were used to construct artificial time series for each variable.

[8] As a check on our results, we also estimated the model using the *short and long procedure* in RATS Version 8.2. The RATS numerical procedure confirmed the results from IV estimation and the numerical differences between the two sets of impulse responses were slight.

[9] This pattern emerged in all specifications including ones that left unrestricted the contemporaneous effect of all the shocks on the relative price of oil.

### 4. Sign Restrictions with P0 Shocks

In addition to the parametric approach, Peersman chose to use the sign restrictions methodology, developed by Faust (1998), Uhlig (2005) and Canova and De Nicoló (2002), to identify the structural shocks. The method starts by obtaining an initial set of shocks that are uncorrelated. Peersman followed traditional practice and obtained these from a recursive model. While this restricts the contemporaneous impacts of the initial shocks, it leaves the long-run impacts unrestricted. In our application, we specify the initial model to preserve the two long-run restrictions that the third and fourth shocks have a zero long-run effect on output and then make the model recursive. In this way, the initial shocks are orthogonal and have the property that the third and fourth shocks do not have a long-run impact on output. Note that in our terminology, the third and fourth shocks are P1 and P0 shocks, respectively, as they can have a long-run effect on oil and consumer prices.

To describe our initial model, we will refer to (31)-(34) for the variable numbers. However, now the relative price of oil has to be replaced by the price of oil, as we are re-considering the results from Peersman's original model with shocks now being identified using sign restrictions.[10] The first equation we set up to generate shocks that are to be the basis of the sign restrictions approach is for output i.e. (37). This has imposed on it the two long-run zero restrictions. But we need a further restriction, and that involves assuming oil prices are ordered after output, so that $a_{21}^0 = 0$. The next equation is for the change in consumer prices (33), and here we assume that oil prices and interest rates are ordered after the general price level, thereby generating the restrictions $a_{31}^0 = 0$ and $a_{34}^0 = 0$. The oil price equation (31) uses the restriction $a_{14}^0 = 0$, that is interest rates are ordered after the oil price. Finally no restrictions are placed on the interest rate equation (34). The model just described is then estimated by IV. In estimation of the consumer price equation, $\hat{\varepsilon}_{2t}$ is used as an instrument; in estimation of the oil price equation, $\hat{\varepsilon}_{2t}$ and $\hat{\varepsilon}_{3t}$ are used as instruments; and in the interest rate equation, $\hat{\varepsilon}_{2t}$, $\hat{\varepsilon}_{3t}$ and $\hat{\varepsilon}_{1t}$ are used as instruments.

In sign restrictions, the initial shocks from the model just described are normalized to have unit variance so they become $\hat{\varepsilon}_{i,t}^* = (\hat{\varepsilon}_{i,t} / \hat{\sigma}_i)$, $i = 1, 2, 3, 4$ and are $i.i.d(0, I_4)$. We focus on the group $\hat{\varepsilon}_{R,t}^* = (\hat{\varepsilon}_{3t}^* \quad \hat{\varepsilon}_{4t}^*)'$ as these are restricted to have a zero long-run effect on output.[11] The next step is

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[10] Again, in the actual application, we follow Peersman and estimate a SVAR with three lags and a constant and time trend in each equation.

[11] Separating the shocks into appropriate groups and applying sign restrictions to each group is the approach taken by Fry and Pagan (2011) for cointegrated systems in which there are both permanent and transitory

to linearly combine these shocks to form a new set of shocks $\hat{\eta}_{R,t}^* = Q\hat{\varepsilon}_{R,t}^*$, where the (2×2) matrix $Q$ is the Givens matrix

$$\begin{bmatrix} \cos\theta_k & -\sin\theta_k \\ \sin\theta_k & \cos\theta_k \end{bmatrix}, \quad \theta_k \in [0,\pi],$$

with the property that $Q'Q = QQ' = I_2$. The $Q$ matrix depends on a 'draw' of $\theta_k$ and, in sign restrictions, the number of draws is large.[12] Note that the new shocks are uncorrelated with each other.

Now let the (4×2) matrix $C_{R,j}$ denote the responses at horizon $j$ of the variables to a one unit innovation in each of the shocks in $\hat{\varepsilon}_{R,t}^*$. Then, for a given draw of the Givens matrix, the responses to a one unit innovation in each of the new shocks, $\hat{\eta}_{R,t}^*$, is $C_{R,j}Q'$. Note that the long-run response of output to $\hat{\eta}_{R,t}^*$ is zero since both elements of the second row of $C_{R,\infty}$ are zero. Sign restrictions are now used to distinguish between the two shocks in $\hat{\eta}_{R,t}^*$.[13] The restrictions we use are taken from Peersman. A positive monetary policy shock raises the interest rate and has a non-positive effect on oil prices, output, and consumer prices. In contrast, if all the responses are non-negative, it is treated as a positive demand shock.[14]

We found that 0.578% of the draws satisfied the sign restrictions for demand and monetary policy shocks. This success rate is a little lower than what Peersman reported (1 in 130 or 0.769%). In both cases however these low retention rates might suggest that the data does not support the sign restrictions. Based on the successful draws, figure 2 reports the median (50th fractile) responses to unit shocks. Demand and monetary policy shocks have a zero long-run effect on output by design but they clearly have long-run effects on the relative price of oil.

In our signs approach, care needs to be exercised in formulating the initial recursive model. Suppose we had decided to order the oil price before output. Then this would mean that the initial third and fourth shocks have a zero contemporaneous effect on oil prices. Now these two shocks

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shocks. As we are making finer distinctions among the shocks, it is natural to adopt a similar approach here, so that the new shocks will retain the features of the initial shocks.

[12] In our application, $\theta_k = k(\pi / 500,000)$, $k = 0,1,2,\ldots,500,000$.

[13] They could also be used to separate the shocks in the group $\hat{\eta}_{U,t}^* = (\hat{\eta}_{1t}^* \quad \hat{\eta}_{2t}^*)'$ but that is not our focus.

[14] In line with Peersman, the time period over which the sign restrictions are binding is for four quarters on the responses of output and consumer prices and only on the instantaneous response of oil prices and the interest rate.

have the requisite zero long-run effects so we linearly combine them together to form new shocks. But this must mean that any new shocks have a zero contemporaneous effect on oil prices. It does not seem reasonable to constrain the demand and monetary shocks to always have such effects. Consequently, this led us to adopt the ordering described where oil prices came after output and the general price level.

**5. Conclusion**

The inclusion of I(0) variables in structural econometric models introduces additional shocks which can be either permanent i.e. have a non-zero long-run effect on at least one I(1) variable, or transitory i.e. have a zero long-run effect on all I(1) variables. We denote the former as P0 shocks and the latter as T0 shocks.

It is common practice for researchers to specify SVARs in the first difference of the I(1) variables and in the levels of the I(0) variables. In this case we show that shocks associated with the I(0) variables can have permanent effects on the I(1) variables i.e. are P0. It was also demonstrated how to set the SVAR up so that shocks are transitory i.e. are made T0, and the method can be seen as a generalization of the Pagan and Pesaran (2008) result. It involves specifying the structural equations for the I(1) variables so that the first difference of the I(0) variables and not their lagged levels appear in these equations. We then derived a general expression for finding the permanent component in an I(1) variable, from which we can see how the I(0) (and error correction) variables in the P0 shock case map into the variable's transitory component. This formula was also used to establish that a method for calculating the permanent component of an I(1) variable by treating the I(0) variable as co-integrating with itself would work, provided that one was careful in choosing the pseudo-SVECM system.

We then turned to some applications, using as the vehicle Peersman's (2005) influential SVAR which features a P0 shock. The latter arises from the presence of an I(0) interest rate variable and is P0 because it is allowed to have a long-run effect on oil and consumer prices, both of which are I(1). In Peersman's SVAR, there are no price or output puzzles, but there is monetary non-neutrality, since the P0 shock affects relative prices in the long-run. When the monetary shock is made transitory i.e. T0, output and price puzzles emerge. We conclude that the absence of price and output puzzles in Peersman's VAR comes about because he allows the P0 shock to have a long-run effect on relative prices; that is, the absence of puzzles comes at the cost of monetary non-neutrality. Finally, we show how to apply sign restrictions to the SVAR for which the two long-run zero restrictions of Peersman are maintained.

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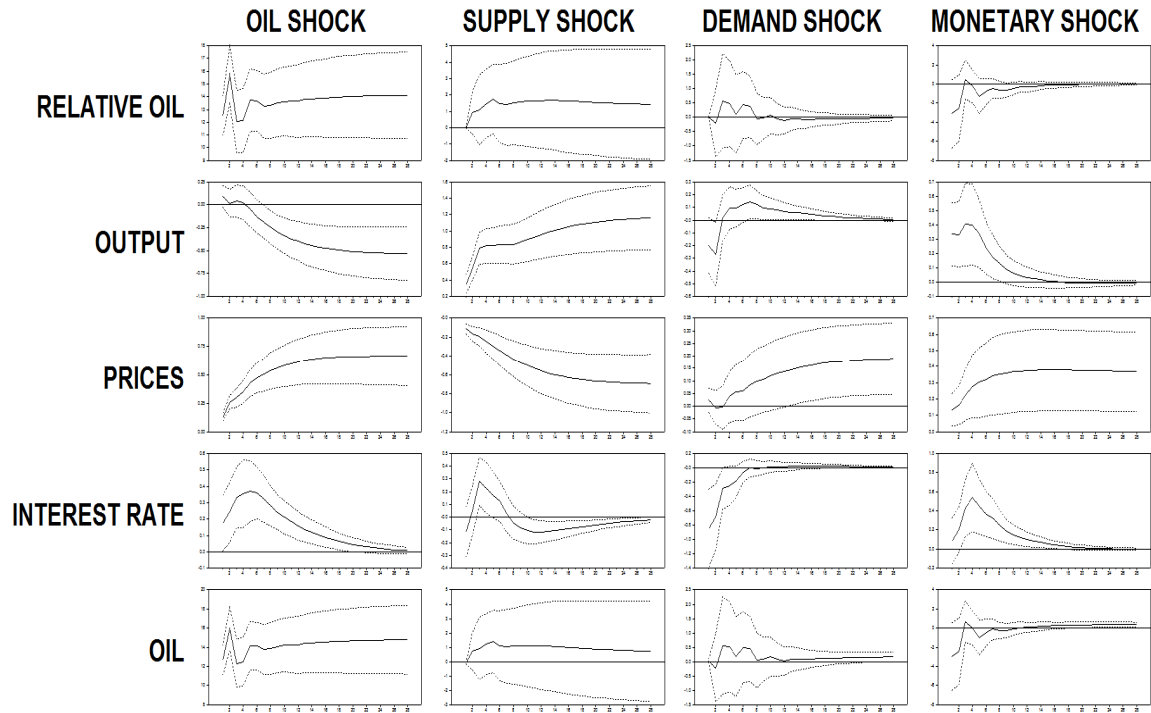
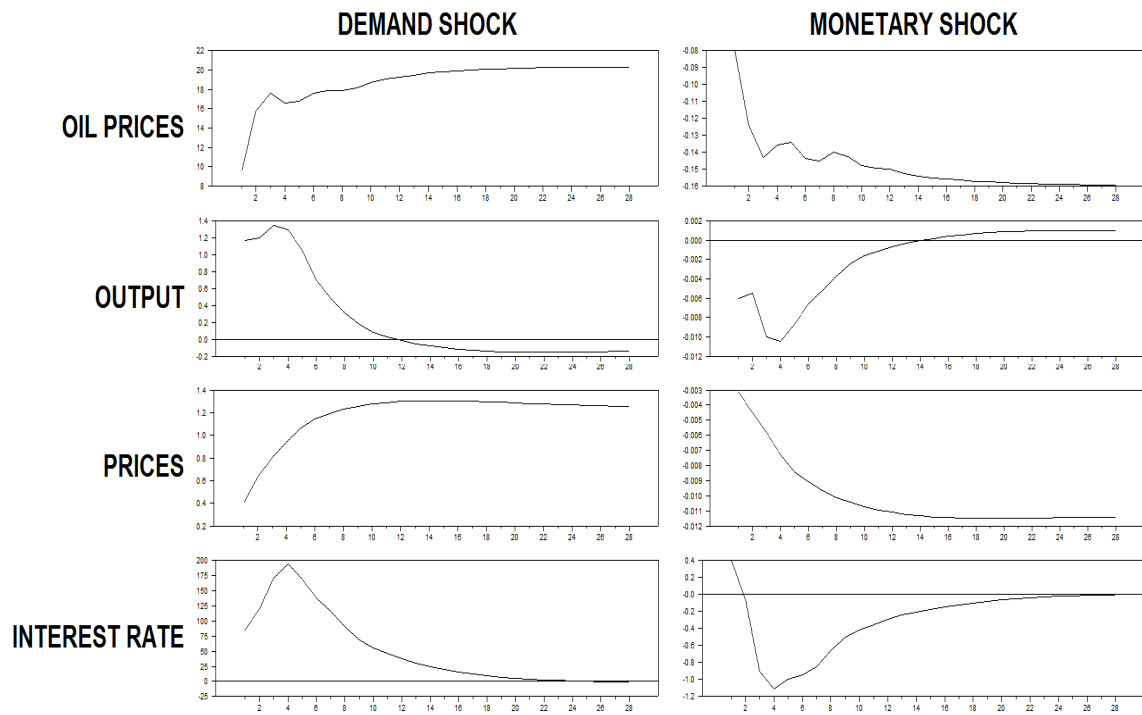**Figure 1**. Impulses responses from relative price model



**Figure 2.** Impulse responses based on signs

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**Appendix 1: Derivation of the Permanent Component in the General Case**

As shown in the text, the following VAR system is considered

$$\Delta y_t = A_1 \Delta y_{t-1} + G z_{t-1} + e_{1t} \tag{A1}$$

$$z_t = F z_{t-1} + \Phi \Delta y_{t-1} + e_{2t}, \tag{A2}$$

where $z_t$ has both any true EC terms as well as the I(0) variables in it. Now the permanent component of $y_t$ is $y_t^p = y_t + E_t \sum_{j=1}^{\infty} \Delta y_{t+j}$, so we need to look at the second term. This will be

$$E_t \sum_{j=1}^{\infty} \Delta y_{t+j} = E_t \sum_{j=1}^{\infty} (A_1 \Delta y_{t-1+j} + G z_{t-1+j} + e_{1t+j}). \tag{A3}$$

Now let us consider $L_t = \sum_{j=1}^{M} \Delta y_{t+j-1}$ and define $K_t = \sum_{j=1}^{M} \Delta y_{t+j}$. Then it is clear that $L_t = K_t + \Delta y_t - \Delta y_{t+M}$. Thus, as $M \to \infty$, $E_t(L_t) = E_t(K_t) + \Delta y_t$. Consequently, when $M \to \infty$, we can write (A3) above as

$$E_t K_t = (A_1 E_t K_t + A_1 \Delta y_t) + G E_t \sum_{j=1}^{\infty} z_{t-1+j}$$

$$\therefore \ E_t K_t = (I - A_1)^{-1} A_1 \Delta y_t + (I - A_1)^{-1} G E_t \sum_{j=1}^{\infty} z_{t-1+j} \tag{A4}$$

This makes sense since, if $G = 0$, then the shocks $e_{2t}$ have no permanent effects.

Now from (A2)

$$E_t \sum_{j=1}^{\infty} z_{t-1+j} = E_t (F \sum_{j=1}^{\infty} z_{t-2+j} + \Phi \sum_{j=1}^{\infty} \Delta y_{t-2+j} + \sum_{j=1}^{\infty} e_{2t+j-1}) = F E_t \sum_{j=1}^{\infty} z_{t-2+j} + \Phi E_t \sum_{j=1}^{\infty} \Delta y_{t-2+j} + e_{2t}. \tag{A5}$$

Using the same methodology as above we let $Q_t = \sum_{j=1}^{M} z_{t-1+j}$, $P_t = \sum_{j=1}^{M} z_{t-2+j}$, so that $P_t = Q_t + z_{t-1} - z_{t+M}$, enabling us to express (A5) as

$$E_t Q_t = F E_t Q_t + F z_{t-1} + E_t \Phi \sum_{j=1}^{\infty} \Delta y_{t-2+j} + e_{2t}$$

$$= F E_t Q_t + F z_{t-1} + \Phi E_t L_t + \Phi \Delta y_{t-1} + e_{2t}$$

$$\therefore E_t Q_t = (I - F)^{-1}(F z_{t-1} + \Phi E_t L_t + \Phi \Delta y_{t-1} + e_{2t})$$
$$= (I - F)^{-1}(F z_{t-1} + \Phi E_t K_t + \Phi \Delta y_t + \Phi \Delta y_{t-1} + e_{2t})$$

Now, replacing $\sum_{j=1}^{\infty} z_{t-1+j}$ by $Q_t$ in (A4), when $M \to \infty$, we get

$$E_t K_t = (I - A_1)^{-1} A_1 \Delta y_t + (I - A_1)^{-1} G E_t Q_t,$$

whereupon using the expression for $E_t Q_t$ gives

$$E_t K_t = (I - A_1)^{-1} A_1 \Delta y_t + (I - A_1)^{-1} G (I - F)^{-1}(F z_{t-1} + \Phi E_t K_t + \Phi \Delta y_t + \Phi \Delta y_{t-1} + e_{2t}).$$

Defining

$$R = (I - A_1)^{-1} G (I - F)^{-1} \tag{A6}$$

we have

$$E_t K_t = (I - A_1)^{-1} A_1 \Delta y_t + R(F z_{t-1} + \Phi E_t K_t + \Phi \Delta y_t + \Phi \Delta y_{t-1} + e_{2t})$$

and so

$$E_t K_t = (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi]\Delta y_t + (I - R\Phi)^{-1} R z_t \tag{A7}$$

We will apply these results to the two simple systems of section 2.3 in the text. The first system had the form

$$\Delta y_t = \delta z_t + \varepsilon_{1t} = \delta \gamma z_{t-1} + \varepsilon_{1t} + \delta \varepsilon_{2t} = \delta \gamma z_{t-1} + e_{1t}$$

$$z_t = \gamma z_{t-1} + \varepsilon_{2t} = \gamma z_{t-1} + e_{2t}.$$

Then $G = \delta \gamma$, $F = \gamma$, $A_1 = 0$, $\Phi = 0$, $R = G(I - F)^{-1}$ so that

$$E_t K_t = R z_t = \frac{\delta \gamma}{1 - \gamma} z_t$$

and

$$\Delta y_t^p = e_{1t} + \frac{\delta \gamma}{1 - \gamma} e_{2t} = \varepsilon_{1t} + \frac{\delta}{1 - \gamma} \varepsilon_{2t},$$

as we found in the text. Thus the second structural shock has a permanent effect.

Looking at the second system, we have

$$\Delta y_t = \delta \Delta z_t + \varepsilon_{1t} = \delta(\gamma - 1)z_{t-1} + \varepsilon_{1t} + \delta \varepsilon_{2t} = \delta(\gamma - 1)z_{t-1} + e_{1t}$$

$$z_t = \gamma z_{t-1} + \varepsilon_{2t} = \gamma z_{t-1} + e_{2t}.$$

Then $G = \delta(\gamma - 1)$, $F = \gamma$, $A_1 = 0$, $\Phi = 0$, $R = G(I - F)^{-1}$ so that

$$E_t K_t = R z_t = -\delta z_t$$

and

$$\Delta y_t^p = e_{1t} - \delta e_{2t} = \varepsilon_{1t}$$

which shows the second structural shock only has a transitory effect, as we said would occur because of the presence of $\Delta z_t$ in the structural equation (17) in the text.

**Appendix 2: Equivalence of the General Formula in Appendix 1 with the Case of Treating the I(0) Variable as Cointegrating with Itself**

Here we derive the expression for the change in permanent component of the I(1) variables from the general formula (A7) of appendix 1 and show that the expression one would use from cointegration analysis is equivalent to it. Recall that,

$$\Delta y_t = A_1 \Delta y_{t-1} + G z_{t-1} + e_{1t}$$
$$z_t = F z_{t-1} + \Phi \Delta y_{t-1} + e_{2t}$$
$$y_t^p = y_t + E_t K_t$$
$$E_t K_t = (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi]\Delta y_t + (I - R\Phi)^{-1} R z_t$$
where $R = (I - A_1)^{-1} G(I - F)^{-1}$

Now

$$\Delta y_t^p = \Delta y_t + E_t \Delta K_t$$
$$= \Delta y_t + (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi]\Delta^2 y_t + (I - R\Phi)^{-1} R \Delta z_t$$

and

$$\Delta^2 y_t = \Delta y_t - \Delta y_{t-1} = (A_1 - I)\Delta y_{t-1} + G z_{t-1} + e_{1t}$$

so that

$$\Delta y_t^p = A_1 \Delta y_{t-1} + G z_{t-1} + e_{1t} +$$

$$(I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi][(A_1 - I)\Delta y_{t-1} + G z_{t-1} + e_{1t}] + (I - R\Phi)^{-1} R\Delta z_t$$

$$= A_1 \Delta y_{t-1} + G z_{t-1} + e_{1t} + (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi][(A_1 - I)\Delta y_{t-1} + G z_{t-1} + e_{1t}] +$$

$$(I - R\Phi)^{-1} R[(F - I)z_{t-1} + \Phi\Delta y_{t-1} + e_{2t}]$$

First, collect terms in $\Delta y_{t-1}$

$$A_1 \Delta y_{t-1} + (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi](A_1 - I)\Delta y_{t-1} + (I - R\Phi)^{-1} R\Phi\Delta y_{t-1}$$

$$= A_1 \Delta y_{t-1} + (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 (A_1 - I)]\Delta y_{t-1} + (I - R\Phi)^{-1} R\Phi(A_1 - I)\Delta y_{t-1} + (I - R\Phi)^{-1} R\Phi\Delta y_{t-1}$$

$$= A_1 \Delta y_{t-1} + (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 (A_1 - I)]\Delta y_{t-1} + (I - R\Phi)^{-1} R\Phi A_1 \Delta y_{t-1}$$

$$= A_1 \Delta y_{t-1} + (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 (A_1 - I) + R\Phi A_1]\Delta y_{t-1}$$

$$= (I - R\Phi)^{-1}[(I - R\Phi)A_1 + (I - A_1)^{-1} A_1 (A_1 - I) + R\Phi A_1]\Delta y_{t-1}$$

$$= (I - R\Phi)^{-1}[A_1 + (I - A_1)^{-1} A_1 (A_1 - I)]\Delta y_{t-1}$$

$$= (I - R\Phi)^{-1}[A_1 - (I - A_1)^{-1} A_1 (I - A_1)]\Delta y_{t-1}$$

$$= (I - R\Phi)^{-1}(I - A_1)^{-1}[(I - A_1)A_1 - A_1(I - A_1)]\Delta y_{t-1}$$

$$= (I - R\Phi)^{-1}(I - A_1)^{-1}[A_1 - A_1^2 - A_1 + A_1^2]\Delta y_{t-1}$$

$$= (0)\Delta y_{t-1} = 0$$

Second, collect terms in $z_{t-1}$

$$(I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi]G z_{t-1} + G z_{t-1} + (I - R\Phi)^{-1} R[(F - I)z_{t-1}]$$

$$= \{G + (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi]G + (I - R\Phi)^{-1} R(F - I)\}z_{t-1}$$

$$= (I - R\Phi)^{-1}[(I - R\Phi)G + (I - A_1)^{-1} A_1 G + R\Phi G + R(F - I)]z_{t-1}$$

$$= (I - R\Phi)^{-1}[G + (I - A_1)^{-1} A_1 G + R(F - I)]z_{t-1}$$

$$= (I - R\Phi)^{-1}[G + (I - A_1)^{-1} A_1 G - R(I - F)]z_{t-1}$$

$$= (I - R\Phi)^{-1}[G + (I - A_1)^{-1} A_1 G - (I - A_1)^{-1} G(I - F)^{-1}(I - F)]z_{t-1}$$

$$= (I - R\Phi)^{-1}[G + (I - A_1)^{-1} A_1 G - (I - A_1)^{-1} G]z_{t-1}$$

$$= (I - R\Phi)^{-1}(I - A_1)^{-1}[(I - A_1)G + A_1 G - G]z_{t-1}$$

$$= (I - R\Phi)^{-1}(I - A_1)^{-1}[G - A_1 G + A_1 G - G]z_{t-1}$$

$$= (0)z_{t-1} = 0$$

Therefore, we are left with

$$\Delta y_t^p = e_{1t} + (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi]e_{1t} + (I - R\Phi)^{-1} R e_{2t}$$

$$= \{I + (I - R\Phi)^{-1}[(I - A_1)^{-1} A_1 + R\Phi]\}e_{1t} + (I - R\Phi)^{-1} R e_{2t}$$

$$= (I - R\Phi)^{-1}[(I - R\Phi) + (I - A_1)^{-1} A_1 + R\Phi]e_{1t} + (I - R\Phi)^{-1} R e_{2t}$$

$$= (I - R\Phi)^{-1}[I + (I - A_1)^{-1} A_1]e_{1t} + (I - R\Phi)^{-1} R e_{2t}$$

$$= (I - R\Phi)^{-1}(I - A_1)^{-1}[(I - A_1) + A_1]e_{1t} + (I - R\Phi)^{-1}Re_{2t}$$

$$= (I - R\Phi)^{-1}(I - A_1)^{-1}e_{1t} + (I - R\Phi)^{-1}Re_{2t}$$

$$= (I - R\Phi)^{-1}(I - A_1)^{-1}e_{1t} + (I - R\Phi)^{-1}(I - A_1)^{-1}G(I - F)^{-1}e_{2t}$$

$$= (I - R\Phi)^{-1}(I - A_1)^{-1}[e_{1t} + G(I - F)^{-1}e_{2t}]$$

which we report as (30) in the text.

In the ECM approach we would have

$$\begin{pmatrix} \Delta y_t^p \\ \Delta z_t^p \end{pmatrix} = \beta_\perp [\alpha_\perp' A(1)\beta_\perp]^{-1}\alpha_\perp' e_t$$

and, noting that $\beta_\perp = \begin{pmatrix} I \\ 0 \end{pmatrix}$, the permanent component of $y_t$ will be $[\alpha_\perp' A(1)\beta_\perp]^{-1}\alpha_\perp' e_t$. Because

$\alpha_\perp' e_t = e_{1t} + G(I - F)^{-1}e_{2t}$ we need to prove that $[\alpha_\perp' A(1)\beta_\perp]^{-1} = (I - R\Phi)^{-1}(I - A_1)^{-1}$ in order to establish equivalence between the two methods.

Now

$$(I - R\Phi)^{-1}(I - A_1)^{-1} = [(I - A_1)(I - R\Phi)]^{-1}$$

$$= [(I - A_1)(I - (I - A_1)^{-1}G(I - F)^{-1}\Phi)]^{-1}$$

$$= [(I - A_1) - G(I - F)^{-1}\Phi]^{-1}$$

Now $\beta' = \begin{bmatrix} 0 & I \end{bmatrix}$, $\beta_\perp = \begin{bmatrix} I \\ 0 \end{bmatrix}$, $A(1) = \begin{bmatrix} I - A_1 & 0 \\ -\Phi & I \end{bmatrix}$, $\alpha_\perp' = \begin{bmatrix} I & G(I - F)^{-1} \end{bmatrix}$ so that

$$[\alpha_\perp' A(1)\beta_\perp]^{-1} = \left[ \begin{pmatrix} I & G(I - F)^{-1} \end{pmatrix} \begin{pmatrix} I - A_1 & 0 \\ -\Phi & I \end{pmatrix} \begin{pmatrix} I \\ 0 \end{pmatrix} \right]^{-1} = \left[ \begin{pmatrix} I & G(I - F)^{-1} \end{pmatrix} \begin{pmatrix} I - A_1 \\ -\Phi \end{pmatrix} \right]^{-1}$$

$$= [I - A_1 - G(I - F)^{-1}\Phi]^{-1}$$

which completes the proof.