To bi, or not to bi? Differences in Spillover Estimates from Bilateral and Multilateral Multi-country Models[*]

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Abstract

Asymptotic analysis and Monte Carlo simulations show that spillover estimates obtained from widely-used bilateral (such as two-country VAR) models are significantly less accurate than those obtained from multilateral (such as global VAR) models. In particular, the accuracy of spillover estimates obtained from bilateral models depends on two aspects of economies' integration with the rest of the world. First, accuracy worsens as direct bilateral transmission channels become less important, for example when the spillover-sender accounts only for a small share of the spillover-recipient's overall integration with the rest of the world. Second, accuracy worsens as indirect higher-order spillovers and spillbacks become more important, for example when the spillover-recipient is more integrated with the rest of the world overall. Empirical evidence on the global output spillovers from US monetary policy is consistent with these generic results: Spillover estimates obtained from two-country VAR models are systematically smaller than those obtained from a global VAR model; and the differences between spillover estimates obtained from two-country VAR models and a global VAR model are more pronounced for economies for which the US accounts for a smaller share of their overall trade and financial integration with the rest of the world, and for economies which are more integrated with the rest of the world overall. The evidence in this paper thus suggests that widely-used two-country models systematically underestimate the magnitude of spillovers from domestic policies and shocks.

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1 Introduction

Over the last decades the global economy has witnessed a dramatic deepening of trade and financial integration. The resulting growing potential for cross-country spillovers has given impetus to academics and practitioners alike to estimate the magnitude of this international transmission of domestic shocks (see IMF, 2014). Examples for the prominence spillovers have gained recently are abundant, including the global effects of the exit from unconventional monetary policy in the US, the implications of the slowdown in China for world growth, or the concerns about the global fallout from the European sovereign debt crisis. Knowing how to estimate the magnitude of spillovers and identify economies which are particularly exposed to shocks from abroad has become critical for policymakers.

Essentially two modelling frameworks have been put forth for the empirical analysis of cross-country spillovers. On the one hand, a number of studies uses bilateral models which only consider the spillover-sender and the spillover-recipient. For example, several papers study the global spillovers from US monetary policy in two-country VAR models that include the US and one non-US economy at a time (Kim, 2001; Canova, 2005; Nobili and Neri, 2006; Mackowiak, 2007; Bluedorn and Bowdler, 2011; Ilzetzki and Jin, 2013).[1] Another set of papers has used two-country VAR models to study the impact of monetary policy on exchange rates (Eichenbaum and Evans, 1995; Cushman and Zha, 1997; Kim and Roubini, 2000; Faust and Rogers, 2003; Faust et al., 2003; Bjørnland, 2009; Voss and Willard, 2009). While bilateral models are easy to implement, they do not capture explicitly higher-order spillovers and spillbacks that reach the spillover-recipient through third and further economies. Despite not explicitly accounting for higher-order geographic channels, it is believed that bilateral models are still able to estimate spillovers consistently.

On the other hand, some studies use multilateral models which consider a large number of economies jointly. For example, the global VAR (GVAR) model developed by Pesaran et al. (2004) has also been used to study the global effects of US monetary policy considering a large number of non-US spillover-receiving economies simultaneously (Chen et al., 2012; Feldkircher and Huber, 2015; Georgiadis, forthcoming). In a similar vein, Canova and Ciccarelli (2009) put forth high-dimensional multi-country VAR models, which they suggest to estimate by Bayesian methods. Mumtaz and Surico (2009) consider an international factor-augmented VAR (FAVAR) model to study the effects of unanticipated rises in foreign interest rates on the UK economy. Moreover, a number of (semi-)structural multi-country models are being developed for the purpose of spillover analysis (Carabenciov et al., 2013; Vitek, 2014).[2]

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[1] Eickmeier (2007) considers a large-dimensional two-country factor model for the US and Germany in order to estimate the spillovers from US shocks. Jannsen and Klein (2011) as well as Kucharcukova et al. (2014) use two-country VAR models in order to examine the spillovers from euro area monetary policy shocks across economies in Europe. Kim and Shin (2015) consider a two-country panel VAR model to study the transmission of global liquidity to emerging market economies.

[2] Large Bayesian VAR models (Banbura et al., 2010) could also be used for spillover analysis in a multilateral

In contrast to bilateral models, multilateral models account for higher-order spillovers and spillbacks explicitly but are technically more difficult to implement, in particular as they are quickly subject to the curse of dimensionality.

The major conceptual difference between bilateral and multilateral models thus is that the former do not account—at least not explicitly—for higher-order, indirect spillover channels. As a consequence, spillover estimates from bilateral models might underestimate the magnitude of spillovers, and could possibly do so more for spillover-recipients which are more susceptible to higher-order, indirect spillovers. An empirical example which motivates this hypothesis is shown in Figure 1, which displays the global output spillovers from a contractionary US monetary policy shock as estimated from a GVAR model and two-country VAR models.[3] The spillover estimates obtained from the GVAR model are statistically and economically significantly larger than those obtained from the two-country VAR models. The literature has not investigated yet whether this difference is random and due to sampling uncertainty, or whether it reflects a systematic bias due to the mis-specification of bilateral models. This paper aims to fill this gap.

This paper advances our understanding of the empirical analysis of cross-country spillovers by investigating whether spillovers are estimated more accurately in a multilateral model than in bilateral models. The main result of the paper is that spillover estimates obtained from bilateral models are in general inconsistent asymptotically and less accurate than those obtained from a multilateral model in finite samples due to omitted variable bias and failure to account for higher-order transmission channels. Moreover, the accuracy of the spillover estimates obtained from bilateral models depends on the relative importance of direct bilateral and indirect higher-order spillover and spillback channels. In particular, spillover estimates obtained from bilateral models are particularly inaccurate relative to those obtained from a multilateral model when (i) the spillover-recipient is more integrated with the rest of the world overall, rendering it more susceptible to higher-order spillovers; and when (ii) the spillover-sender accounts only for a small share of the spillover-recipient's overall integration with the rest of the world, implying relatively less important direct bilateral spillovers.

I arrive at these conclusions in three steps. First, I explore whether the parameter and spillover estimates obtained from a bilateral model are asymptotically consistent if the true data-generating process is given by a multilateral model involving N economies—arguably the most plausible data-generating process for macroeconomic variables in an era of unprecedented trade and financial globalisation. The results suggest that the spillover estimates obtained from the bilateral model are in general inconsistent asymptotically due to omitted variable bias and failure to account for higher-order spillovers. Moreover, I find that the spillover-recipient's international integration properties determine the magnitude of the bias

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multi-country framework, but have so far to the best of my knowledge been applied only to study the domestic transmission of shocks.

[3]The underlying model specifications are discussed in more detail in Section 4.

in the spillover estimates obtained from the bilateral model. In particular, the bias rises with the spillover-recipient's overall integration with the rest of the world, and thereby its sensitivity to higher-order spillovers; and it falls with the relative importance of the spillover-sender in the spillover-recipient's overall integration with the rest of the world, and thereby the relative importance of direct bilateral spillovers.

Second, in order to evaluate the properties of spillover estimates obtained from bilateral models in finite samples and to assess how a bilateral model may be expected to perform relative to the alternative of a multilateral model I carry out a Monte Carlo experiment. Specifically, I simulate data based on a multilateral data-generating process and estimate spillovers using bilateral two-country VAR models and multilateral GVAR or factor-augmented VAR models. Consistent with the asymptotic results, I find that the finite sample bias of the spillover estimates obtained from the bilateral model rises relative to that from the multilateral model with the spillover-recipient's overall integration with the rest of the world, and that it decreases with the relative importance of the spillover-sender in the spillover-recipient's overall integration.[4] I obtain these Monte Carlo results both for simulations with data-generating processes based on a reduced-form and a structural macroeconomic multi-country model.

Finally, I illustrate the possible practical consequences of using bilateral models instead of a multilateral model by estimating the global output spillovers from US monetary policy using two-country VAR models and a GVAR model. Specifically, I find that the GVAR model produces spillover estimates which are economically and statistically significantly larger than those from the two-country VAR models. In line with the asymptotic and Monte Carlo results, the differences between the spillover estimates obtained from the two-country VAR models and the GVAR model are larger for economies which are more integrated with the rest of the world overall, and for which the US accounts for a smaller share in their overall integration. Moreover, consistent with the hypothesis that higher-order spillovers and spillbacks are not captured well by bilateral models, I also find that the differences between the spillover estimates obtained from the two-country VAR models and the GVAR model are also larger for economies which are centrally located in the global trade network; whose trade is concentrated on economies which trade substantially with the US; which are located upstream in the global value chain; and for which direct bilateral spillovers are less important due to higher transaction costs as measured by the distance to the US or in the presence of a flexible exchange rate vis-à-vis the US dollar.

This paper is related to existing work. First, Chudik and Pesaran (2011) consider the estimation of VAR models in which both $N \longrightarrow \infty$ and $T \longrightarrow \infty$. Specifically, they assume that economies $2, 3, \ldots, N$ can be classified either as "neighbours" or "non-neighbours" of economy 1 based on the magnitude of their effect on economy 1 as $N \longrightarrow \infty$: While the

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[4]These results are consistent with the finding of Dovern et al. (2015) that multilateral models produce more accurate macroeconomic forecasts, in particular due to capturing spillover effects.

impact of each individual neighbour does not vanish as $N \longrightarrow \infty$, the effect of *individual* non-neighbours vanishes, even if their *joint* effect does not vanish in the limit. The main result of Chudik and Pesaran (2011) is that the neighbourhood effects can be estimated consistently in a model which omits the non-neighbour economies. While the work of Chudik and Pesaran (2011) suggests that under specific conditions it is admissible to disregard some economies from the empirical model, they do not recommend bilateral models in general. Specifically, their results suggest that when the set of neighbours comprises more than a single economy the appropriate model framework for spillover analysis is multilateral. Moreover, in order for the estimates of the effects of neighbour economies to be consistent it is critical to know which economies are non-neighbours, which suggests one should be cautious in omitting economies from the model. Relative to the work of Chudik and Pesaran (2011) this paper studies the properties of the bias in spillover estimates that arises when bilateral models that disregard economies without pondering whether the latter are non-neighbours or not are used for spillover analysis. Moreover, in this paper the focus is on settings with fixed N, arguably an empirically more relevant context than $N \longrightarrow \infty$.

Second, Chudik and Straub (forthcoming) investigate the role of trade integration for an economy's sensitivity to foreign shocks and the relationship to the widely-used small open-economy concept in international macroeconomics. In particular, Chudik and Straub (forthcoming) consider a structural multi-country model in which they let the number of economies $N \longrightarrow \infty$, finding that the diversification of economies' trade across trading partners is critical for their international macroeconomic interdependence. More specifically, if an economy diversifies its trade across partners and no economy is regionally or globally dominant, then as $N \longrightarrow \infty$ the equilibrium solution for the domestic endogenous variables does not depend on the idiosyncratic shocks in foreign economies. In contrast, if some economies are regionally or globally dominant, then it is not admissible to treat economies individually and as if they were closed; instead, sets of economies need to be modelled jointly in a multilateral framework based on the structure of direct bilateral and indirect higher-order trade linkages. The implications of the analysis of Chudik and Straub (forthcoming) for the choice of empirical spillover modelling frameworks are thus consistent with those of this paper. The major differences of this paper relative to Chudik and Straub (forthcoming) are that I study the properties of the bias in the spillover estimates obtained from bilateral models in the context of fixed N, and in particular the role of economies' integration with the rest of the world and the relative strength of bilateral country linkages therein.

It is also worthwhile to distinguish this paper from existing work that has studied the identification of structural shocks in under-specified empirical models (Bernanke et al., 2005; Christiano et al., 2005; Stock and Watson, 2005; Giannone and Reichlin, 2006; Canova and Ciccarelli, 2013; Forni and Gambetti, 2014). Specifically, in order to isolate the effect of using under-specified bilateral rather than multilateral models that do not account explicitly for higher-order spillovers and spillbacks for the global *propagation* of shocks beyond the prob-

lems of identification, in this paper I assume that the structural shock has been identified correctly. Technically, I implement this assumption by examining the spillovers from shocks to a variable that is exogenous to the system. Moreover, the empirical setting that motivates the analysis in this paper refers to the global economy and the global propagation of shocks through cross-country spillovers, rather than to the transmission of country-specific shocks across different variables within the domestic economy as typically studied in this literature.

The rest of the paper is organised as follows. Section 2 derives the probability limit of the parameter estimates in the bilateral model. In Section 3 I carry out a Monte Carlo experiment to assess the relative accuracy of spillover estimates obtained from bilateral and multilateral models. Section 4 illustrates the practical differences between spillover estimates from bilateral and multilateral models using two-country VAR models and a GVAR model for the case of the global impact of US monetary policy shocks. Finally, Section 5 concludes.

2 Asymptotic Analysis

2.1 Conceptual Framework

Consider a data-generating process given by a stationary multilateral VAR model

$$
\begin{bmatrix} x_{1t} \\ x_{2t} \\ \boldsymbol{x}_{3t} \end{bmatrix} \equiv \begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{z}_t \end{bmatrix} \equiv \boldsymbol{y}_t = \boldsymbol{\Gamma}_0 \boldsymbol{y}_t + \boldsymbol{\Gamma}_1 \boldsymbol{y}_{t-1} + \boldsymbol{\Psi} s_t + \boldsymbol{\nu}_t, \quad \boldsymbol{\nu}_t \overset{i.i.d.}{\sim} (\boldsymbol{0}, \boldsymbol{I}), \text{ and } s_t \overset{i.i.d.}{\sim} (0, \sigma_s^2), \quad (1)
$$

where $\boldsymbol{z}_t = \boldsymbol{x}_{3t}$, $\boldsymbol{x}_t = (x_{1t}, x_{2t})'$, x_{1t} and x_{2t} are scalar variables of economies 1 and 2, $\boldsymbol{x}_{3t}$ is an $(N-2)$-dimensional vector of variables pertaining to economies $3, 4, \ldots, N$, $\boldsymbol{\Psi} = (1, 0, \ldots, 0)'$, and $Cov(s_t, \boldsymbol{\nu}_t) = \boldsymbol{0}$. I consider an exogenous variable s_t as a shock in economy 1 in order to abstract from issues of identification of structural shocks. It is important to clarify the role of the latter modelling choice. Specifically, in this paper I intend to examine the consequences of considering an under-specified bilateral rather than a multilateral model for the estimation of the global *propagation* of shocks. The assumption that the shock s_t has been identified correctly is critical in order to isolate problems in the estimation of the *propagation* of shocks from issues of *identification* in under-specified models. In practice, the structural shocks are typically not known and have to be identified. As a result, the problems of identification in under-specified bilateral models will exacerbate those that arise from their use for the estimation of the propagation of the shocks.

The reduced form of the model in Equation (1) is given by

$$
\begin{aligned}
(\boldsymbol{I} - \boldsymbol{\Gamma}_0)\boldsymbol{y}_t &= \boldsymbol{\Gamma}_1 \boldsymbol{y}_{t-1} + \boldsymbol{\Psi} s_t + \boldsymbol{\nu}_t \\
\boldsymbol{y}_t &= (\boldsymbol{I} - \boldsymbol{\Gamma}_0)^{-1} \boldsymbol{\Gamma}_1 \boldsymbol{y}_{t-1} + (\boldsymbol{I} - \boldsymbol{\Gamma}_0)^{-1} \boldsymbol{\Psi} s_t + (\boldsymbol{I} - \boldsymbol{\Gamma}_0)^{-1} \boldsymbol{\nu}_t \\
&= \boldsymbol{\Phi} \boldsymbol{y}_{t-1} + \boldsymbol{\Omega} s_t + \boldsymbol{u}_t,
\end{aligned}
\tag{2}
$$

with $\boldsymbol{u}_t \overset{i.i.d.}{\sim} (\boldsymbol{0}, \boldsymbol{\Sigma}^u)$, $\boldsymbol{\Sigma}^u = (\boldsymbol{I} - \boldsymbol{\Gamma}_0)^{-1}(\boldsymbol{I} - \boldsymbol{\Gamma}_0)^{-1\prime}$. For future reference define

$$
\boldsymbol{\Sigma}^y = \begin{bmatrix} \boldsymbol{\Sigma}^y_{xx} & \boldsymbol{\Sigma}^y_{xz} \\ \boldsymbol{\Sigma}^y_{zx} & \boldsymbol{\Sigma}^y_{zz} \end{bmatrix} \equiv Var(\boldsymbol{y}_t) = \sum_{j=0}^{\infty} \boldsymbol{\Phi}^j \boldsymbol{\Omega} \boldsymbol{\Omega}' \boldsymbol{\Phi}^{j\prime} \cdot \sigma_s^2 + \sum_{j=0}^{\infty} \boldsymbol{\Phi}^j \boldsymbol{\Sigma}^u \boldsymbol{\Phi}^{j\prime},
\tag{3}
$$

and

$$
\begin{bmatrix} \boldsymbol{\Sigma}_{xs} \\ \boldsymbol{\Sigma}_{zs} \end{bmatrix} \equiv \begin{bmatrix} Cov(\boldsymbol{x}_t, s_t) \\ Cov(\boldsymbol{z}_t, s_t) \end{bmatrix} = \boldsymbol{\Omega} \sigma_s^2.
\tag{4}
$$

For the parameter matrices in Equation (2) assume the partitions

$$
\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_{xx} & \boldsymbol{\Phi}_{xz} \\ \boldsymbol{\Phi}_{zx} & \boldsymbol{\Phi}_{zz} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_x \\ \boldsymbol{\Omega}_z \end{bmatrix}.
\tag{5}
$$

Notice that in the context of studying spillovers from shocks in economy 1 to economy 2 the matrix $\boldsymbol{\Phi}_{xz}$ is critical, as it reflects the existence of higher-order spillovers and spillbacks: Shocks to economy 1 affect economies $3, 4, \ldots, N$ through $\boldsymbol{\Omega}_z$ and $\boldsymbol{\Phi}_{zx}$, and then spillover and back to economies 1 and 2 if $\boldsymbol{\Phi}_{xz} \neq \boldsymbol{0}$ in second and further rounds. In the following I assume that $\boldsymbol{\Phi}_{xz} \neq \boldsymbol{0}$, which excludes the trivial case in which there are no higher-order spillovers.

A typical object of interest in empirical applications is the impulse response function of the endogenous variables $\boldsymbol{y}_t$, which also represents the spillovers in an international context. Specifically, for the multilateral VAR model in Equation (2) the impulse response functions to the exogenous variable s_t in economy 1 are given by

$$
IRF(h) = \begin{bmatrix} IRF_x(h) \\ IRF_z(h) \end{bmatrix} \equiv \begin{bmatrix} \frac{\partial \boldsymbol{x}_{t+h}}{\partial s_t} \\ \frac{\partial \boldsymbol{z}_{t+h}}{\partial s_t} \end{bmatrix} = \boldsymbol{\Phi}^h \boldsymbol{\Omega}, \quad h = 0, 1, 2, \ldots .
\tag{6}
$$

Now suppose that rather than estimating the full N-dimensional multilateral VAR model in Equation (2), a smaller bilateral VAR model in which the variables of economies $3, 4, \ldots, N$ in $\boldsymbol{z}_t$ are omitted is considered. Specifically, using the partitions in Equation (5), consider the bilateral VAR model

$$
\boldsymbol{x}_t = \boldsymbol{\Phi}_{xx} \boldsymbol{x}_{t-1} + \boldsymbol{\Omega}_x s_t + \underbrace{(\boldsymbol{u}_t^x + \boldsymbol{\Phi}_{xz} \boldsymbol{z}_{t-1})}_{\equiv \boldsymbol{\epsilon}_t},
\tag{7}
$$

6

with impulse response functions

$$IRF^{bl}(h) = \mathbf{\Phi}_{xx}^h \mathbf{\Omega}_x. \tag{8}$$

The main question of this paper is whether the impulse response functions for a shock in the spillover-sending economy 1 to the spillover-receiving economy 2 in Equation (6) can be estimated consistently in the bilateral VAR model in Equation (7), that is whether

$$\plim_{T \to \infty} \widehat{IRF}^{bl}(h) = IRF_x(h). \tag{9}$$

2.2 Consistency of Spillover Estimates in the Bilateral Model

Notice that from Equations (5) and (6) it follows that the true spillovers are given by

$$
\begin{align}
IRF_x(0) &= \mathbf{\Omega}_x, \tag{10}\\
IRF_x(1) &= \mathbf{\Phi}_{xx}\mathbf{\Omega}_x + \mathbf{\Phi}_{xz}\mathbf{\Omega}_z, \tag{11}\\
IRF_x(2) &= (\mathbf{\Phi}_{xx}^2 + \mathbf{\Phi}_{xz}\mathbf{\Phi}_{zx})\mathbf{\Omega}_x + (\mathbf{\Phi}_{xx}\mathbf{\Phi}_{xz} + \mathbf{\Phi}_{xz}\mathbf{\Phi}_{zz})\mathbf{\Omega}_z, \tag{12}\\
IRF_x(3) &= (\mathbf{\Phi}_{xx}^3 + \mathbf{\Phi}_{xz}\mathbf{\Phi}_{zx}\mathbf{\Phi}_{xx} + \mathbf{\Phi}_{xx}\mathbf{\Phi}_{xz}\mathbf{\Phi}_{zx} + \mathbf{\Phi}_{xz}\mathbf{\Phi}_{zz}\mathbf{\Phi}_{zx})\mathbf{\Omega}_x \\
&\quad + (\mathbf{\Phi}_{xx}^2\mathbf{\Phi}_{xz} + \mathbf{\Phi}_{xz}\mathbf{\Phi}_{zx}\mathbf{\Phi}_{xz} + \mathbf{\Phi}_{xx}\mathbf{\Phi}_{xz}\mathbf{\Phi}_{zz} + \mathbf{\Phi}_{xz}\mathbf{\Phi}_{zz}^2)\mathbf{\Omega}_z. \tag{13}\\
&\vdots
\end{align}
$$

Equations (10) to (13) suggest that consistency of the parameter estimates in the bilateral model implies inconsistent spillover estimates, that is

$$\plim_{T \to \infty} \widehat{\mathbf{\Phi}}_{xx} = \mathbf{\Phi}_{xx} \;\wedge\; \plim_{T \to \infty} \widehat{\mathbf{\Omega}}_x = \mathbf{\Omega}_x \implies \plim_{T \to \infty} \widehat{IRF}^{bl}(h) \neq IRF_x(h), \quad h > 0. \tag{14}$$

The intuition for this finding is that using the true values for $\mathbf{\Phi}_{xx}$ and $\mathbf{\Omega}_x$ for calculating the spillovers in the bilateral model according to Equation (8) does not yield the true spillovers, because the higher-order spillovers arising through the terms involving $\mathbf{\Phi}_{xz}$ in the true spillovers in Equation (6) are not accounted for. In other words, using the true parameter values results in the wrong spillovers in the bilateral model due to failure to account for higher-order spillovers and spillbacks. However, if the parameter estimates obtained from the bilateral model were inconsistent, it could in principle be that the asymptotic bias is such that it offsets the bias in the spillover estimates that arises due to the failure to account for higher-order spillovers and spillbacks. In order to determine whether the spillover estimates obtained from the bilateral model are consistent, it is thus crucial to determine the probability limits of the parameter estimates in Equation (7).

Denoting by

$$X \equiv \begin{bmatrix} \boldsymbol{x}_0, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_{T-1} \\ s_1, s_2, \ldots, s_T \end{bmatrix}, \quad Y \equiv [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_T], \quad \boldsymbol{\epsilon} \equiv [\boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2, \ldots, \boldsymbol{\epsilon}_T], \quad \boldsymbol{B} \equiv [\boldsymbol{\Phi}_{xx}, \boldsymbol{\Omega}_x],$$

the ordinary least squares estimator of the bilateral VAR model in Equation (7) delivers

$$
\begin{aligned}
\widehat{\boldsymbol{B}} &= \boldsymbol{Y}\boldsymbol{X}'(\boldsymbol{X}\boldsymbol{X}')^{-1} = \boldsymbol{B} + \boldsymbol{\epsilon}\boldsymbol{X}'(\boldsymbol{X}\boldsymbol{X}')^{-1} \\
&= \boldsymbol{B} + \left[\sum \boldsymbol{\epsilon}_t \boldsymbol{x}'_{t-1}, \sum \boldsymbol{\epsilon}_t s_t \right] \begin{bmatrix} \sum \boldsymbol{x}_{t-1}\boldsymbol{x}'_{t-1} & \sum \boldsymbol{x}_{t-1}s_t \\ \sum \boldsymbol{x}'_{t-1}s_t & \sum s_t^2 \end{bmatrix}^{-1} \\
&= \boldsymbol{B} + \left[\sum (\boldsymbol{u}_t^x + \boldsymbol{\Phi}_{xz}\boldsymbol{z}_{t-1})\boldsymbol{x}'_{t-1}, \sum (\boldsymbol{u}_t^x + \boldsymbol{\Phi}_{xz}\boldsymbol{z}_{t-1})s_t \right] \begin{bmatrix} \sum \boldsymbol{x}_{t-1}\boldsymbol{x}'_{t-1} & \sum \boldsymbol{x}_{t-1}s_t \\ \sum \boldsymbol{x}'_{t-1}s_t & \sum s_t^2 \end{bmatrix}^{-1} \quad (15)
\end{aligned}
$$

with summations running from $t = 1$ to T. Under standard assumptions for stationary VAR models (see Lütkepohl, 2007, chpt. 3) we have

$$\underset{T \to \infty}{plim} \; \widehat{\boldsymbol{B}} = \boldsymbol{B} + [\boldsymbol{\Phi}_{xz}\boldsymbol{\Sigma}_{zx}^y, \boldsymbol{0}] \begin{bmatrix} \boldsymbol{\Sigma}_{xx}^y & \boldsymbol{\Sigma}_{xs} \\ \boldsymbol{\Sigma}_{sx} & \sigma_s^2 \end{bmatrix}^{-1}. \quad (16)$$

Applying the partitioned inverse we obtain

$$\underset{T \to \infty}{plim} \; \widehat{\boldsymbol{\Phi}}_{xx} = \boldsymbol{\Phi}_{xx} + \boldsymbol{\Phi}_{xz}\boldsymbol{\Sigma}_{zx}^y \left(\boldsymbol{\Sigma}_{xx}^y - \boldsymbol{\Sigma}_{xs}\boldsymbol{\Sigma}_{sx}\sigma_s^{-2} \right)^{-1}, \quad (17)$$

$$\underset{T \to \infty}{plim} \; \widehat{\boldsymbol{\Omega}}_x = \boldsymbol{\Omega}_x - \boldsymbol{\Phi}_{xz}\boldsymbol{\Sigma}_{zx}^y (\boldsymbol{\Sigma}_{xx}^y)^{-1} \boldsymbol{\Sigma}_{xs} \left[\sigma_s^2 - \boldsymbol{\Sigma}_{sx}(\boldsymbol{\Sigma}_{xx}^y)^{-1}\boldsymbol{\Sigma}_{xs} \right]^{-1}. \quad (18)$$

Equations (17) and (18) suggest that the parameter estimates obtained from the bilateral VAR model are in general inconsistent asymptotically: In the presence of higher-order spillovers, $\boldsymbol{\Phi}_{xz} \neq 0$, the error term $\boldsymbol{\epsilon}_t$ in Equation (7) and $\boldsymbol{x}_{t-1}$ are correlated due to the omission of the rest of the world $\boldsymbol{z}_{t-1}$.[5] Moreover, and more importantly, plugging in the probability limits in Equations (17) and (18) in Equations (10) to (12) it can easily be seen that it is *not* the case that the asymptotic bias in the parameter estimates for $\boldsymbol{\Phi}_{xx}$ and $\boldsymbol{\Omega}_x$ in the bilateral model is such that it compensates for the failure to account for higher-order spillovers and spillbacks in the calculation of the spillovers according to Equation (8) rather than Equation (6). As a result, in the presence of higher-order spillovers the spillover estimates obtained from a bilateral model are inconsistent due to omitted variable bias and the failure to account for higher-order spillovers and spillbacks.

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[5]In the context of the forward premium puzzle a similar omitted variable bias is discussed in Binder et al. (2010).

2.3 Determinants of the Asymptotic Bias

If we think of the multilateral model in Equation (1) as a macroeconomic model of the world economy, a natural question to ask is how the inconsistency in the spillover estimates obtained from the bilateral VAR model in Equations (17) and (18) is related to economies' international integration patterns. In order to shed light on this question, consider the contemporaneous impact matrix $\mathbf{\Gamma}_0$ from the structural form of the model in Equation (1), define $\gamma_{0,ij} \equiv [\mathbf{\Gamma}_0]_{ij}$, $\bar{\gamma}_{0,i} \equiv \sum_j \gamma_{0,ij}$ and $w_{ij} \equiv \gamma_{0,ij}/\bar{\gamma}_{0,i}$, and write

$$
\begin{aligned}
\mathbf{\Gamma}_0 &= \begin{bmatrix} 0 & \gamma_{0,12} & \cdots & \gamma_{0,1N} \\ \gamma_{0,21} & 0 & & \gamma_{0,2N} \\ \vdots & & \ddots & \vdots \\ \gamma_{0,N1} & \cdots & \gamma_{0,NN-1} & 0 \end{bmatrix} \\[2mm]
&= \begin{bmatrix} 0 & \bar{\gamma}_{0,1}\gamma_{0,12}/\bar{\gamma}_{0,1} & \cdots & \bar{\gamma}_{0,1}\gamma_{0,1N}/\bar{\gamma}_{0,1} \\ \bar{\gamma}_{0,2}\gamma_{0,21}/\bar{\gamma}_{0,2} & 0 & & \bar{\gamma}_{0,2}\gamma_{0,2N}/\bar{\gamma}_{0,2} \\ \vdots & & \ddots & \vdots \\ \bar{\gamma}_{0,N}\gamma_{0,N1}/\bar{\gamma}_{0,N} & \cdots & \bar{\gamma}_{0,N}\gamma_{0,NN-1}/\bar{\gamma}_{0,N} & 0 \end{bmatrix} \\[2mm]
&= \begin{bmatrix} 0 & \bar{\gamma}_{0,1}w_{12} & \cdots & \bar{\gamma}_{0,1}w_{1N} \\ \bar{\gamma}_{0,2}w_{21} & 0 & & \bar{\gamma}_{0,2}w_{2N} \\ \vdots & & \ddots & \vdots \\ \bar{\gamma}_{0,N}w_{N1} & \cdots & \bar{\gamma}_{0,N}w_{2N-1} & 0 \end{bmatrix} \\[2mm]
&= \begin{bmatrix} 0 & w_{12} & \cdots & w_{1N} \\ w_{21} & 0 & & w_{2N} \\ \vdots & & \ddots & \vdots \\ w_{N1} & \cdots & w_{2N-1} & 0 \end{bmatrix} \odot \begin{bmatrix} \bar{\gamma}_{0,1} & \bar{\gamma}_{0,1} & \cdots & \bar{\gamma}_{0,1} \\ \bar{\gamma}_{0,2} & \bar{\gamma}_{0,2} & \cdots & \bar{\gamma}_{0,2} \\ \vdots & \vdots & \vdots & \vdots \\ \bar{\gamma}_{0,N} & \bar{\gamma}_{0,N} & \cdots & \bar{\gamma}_{0,N} \end{bmatrix},
\end{aligned} \tag{19}
$$

where $\odot$ represents element-wise multiplication. Thus, without loss of generality we can rewrite the matrices $\mathbf{\Gamma}_\ell$ in Equation (1) as

$$
\mathbf{\Gamma}_0 = \boldsymbol{W} \odot \left(\boldsymbol{\iota}' \otimes \boldsymbol{\gamma}_0 \right), \tag{20}
$$

$$
\mathbf{\Gamma}_1 = \mathbf{\Gamma}_1^{(d)} + \boldsymbol{W} \odot \left(\boldsymbol{\iota}' \otimes \boldsymbol{\gamma}_1 \right), \tag{21}
$$

where $\boldsymbol{\iota}$ is an $N \times 1$ vector of ones, $[\boldsymbol{\gamma}_\ell]_i \equiv \bar{\gamma}_{\ell,i}$, $\mathbf{\Gamma}_1^{(d)}$ is a diagonal matrix, and the matrix $\boldsymbol{W}$, $[\boldsymbol{W}]_{ij} = w_{ij}$, has zeros on its diagonal and its row sum is unity. The matrix $\boldsymbol{W}$ thus reflects a bilateral weight matrix and w_{ij} the importance of economy j to economy i relative to the other economies $k \neq j$. For example, w_{ij} could be related to the share of economy j in economy i's overall trade and financial integration with the rest of the world. In turn, $[\boldsymbol{\gamma}_\ell]_i$ reflects the overall sensitivity of economy i to developments in the rest of the world. For example, $[\boldsymbol{\gamma}_\ell]_i$ could be related to economy i's overall trade and financial integration with the

rest of the world.

The assumptions in Equations (20) and (21) imply

$$\mathbf{\Phi}_{xz} = \mathbf{\Phi}_{xz}(\mathbf{W}, \boldsymbol{\gamma}_0, \boldsymbol{\gamma}_1), \tag{22}$$

$$\mathbf{\Sigma}_{zx}^y = \mathbf{\Sigma}_{zx}^y(\mathbf{W}, \boldsymbol{\gamma}_0, \boldsymbol{\gamma}_1, \mathbf{\Psi}, \mathbf{\Sigma}_u, \sigma_s^2), \tag{23}$$

$$\mathbf{\Sigma}_{xx}^y = \mathbf{\Sigma}_{xx}^y(\mathbf{W}, \boldsymbol{\gamma}_0, \boldsymbol{\gamma}_1, \mathbf{\Psi}, \mathbf{\Sigma}_u, \sigma_s^2), \tag{24}$$

$$\mathbf{\Sigma}_{sx} = \mathbf{\Sigma}_{sx}(\mathbf{W}, \boldsymbol{\gamma}_0, \boldsymbol{\gamma}_1, \mathbf{\Psi}, \sigma_s^2). \tag{25}$$

The bias in the estimates $\widehat{\mathbf{\Phi}}_{xx}$ and $\widehat{\mathbf{\Omega}}_x$ obtained from the bilateral VAR model in Equations (17) and (18) thus depends on economies' bilateral integration patterns reflected by the weight matrix $\mathbf{W}$, and on their overall sensitivity to developments in the rest of the world reflected by $\boldsymbol{\gamma}_\ell$, $\ell = 0, 1$.

While the relationships in Equations (22) to (25) are too complex to read off directly the impact of $\mathbf{W}$ and $\boldsymbol{\gamma}_\ell$ on the asymptotic bias in the spillover estimates obtained from the bilateral model, it can be explored numerically. The effects of differences in economy 2's overall integration with the rest of the world and the relative importance of economy 1 therein on the asymptotic bias in the spillover estimates can be gauged by varying $[\boldsymbol{\gamma}_\ell]_2$ and w_{21}. Specifically, consider the parametrisation

$$[\mathbf{\Gamma}_1^{(d)}]_{ii} \sim N(0.6, 0.05^2), \tag{26}$$

$$[\boldsymbol{\gamma}_\ell]_2 = \bar{\gamma} \quad \text{for } \ell = 0, 1, \tag{27}$$

$$[\boldsymbol{\gamma}_0]_i \sim N\left(0.1, 0.025^2\right) \quad \text{for } i \neq 2, \tag{28}$$

$$[\boldsymbol{\gamma}_1]_i \sim N\left(0.2, 0.025^2\right) \quad \text{for } i \neq 2, \tag{29}$$

$$s_t \sim N(0, 1^2), \tag{30}$$

$$w_{21} = \bar{\omega}, \tag{31}$$

$$w_{ij} = \widetilde{w}_{ij} / \sum_j \widetilde{w}_{ij}, \ \ \widetilde{w}_{ij} \sim N(1/N, N^{-2}), \ \ \widetilde{w}_{ij} \geq 0, \ \text{for } i \neq 2 \wedge j \neq 1. \tag{32}$$

Figure 2 displays the true impulse response functions to a shock s_t in economy 1 based on the parametrisation in Equations (26) to (32). In particular, Figure 2 displays the domestic effects in the spillover-sending economy 1 and the spillovers to economy 2 for small and large values of $\bar{\gamma}$ and $\bar{\omega}$. The magnitudes of the spillovers range from being hardly discernible to being as large as the domestic effects in the spillover-sender. Thus, the parametrisation in Equations (26) to (32) produces an empirically relevant quantitative range of spillovers.

Based on the parametrisation in Equations (26) to (32) and the probability limits of the parameter estimates in Equations (17) and (18), I calculate the asymptotic bias in the spillover estimates obtained from the bilateral model for different values of $\bar{\gamma}$ and $\bar{\omega}$. In particular, denote by $IRF_{21}(h)$ the true impulse response function of the spillover-receiving economy 2

to the shock s_t in the spillover-sending economy 1 at horizon h. I consider the asymptotic
bias over all impulse response horizons and at a fixed horizon $\bar{h}$

$$bias_{average}^{asympt} \quad = \quad H^{-1} \sum_{h=1}^{H} \left[\plim_{T \to \infty} \widehat{IRF}_{21}^{bl}(h) - IRF_{21}(h) \right] \bigg/ \sum_{h=1}^{H} IRF_{21}(h), \qquad (33)$$

$$bias_{fixhor}^{asympt} \quad = \quad \left[\plim_{T \to \infty} \widehat{IRF}_{21}^{bl}(\bar{h}) - IRF_{21}(\bar{h}) \right] \bigg/ IRF_{21}(\bar{h}). \qquad (34)$$

Figure 3 suggests that the different versions of the asymptotic bias in the spillover esti-
mates obtained from the bilateral model rise monotonously with rising $\bar{\gamma}$, i.e. when the
spillover-receiving economy 2's overall sensitivity to developments in the rest of the world
rises. Moreover, the different versions of the asymptotic bias rise with falling $\bar{\omega}$, i.e. when
the spillover-sending economy 1 accounts for a decreasing share of the spillover-receiving
economy 2's overall integration with the rest of the world. These results are consistent with
the hypothesis that as a spillover-recipient's overall integration with the rest of the world
rises, the spillovers it receives increasingly occur through indirect higher-order spillovers and
spillbacks, which a bilateral model fails to capture. Moreover, the results are consistent with
the hypothesis that as the spillover-sender's importance in the spillover-recipient's overall
integration with the rest of the world rises, spillovers occur less through indirect and more
through direct bilateral channels.

These asymptotic results provide some indications regarding the pitfalls of using bilateral
models for spillover analysis. However, for empirical applications it is important to under-
stand whether finite sample issues exacerbate the asymptotic bias, how it depends on the
sample size N and T, and how spillover estimates obtained from bilateral models can be
expected to perform relative to those obtained from multilateral models. In the next section
I consider a Monte Carlo experiment to shed light on these questions.

3   Finite Sample Evidence

I carry out a Monte Carlo experiment in which I generate data from a multilateral VAR
model and estimate the spillovers from shocks that occur in economy 1 to economy 2 using a
bilateral VAR model and a multilateral GVAR model. The data-generating process is given by
Equation (1), Equations (20) and (21), and the parametrisation in Equations (26) to (32). As
in the analysis of the determinants of the asymptotic bias in Section 2.3, I consider variations
in the parametrisation regarding the overall sensitivity of the spillover-receiving economy 2
to developments in all other economies reflected by $[\boldsymbol{\gamma}_\ell]_2$, and the relative importance of the
spillover-sending economy 1 in spillover-receiving economy 2's overall integration with the
rest of the world reflected by w_{21}.

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3.1 The Bilateral Model

The bilateral model I estimate on the simulated data is given by

$$\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} + \boldsymbol{B}s_t + \boldsymbol{e}_t. \tag{35}$$

The impulse response functions for the bilateral model are given by

$$IRF^{bl}(h) \quad = \quad \boldsymbol{A}^h \boldsymbol{B}. \tag{36}$$

3.2 The Multilateral Model

For the multilateral model I consider a GVAR model. In particular, for each economy I estimate a country-specific model

$$x_{it} = a_{ii}x_{i,t-1} + a_{0,i}^* x_{it}^* + a_{1,i}^* x_{i,t-1}^* + b_i s_t + e_{it}, \tag{37}$$

where $x_{it}^* \equiv \sum_j w_{ij} x_{jt}$. Each economy's model in Equation (37) can be re-written as

$$[1, -a_{0,i}^*] \begin{bmatrix} x_{it} \\ x_{it}^* \end{bmatrix} \quad = \quad [a_{ii}, a_{1,i}^*] \begin{bmatrix} x_{i,t-1} \\ x_{i,t-1}^* \end{bmatrix} + b_i s_t + e_{it},$$

$$[1, -a_{0,i}^*] \boldsymbol{L}_i \boldsymbol{y}_t \quad = \quad [a_{ii}, a_{1,i}^*] \boldsymbol{L}_i \boldsymbol{y}_{t-1} + b_i s_t + e_{it}, \tag{38}$$

where $\boldsymbol{L}_i$ are link matrices that contain the weights w_{ij} for the construction of the "foreign" variables so that $(x_{it}, x_{it}^*)' = \boldsymbol{L}_i \boldsymbol{y}_t$. In stacked form the GVAR model is given by

$$\begin{bmatrix} (1, -a_{0,1}^*)\boldsymbol{L}_1 \\ (1, -a_{0,2}^*)\boldsymbol{L}_2 \\ \vdots \\ (1, -a_{0,N}^*)\boldsymbol{L}_N \end{bmatrix} \boldsymbol{y}_t \quad = \quad \begin{bmatrix} (a_{11}, a_{1,2}^*)\boldsymbol{L}_1 \\ (a_{12}, a_{1,3}^*)\boldsymbol{L}_2 \\ \vdots \\ (a_{1N}, a_{1,N}^*)\boldsymbol{L}_N \end{bmatrix} \boldsymbol{y}_{t-1} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} s_t + \begin{bmatrix} e_{1t} \\ e_{2t} \\ \vdots \\ e_{Nt} \end{bmatrix}, \tag{39}$$

and can be written more compactly as

$$\begin{aligned} \boldsymbol{\mathcal{A}}_0 \boldsymbol{y}_t \quad &= \quad \boldsymbol{\mathcal{A}}_1 \boldsymbol{y}_{t-1} + \boldsymbol{\mathcal{B}}s_t + \boldsymbol{\epsilon}_t, \\ \boldsymbol{y}_t \quad &= \quad \boldsymbol{\mathcal{A}}_0^{-1}\boldsymbol{\mathcal{A}}_1 \boldsymbol{y}_{t-1} + \boldsymbol{\mathcal{A}}_0^{-1}\boldsymbol{\mathcal{B}}s_t + \boldsymbol{\mathcal{A}}_0^{-1}\boldsymbol{\epsilon}_t \\ &= \quad \boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{B}s_t + \boldsymbol{e}_t. \end{aligned} \tag{40}$$

The impulse response functions from the GVAR model are given by

$$IRF^{ml}(h) \quad = \quad \boldsymbol{A}^h \boldsymbol{B}. \tag{41}$$

3.3 Simulation Results

3.3.1 Finite Sample Bias and Root Mean Square Error

Denote the finite sample bias in the spillover estimates obtained from the bilateral and the multilateral models $j \in \{ml, bl\}$ by

$$bias^j_{average} = R^{-1} \sum_{r=1}^{R} H^{-1} \left[\sum_{h=1}^{H} \left(\widehat{IRF}^j_r(h) - IRF_r(h) \right) \right] \Big/ \sum_{h=1}^{H} IRF_r(h), \qquad (42)$$

$$bias^j_{fixhor} = R^{-1} \sum_{r=1}^{R} \left(\widehat{IRF}^j_r(\bar{h}) - IRF_r(\bar{h}) \right) \Big/ IRF_r(\bar{h}), \qquad (43)$$

and the corresponding RMSEs by

$$rmse^j_{average} = \sqrt{ R^{-1} \sum_{r=1}^{R} H^{-1} \left[\sum_{h=1}^{H} \left(\widehat{IRF}^j_r(h) - IRF_r(h) \right)^2 \right] \Big/ \sum_{h=1}^{H} IRF_r(h)}, \quad (44)$$

$$rmse^j_{fixhor} = \sqrt{ R^{-1} \sum_{r=1}^{R} \left(\widehat{IRF}^j_r(\bar{h}) - IRF_r(\bar{h}) \right)^2 \Big/ IRF_r(\bar{h})}, \qquad (45)$$

where R represents the total number of replications in the Monte Carlo experiment.

In order to compare the properties of the spillover estimates obtained from the bilateral and the multilateral models, I consider the difference between their absolute biases and their RMSEs as defined in Equations (42) to (45), that is

$$\Delta bias_m = |bias^{ml}_m| - |bias^{bl}_m|, \quad m \in \{average, fixhor\}, \qquad (46)$$

$$\Delta rmse_m = rmse^{ml}_m - rmse^{bl}_m, \quad m \in \{average, fixhor\}, \qquad (47)$$

across different values of $[\boldsymbol{\gamma}_\ell]_2$ and w_{21}. Notice that a negative value for $\Delta bias_m$ implies that the finite sample bias of the spillover estimates obtained from the bilateral model is larger in absolute terms than that of the spillover estimates obtained from the multilateral model. Similarly, a negative value for $\Delta rmse_m$ implies that the RMSE of the spillover estimates obtained from the bilateral model is larger than that of the spillover estimates obtained from the multilateral model.

The results for the differences between the finite sample bias and the RMSE of the spillover estimates obtained from the bilateral and the multilateral models are displayed in Figure 4.[6] In particular, the results suggest that the finite sample bias and the RMSE of the

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[6]The results are based on $N = 50$ and $T = 150$. See Section 3.3.6 for a discussion of the effect of N and T on the bias and the RMSE.

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spillover estimates obtained from the bilateral model rise relative to those of the spillover estimates obtained from the multilateral model with increasing $[\boldsymbol{\gamma}_\ell]_2$ and decreasing w_{21}. Thus, the finite sample bias and the RMSE of the spillover estimates obtained from the bilateral model rise relative to those of the spillover estimates obtained from the multilateral model as the spillover-recipient becomes more susceptible to developments in the rest of the world overall, and as the spillover-sender becomes less important in the spillover-recipient's overall integration with the rest of the world. These findings are consistent with those for the asymptotic bias in the spillover estimates obtained from the bilateral model in Section 2.

3.3.2 Calibrated Weight Matrix

One could argue that the multilateral GVAR model considered in the Monte Carlo experiment is favoured relative to the bilateral model as it is estimated using the true weights in the matrix $\boldsymbol{W}$ for the link matrices $\boldsymbol{L}_i$ in Equation (38). In particular, in practice the true weights are not available so that they have to be calibrated, typically using bilateral trade flow or financial exposure data.[7] In order to render the Monte Carlo experiment more realistic in this regard, for the estimation of the GVAR model I consider a calibrated weight matrix $\boldsymbol{C}$, whose elements $c_{ij} \equiv [\boldsymbol{C}]_{ij}$ are given by

$$\widetilde{c}_{ij} = w_{ij} + \varsigma_{ij}, \qquad \varsigma_{ij} \sim N\left[0, (w_{ij}/\tau)^2\right], \tag{48}$$

$$c_{ij} = \widetilde{c}_{ij} / \sum_j \widetilde{c}_{ij}. \tag{49}$$

The parameter τ can be interpreted as the accuracy of the calibrated weights. For $\tau{=}5$, Figure 5 shows the difference between the finite sample bias and the RMSE of the spillover estimates obtained from the bilateral and the multilateral models if estimation of the latter is carried out using the calibrated weight matrix $\boldsymbol{C}$. The results are very similar to those from the baseline in Figure 4.

3.3.3 Factor-augmented VAR

One could also argue that it is quite natural for a much more strongly parameterised multilateral model to outperform a more parsimonious bilateral model. In order to examine whether the finding that a multilateral model produces more accurate spillover estimates relative to the bilateral model is not specific to the GVAR model, I consider a simple FAVAR model as an alternative multilateral framework (see, for example, Mumtaz and Surico, 2009). Specifically, denote by p_t the first principal component of the variables $x_{3t}, x_{4t}, \ldots, x_{Nt}$ in $\boldsymbol{z}_t$ of

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[7]Gross (2013) has put forth approaches to estimate the weights.

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economies $3, 4, \ldots, N$. Then, consider the FAVAR model

$$\begin{bmatrix} x_{1t} \\ x_{2t} \\ p_t \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ p_{t-1} \end{bmatrix} + \boldsymbol{B} s_t + \boldsymbol{\zeta}_t, \tag{50}$$

with impulse response functions

$$IRF^{favar}(h) \quad = \quad \boldsymbol{A}^h \boldsymbol{B}. \tag{51}$$

Figure 6 displays the differences between the finite sample bias and the RMSE of the spillover estimates obtained from the bilateral and the FAVAR model. The results suggest that the finite sample bias and the RMSE of spillover estimates obtained from the bilateral model rise relative to that of the FAVAR model for increasing $[\boldsymbol{\gamma}_\ell]_2$ and decreasing w_{21}. Thus, the finding that the bilateral model delivers inferior spillover estimates relative to a multilateral model is not specific to choosing the GVAR model as multilateral benchmark.

3.3.4 Bilateral Model with Global Variables

One could further argue that the omitted variable bias in the parameter estimates in the bilateral model could be addressed by including global variables to proxy for $\boldsymbol{z}_{t-1}$ in Equation (7). Specifically, consider the bilateral VAR model augmented by the first principal component of the variables $x_{3t}, x_{4t}, \ldots, x_{Nt}$ in $\boldsymbol{z}_t$ of economies $3, 4, \ldots, N$

$$\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} + \boldsymbol{C} p_{t-1} + \boldsymbol{B} s_t + \boldsymbol{\zeta}_t. \tag{52}$$

Figure 7 displays the difference between the finite sample bias and the RMSE of the spillover estimates obtained from the multilateral model and those from the bilateral model augmented by the global variable p_t. The results for the bias suggest that including a global variable does not lead to an improvement in the accuracy of the spillover estimates obtained from a bilateral model. The reason for this finding is that while the inclusion of the global variable may lead to consistent estimates of the parameters $\boldsymbol{\Phi}_{xx}$ and $\boldsymbol{\Omega}_x$ in the bilateral model in Equation (7), it does not address the failure to account for higher-order spillovers and spillbacks in the calculation of the impulse response functions based on Equation (8) rather than Equation (6).[8]

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[8]A parsimonious approach to addressing the bias in the spillover estimates that arises from the failure to account for higher-order transmission channels in the bilateral model is as follows: First, estimate the bilateral model with the rest of the world's variables included. Second, use the consistent parameter estimates to identify the structural shocks. Third, use the time-series of consistently estimated structural shocks in local projections or regressions of truncated infinite-order moving-average representations to obtain impulse responses (see Romer and Romer, 2004; Jorda, 2005).

3.3.5 Bilateral Model with Higher Lag Orders

Finally, one could argue that including higher lag orders in the bilateral model may address the omission of the rest of the world in Equation (7). Specifically, consider separately the blocks for economies 1 and 2 and the rest of the world in Equation (2) using the partitions in Equation (5)

$$
\begin{aligned}
\boldsymbol{x}_t &= \boldsymbol{\Phi}_{xx}\boldsymbol{x}_{t-1} + \boldsymbol{\Phi}_{xz}\boldsymbol{z}_{t-1} + \boldsymbol{\Omega}_x s_t + \boldsymbol{u}_t^x, &\text{(53)} \\
\boldsymbol{z}_t &= \boldsymbol{\Phi}_{zx}\boldsymbol{x}_{t-1} + \boldsymbol{\Phi}_{zz}\boldsymbol{z}_{t-1} + \boldsymbol{\Omega}_z s_t + \boldsymbol{u}_t^z. &\text{(54)}
\end{aligned}
$$

Then solve for $\boldsymbol{z}_t$ in Equation (54)

$$
\boldsymbol{z}_t = (\boldsymbol{I} - \boldsymbol{\Phi}_{zz}L)^{-1} \cdot \left(\boldsymbol{\Phi}_{zx}\boldsymbol{x}_{t-1} + \boldsymbol{\Omega}_z s_t + \boldsymbol{u}_t^z\right), \tag{55}
$$

where L denotes the lag operator, and plug the solution for $\boldsymbol{z}_t$ in Equation (55) lagged by one period into Equation (53), resulting in

$$
\begin{aligned}
\boldsymbol{x}_t &= \boldsymbol{\Phi}_{xx}\boldsymbol{x}_{t-1} + \boldsymbol{\Phi}_{xz}\left[(\boldsymbol{I} - \boldsymbol{\Phi}_{zz}L)^{-1}\left(\boldsymbol{\Phi}_{zx}\boldsymbol{x}_{t-2} + \boldsymbol{\Omega}_z s_{t-1} + \boldsymbol{u}_{t-1}^z\right)\right] + \boldsymbol{u}_t^x \\
&= \boldsymbol{\Phi}_{xx}\boldsymbol{x}_{t-1} + \boldsymbol{\Phi}_{xz}\sum_{j=0}^{\infty}\boldsymbol{\Phi}_{zz}^j\left(\boldsymbol{\Phi}_{zx}\boldsymbol{x}_{t-2-j} + \boldsymbol{\Omega}s_{t-1-j} + \boldsymbol{u}_{t-1-j}^z\right) + \boldsymbol{u}_t^x \\
&= \boldsymbol{\Pi}(L)\boldsymbol{x}_{t-1} + \boldsymbol{\Theta}(L)s_t + \boldsymbol{u}_t^x + \boldsymbol{\Upsilon}(L)\boldsymbol{u}_{t-1}^z. &\text{(56)}
\end{aligned}
$$

For economies 1 and 2, the true model can thus be re-written as an infinite-order bilateral VARMA model (see Zellner and Palm, 1974). The existing literature on cross-country spillovers using bilateral models has indeed considered higher lag orders for the endogenous variables, but it has not considered moving-average components. Therefore, as an alternative bilateral model in the Monte Carlo experiment I consider the bilateral model

$$
\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} = \sum_{m=1}^{p} \boldsymbol{A_m} \begin{bmatrix} x_{1,t-m} \\ x_{2,t-m} \end{bmatrix} + \boldsymbol{B}s_t + \boldsymbol{\zeta}_t, \tag{57}
$$

where I set $p = 4$. Figure 8 displays the difference between the finite sample bias and the RMSE of the spillover estimates obtained from the multilateral model and those from the bilateral model with higher lag orders of the endogenous variables. The results for the bias are again similar to those from the baseline in Figure 4. In contrast, the RMSE of the spillover estimates obtained from the bilateral model relative to that of the spillover estimates from the GVAR model either does not vary much with $[\boldsymbol{\gamma}_\ell]_2$ or even approaches the latter with increasing $[\boldsymbol{\gamma}_\ell]_2$.

3.3.6 Response Surface Regressions

While one could attempt to gauge the impact of N and T on the relative accuracy of the spillover estimates obtained from the bilateral and the multilateral model from inspecting Figure 4 for alternative sample sizes, this would be rather cumbersome and unsystematic. An alternative is to run response surface regressions (see MacKinnon, 1994). Specifically, consider the regressions

$$
\begin{aligned}
|bias_{m,\ell}^{ml}| - |bias_{m,\ell}^{bl}| &= \alpha_{m,0} + \alpha_{m,1} \, log(T_\ell) + \alpha_{m,2} \, log(N_\ell) + \alpha_{m,3} \, \bar{\gamma}_\ell + \alpha_{m,4} \, \bar{\omega}_\ell + \epsilon_\ell, &(58) \\
rmse_{m,\ell}^{ml} - rmse_{m,\ell}^{bl} &= \rho_{m,0} + \rho_{m,1} \, log(T_\ell) + \rho_{m,2} \, log(N_\ell) + \rho_{m,3} \, \bar{\gamma}_\ell + \rho_{m,4} \, \bar{\omega}_\ell + \varsigma_\ell, &(59)
\end{aligned}
$$

where ℓ refers to runs of Monte Carlo experiments with different specifications for $T \in \{100, 150, 500\}$, $N \in \{25, 50, 100\}$, $\bar{\omega}$ and $\bar{\gamma}$, and $m \in \{average, fixhor\}$. The coefficients $\alpha_{m,1}, \rho_{m,1}$ ($\alpha_{m,2}, \rho_{m,2}$) reflect the impact of T (N) on the relative finite sample bias and the RMSE of the spillover estimates obtained from the bilateral and the multilateral model.

Consistent with the results for the Monte Carlo experiment displayed in Figure 4, the estimation results for the response surface regressions reported in Tables 1 and 2 suggest that the difference between the finite sample bias and RMSE of the spillover estimates obtained from the multilateral and the bilateral model rises with $\bar{\gamma}$ and falls with $\bar{\omega}$ (column (1)). The difference between the finite sample biases obtained from the multilateral and the bilateral models rises with increasing T, as the estimates obtained from the multilateral model converge to the true values, while those from the bilateral model converge to the inconsistent probability limit (column (2)). In contrast, the difference between the biases decreases with increasing N, since the magnitude of higher-order spillovers falls as the importance of economies other than the spillover-sending economy 1 for the spillover-receiving economy 2 vanishes (column (2)); this result is consistent with the findings in Chudik and Straub (forthcoming), according to which a bilateral model is obtained as $N \longrightarrow \infty$ if the spillover-sending economy is the only regionally dominant economy for the spillover-receiving economy, and is regionally dominant only for economy 1. These results continue to hold when $\bar{\gamma}$ and $\bar{\omega}$ are entered in the regression in non-linear terms (columns (3) to (5)). The results for the effects of $\bar{\omega}$ and $\bar{\gamma}$ on the difference between the RMSEs of the spillover estimates obtained from the multilateral and the bilateral model are similar to those for the finite sample bias. However, in contrast to the results for the bias, the difference in the RMSEs also falls with increasing T, consistent with the convergence of the estimates from both models to their respective probability limits.

3.4 Finite Sample Evidence Based on a Structural Macro-model

As opposed to a structural macroeconomic model, the data-generating process considered thus far (see Equation (35)) is intuitive and simple, allowing me to derive closed-form solutions

for the asymptotic bias in the parameter and spillover estimates. Moreover, while important advances have been made in structural macroeconomic modelling over the last decade, most existing models continue to be rather limited in their country coverage and/or they abstract from salient features of the global economy that give rise to more complex spillover channels. For example, existing structural models tyically focus on cross-country linkages through trade, but lack transmission channels for financial spillovers; the models do not include spillover channels through global value chains; and economies are typically assumed to trade similar goods, implying they assume a similar position in the global trade network. Finally, exploring ranges of parameterisations in structural models is often not straightforward as many of the former imply unstable dynamics or multiple equilibria.

At the same time, one could argue it is not clear if changes in the deep structural parameters reflecting economies' overall sensitivity to developments in the rest of the world and the relative importance of a spillover-sender therein would alter the parametrisation of the reduced-form of the model in Equation (35) in the way laid out in Equations (20) and (21). Therefore, in this section I estimate spillovers using a bilateral and a multilateral model on data simulated based on a structural multi-country model. In particular, I consider the semi-structural model of Coenen and Wieland (2002) which includes the US, the euro area and Japan. The components of the model are not derived explicitly from micro-founded optimisation problems, but are very similar to what is obtained in more rigorously constructed structural models. In particular, the core building blocks of the model of Coenen and Wieland (2002) are an IS-curve, a Phillips curve, a Taylor-rule, and a term structure defining long-term interest rates.

For $i \in \{us, ea, ja\}$, the IS-curve for the domestic output gap q_{it} is given by

$$q_{it} = \sum_{j=1}^{3} \delta_{ij}^q q_{i,t-j} + \sum_{j=1}^{3} \delta_{ij}^{q^*} q_{i,t-j}^* + \delta_i^z z_{it} + \delta_i^{rl}(rl_{i,t-1} - \overline{rl}_i) + \sigma^{e^d} e_{it}^d, \tag{60}$$

where $z_{it} = \sum_{j=1,j\neq i}^{N} w_{ij}\omega_{ij,t}$ is an economy's real effective exchange rate with w_{ij} representing bilateral trade shares and $\omega_{ij,t}$ bilateral exchange rates; $r_{it}^{(l)}$ is the real long-term interest rate; q_{it}^* is the foreign output gap defined as a trade-weighted average of the other economies' output gaps, $q_{it}^* = \sum_{j=1,j\neq i}^{N} w_{ij}q_{jt}$; and e_{it}^d is a demand shock. Output spillovers thus arise through changes in foreign demand driven by changes in economies' in competitiveness as reflected by the real effective exchange rate, as well as through changes in foreign demand as reflected by the foreign output gap and which are unrelated to changes in the domestic economy's competitiveness and proxy for financial spillovers.[9] Quarter-on-quarter inflation

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[9]The latter element is not part of the original model of Coenen and Wieland (2002). I include it in order to generate spillovers in response to a US monetary policy shock that are of similar magnitudes as those observed in the data (see Georgiadis, forthcoming; Dedola et al., 2015; Banerjee et al., 2015). In fact, notice that spillovers from a contractionary US monetary policy shock arising exclusively through changes in economies' competitiveness in the original model of Coenen and Wieland (2002) are positive: The euro real

is determined in a backward-looking Phillips-curve

$$\pi_{it} = \left(\sum_{j=1}^{3} \phi_{ji}\right)^{-1} \left(\sum_{j=0}^{3} \phi_{ji} cwp_{i,t-j} - (\phi_{2i} + \phi_{3i})\,\pi_{i,t-1} - \phi_{3i}\pi_{i,t-2}\right), \tag{61}$$

where cwp_{it} is the contract wage. Based on specification tests Coenen and Wieland (2002) specify fixed-duration Taylor-style wage contracts for the euro area and Japan

$$cwp_{it} = (\phi_{1i} + \phi_{2i} + \phi_{3i})\,E_t\pi_{i,t+1} + (\phi_{2i} + \phi_{3i})\,E_t\pi_{i,t+2} + \phi_{3i}E_t\pi_{i,t+3}$$
$$+ \gamma_i \sum_{j=0}^{3} \phi_{ji} E_t q_{i,t+j} + \sigma_i^{cw} e_{it}^{cw}, \qquad i \in \{ea, ja\}, \tag{62}$$

and relative real wage contracts for the US

$$cwp_{us,t} = \sum_{j=0}^{3} \phi_{j,us} E_t \varpi_{us,t+j} + \gamma_{us} \sum_{j=0}^{3} \phi_{j,us} E_t q_{us,t+j} + \sigma_{us}^{cw} e_{us,t}^{cw},$$
$$\varpi_{us,t} = \sum_{j=0}^{3} \phi_{j,us} cwp_{us,t-j}. \tag{63}$$

The model is closed by monetary policy rules which determine the nominal short-term interest rate $i_{it}^{(s)}$ according to

$$i_{it}^{(s)} = \rho_{i^s} i_{i,t-1}^{(s)} + \alpha_i \left(\pi_{it}^{(4)} - \pi_i^T\right) + \beta_i q_{it} + (1 - \rho_i) \left(\overline{r}_i^{(l)} + \pi_{it}^{(4)}\right) + \sigma_i^{i^s} e_{it}^{mp}, \tag{64}$$

where π_i^T represents the inflation target and e_{it}^{mp} a monetary policy shock. Year-on-year inflation is given by

$$\pi_{it}^{(4)} = \sum_{j=0}^{3} \pi_{i,t-j}, \tag{65}$$

and the nominal and real long-term interest rate are defined through the term structure as

$$i_{it}^{(l)} = \frac{1}{8} \sum_{j=0}^{8} E_t i_{i,t+j}^{(s)}, \tag{66}$$

$$r_{it}^{(l)} = i_{it}^{(l)} - 0.5 \sum_{j=1}^{8} E_t \pi_{i,t+j}. \tag{67}$$

Figure 9 displays the responses of the US and euro area output gap to a contractionary US monetary policy shock for combinations of high and low values of the euro area's overall sensitivity to developments in the rest of the world ($\delta_{ea,j}^{q^*}$ in Equation (60)) and for high

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exchange rate depreciates relative to the US dollar, boosting euro area exports and dampening imports.

and low values of the share of the US in the euro area's overall integration with the rest of the world ($w_{ea,us}$ in Equation (60)). Figure 9 suggests that the ranges of parameter values I consider include all empirically relevant constellations, namely the case of rather small spillovers as well as the case in which the spillovers are larger than the domestic effects in the US.

For the Monte Carlo experiment I simulate data based on the model of Coenen and Wieland (2002) as described above for different values of $\delta_{ea,j}^{q^*}$ and $w_{ea,us}$. For each simulated dataset, I estimate the spillovers from a contractionary US monetary policy shock to the euro area using a two-country VAR and a GVAR model. In both models, I include the output gap q_{it}, quarter-on-quarter inflation π_{it}, and the nominal short-term interest rate $i_{it}^{(s)}$ as endogenous variables. As in the previous sections, I introduce the true time-series of US monetary policy shocks as observed exogenous variable in the two-country VAR and the GVAR models in order to abstract from issues of identification.[10] In order to give the two-country VAR model the best chances to recover the true spillovers, I consider two lags of the endogenous variables and I add the corresponding variables for Japan as exogenous variables (see Sections 3.3.4 and 3.3.5). Finally, I calculate the bias in the spillover estimates obtained from the two-country VAR and the GVAR model by comparing the spillover estimates to the true impulse response functions implied by the structural model.

Figure 10 displays the difference between the absolute bias of the spillover estimates obtained from the GVAR model and the two-country VAR model. As in the previous sections, the accuracy of the spillover estimates obtained from the two-country VAR model deteriorates relative to that of the spillover estimates obtained from the GVAR model when the sensitivity of the euro area to developments in the rest of the world rises (higher $\delta_{ea,j}^{q^*}$ in Equation (60)); moreover, the relative accuracy of the spillover estimates obtained from the two-country VAR model deteriorates when the US accounts for a smaller share of the euro area's overall integration with the rest of the world (lower bilateral trade share $w_{ea,us}$).

4   Global Spillovers from US Monetary Policy

As an empirical illustration of the possible differences between the spillover estimates obtained from bilateral and multilateral models and their determinants, I consider the global output spillovers from US monetary policy. In particular, I estimate the spillovers from US

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[10]The fit of the estimated models may change for different choices of $\delta_{ea,j}^{q^*}$ and $w_{ea,us}$ as the US monetary policy shock—which enters the empirical models as exogenous variable—accounts for a different share of the variance of the euro area output gap in the data generating process. In order to preclude that results are driven by variations in the variance share, in the simulations I change the variance of the US monetary policy shock for different choices of $\delta_{ea,j}^{q^*}$ and $w_{ea,us}$ so that the share of the variance of the euro area output gap accounted for by the US monetary policy shock stays constant.

monetary policy shocks using two-country VAR models and a GVAR model.[11] In line with the previous analysis, I circumvent the problem of identifying US monetary policy shocks by using the time series of shocks constructed by Romer and Romer (2004), Sims and Zha (2006), Bernanke and Kuttner (2005), Barakchian and Crowe (2013), Smets and Wouters (2007), and financial market survey data of future monetary policy rates.[12,13] Specifically, the monetary policy shocks constructed by Romer and Romer (2004) are based on the seminal "narrative approach"; those from Sims and Zha (2006) are structural shocks implied by a non-recursively identified VAR model for the US economy; those from Bernanke and Kuttner (2005) as well as Barakchian and Crowe (2013) build on the difference between lagged futures and the actual values of the federal funds rate; the monetary policy shocks from Smets and Wouters (2007) are smoothed structural shocks from an estimated dynamic stochastic general equilibrium model for the US economy; finally, I construct shock time series based on the difference between lagged survey expectations of future short-term interest rates and their actual values using data from Consensus Economics or the Survey of Professional Forecasters. Consistent with the exposition in the previous sections, the US economy is represented by the unit with subscript $i = 1$.

4.1 The Bilateral Model

The two-country VAR models are given by

$$\begin{bmatrix} \boldsymbol{x}_{1t} \\ \boldsymbol{x}_{it} \end{bmatrix} = \sum_{m=1}^{p} \boldsymbol{A}_{im} \begin{bmatrix} \boldsymbol{x}_{1,t-m} \\ \boldsymbol{x}_{i,t-m} \end{bmatrix} + \boldsymbol{B}_i s_t^j + \sum_{m=0}^{q} \boldsymbol{C}_{im} g_{t-m} + \boldsymbol{e}_{it}, \quad i = 2, 3, \ldots, N, \qquad (68)$$

where the vector of endogenous variables $\boldsymbol{x}_{it}$ for non-US economies includes output growth, inflation, short-term interest rates and the nominal bilateral exchange rate vis-à-vis the US dollar. The US variables in $\boldsymbol{x}_{1t}$ include US output growth, inflation and short-term interest

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[11]Based on the Monte Carlo results in Section 3 the FAVAR model would be an alternative multilateral benchmark. However, in the dataset considered for this empirical application the first principal component captures only a relatively small share of the variation in the data across economies. For example, the first principal components in the data for inflation, output growth and changes in interest rates capture only around 40% of the total variation across economies. As a result, in the particular application considered here the FAVAR model produces spillover estimates which are rather similar to those obtained from the two-country VAR models.

[12]Georgiadis (forthcoming) shows that the spillover estimates obtained on the basis of these shocks are very similar to those obtained from applying sign restrictions on short-term interest rates, inflation and the nominal effective exchange rate (as well as output growth, oil prices and money growth) in order to identify US monetary policy shocks.

[13]One could argue that if the monetary policy shocks are known one could simply estimate a truncated MA(∞) representation of the model or use local projections in order to obtain the spillovers (see, for example, Romer and Romer, 2004; Jorda, 2005). However, recall that the purpose of the exercise carried out in this section is not to obtain generic estimates of the global spillovers from US monetary policy. Rather, the purpose of this exercise is to illustrate that even beyond the problem of identification bilateral models deliver less accurate spillover estimates relative to a multilateral model because they fail to capture correctly the global *propagation* of—even correctly identified—US monetary policy shocks.

rates. g_t is a global variable and includes oil price inflation.

4.2 The Multilateral Model

The GVAR model is adopted from Georgiadis (2015, forthcoming) and consists of unit-specific VAR models given by

$$\boldsymbol{x}_{it} = \sum_{m=1}^{p} \boldsymbol{A}_{im} \boldsymbol{x}_{i,t-m} + \sum_{m=0}^{p^*} \boldsymbol{A}_{im}^* \boldsymbol{x}_{i,t-m}^* + \boldsymbol{B}_{i0} s_t^j + \boldsymbol{e}_{it}, \quad i = 1, 2, 3, \ldots, N, \qquad (69)$$

in which the vector of domestic endogenous variables $\boldsymbol{x}_{it}$ includes output growth and inflation for all economies; for non-euro area economies, it also includes short-term interest rates and the nominal bilateral exchange rate vis-à-vis the euro. In the GVAR model of Georgiadis (2015) one unit represents the ECB's monetary policy by an autoregressive model in which euro area short-term interest rates are determined as a function of GDP-weighted aggregate euro area output growth and inflation. Moreover, another unit refers to an oil block in which oil price inflation is determined endogenously as a function of GDP-weighted world output growth, inflation and interest rates. For all economies in the GVAR model, the vector of foreign variables $\boldsymbol{x}_{it}^*$ includes oil price inflation as well as trade-weighted averages of global output growth, inflation and interest rates. For the euro area economies, the vector of "foreign" variables additionally includes euro area short-term interest rates which are determined in the ECB's model. The VAR models are estimated unit-by-unit, followed by the derivation of the global solution (see Equation (40)) which is used for the construction of impulse response functions.

4.3 Baseline Results

Upon estimation of the two-country VAR models and the GVAR model on quarterly data over the time period from 1999 to 2009 for 61 economies I calculate the impulse response functions of output to the US monetary policy shock $s_t^j, j \in \{RR, BK, SZ, BC, SW, CONSENSUS, SPF\}$, and consider the average response over 12 quarters.[14] Figure 1 presents a scatter plot of the output spillover estimates obtained from the two-country VAR models against those obtained from the GVAR model when using the monetary policy shocks constructed by Bernanke and Kuttner (2005). The red dashed line represents the fit of the regression

$$\widehat{\overline{s}}_i^{gvar,BK} = \beta_0^{BK} + \beta_1^{BK} \widehat{\overline{s}}_i^{tcvar,BK} + e_i^{BK}, \qquad (70)$$

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[14]The lag orders for both the two-country VAR and the country-specific VAR models in the GVAR model are determined using the Akaike information criterion.

where $\widehat{\overline{s}}_i^{gvar,BK}$ and $\widehat{\overline{s}}_i^{tcvar,BK}$ denote the average output spillover estimates from the US monetary policy shock over 12 quarters obtained from the GVAR and the two-country VAR models; the black solid line represents the 45-degree line. Two observations stand out: First, as reflected by the statistically significant intercept estimate, the global output spillovers from US monetary policy obtained from the two-country VAR models are systematically smaller (in absolute terms) by about 20 basis points compared to those obtained from the GVAR model. Second, as reflected by the R-squared below unity, while the spillover estimates obtained from the two-country VAR models and those from the GVAR model are similar, the correspondence is not perfect.[15] And the regression results in Table 3 document that these findings are not specific to the use of the monetary policy shock time series constructed by Bernanke and Kuttner (2005).[16] Together with the results from the asymptotic and the Monte Carlo analysis in Section 3 this evidence points to a statistically and economically significant mis-measurement of the global spillovers from US monetary policy based on two-country VAR models.

The evidence in Sections 2 and 3 suggests that the difference between the spillover estimates obtained from the GVAR model and the two-country VAR models should be related to (i) spillover-recipients' overall integration with the rest of the world that renders them susceptible to developments abroad, and (ii) the importance of the US in spillover-recipients' overall integration with the rest of the world. In particular, one would expect the two-country VAR models to deliver spillover estimates that are close to those obtained from the GVAR model if (i) an economy is less integrated in global trade and finance overall, and if (ii) the US accounts for a large share in spillover-recipients' overall trade and financial integration with the rest of the world.

In order to shed light on whether these predictions are borne out by the data, I exploit information on (i) economies' overall trade and financial integration with the rest of the world, as well as on (ii) the relative importance of the US in economies' overall integration. In particular, I consider the sum of imports and exports to GDP ($tradeopenn_i$) as a measure of economies' overall trade integration; the ratio of gross foreign assets and liabilities to GDP ($finopenn_i$) as a measure of economies' overall financial integration; the share of imports

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[15]Another observation that stands out is that the spillover estimates obtained from the two-country VAR models are positive for many economies. Positive spillover estimates from a US monetary policy tightening are theoretically possible for two reasons. First, the spillovers are estimated imprecisely, not allowing one to reject the hypothesis that the true spillover is negative. Second, the true spillover can be positive or negative depending on whether expenditure-reducing or expenditure-switching effects dominate: On the one hand, the drop in US output leads to a fall in the spillover-receiving economy's foreign demand; on the other hand, the appreciation of the US dollar stimulates US demand for the spillover-receiving economy's goods. While theoretically possible, positive spillovers from a US monetary policy tightening are rather inconsistent with conventional wisdom according to which the US is an important driver of the global business cycle.

[16]In order to preclude that the spillover estimates based on a particular monetary policy shock time series have a disproportionate influence on the coefficient estimates, I standardise the differences between the spillover estimates obtained from the two-country VAR models and those from the GVAR model for a given shock time series j whenever I run pooled regressions.

from and exports to the US in an economy's total trade ($tradeshareUS_i$) as a measure of the importance of the US in economies' overall trade integration; and the sum of US financial assets held by an economy's residents and an economy's foreign liabilities held by US residents relative to the economy's total foreign assets and liabilities ($finshareUS_i$) as a measure of the relative importance of the US in economies' overall financial integration with the rest of the world.[17] Figure 11 displays the data and shows that there are pronounced cross-country differences in economies' overall trade and financial integration with the rest of the world and the relative importance of the US therein.

The top and middle panels of Figure 12 present scatterplots of the difference between the spillover estimates obtained from the GVAR model and those from the two-country VAR models on the one hand, and economies' overall integration with the rest of the world (left-hand side panels) as well as the relative importance of the US therein (right-hand side panels) on the other hand.[18] The bottom panels present scatterplots of the differences between the spillover estimates and the first principal component of economies' overall trade and financial integration with the rest of the world ("Multilateral integration") as well as of the relative importance of the US therein ("Bilateral integration"). In line with the results from Sections 2 and 3, the scatterplots suggest that spillover estimates obtained from the GVAR model are systematically larger (in absolute terms) than those obtained from the two-country VAR models for economies which exhibit a stronger overall trade and financial integration with the rest of the world. Also in line with the results from Sections 2 and 3, the differences between the spillover estimates obtained from the GVAR and the two-country VAR models are smaller if the US accounts for a large share of economies' overall trade and financial integration.

To move beyond unconditional correlations, I run the regression

$$\widehat{s}_i^{gvar,j} - \widehat{s}_i^{tcvar,j} = \beta_0 + \beta_1 \cdot tradeopenn_i + \beta_2 \cdot tradeshareUS_i +$$
$$+ \beta_3 \cdot finopenn_i + \beta_4 \cdot finshareUS_i + \beta_5 \cdot contiguityUS_i + e_i, \quad (71)$$

where $contiguityUS_i$ reflects a dummy variable that equals unity for Mexico and Canada,

[17]The data on total trade are taken from the World Development Indicators, those on bilateral trade from the IMF Direction of Trade Statistics, those on total gross foreign assets and liabilities from Lane and Milesi-Ferretti (2007), and those on bilateral foreign assets and liabilities from the IMF Coordinated Portfolio Investment Survey. I take the logarithm of one plus the time averages over 1999 to 2009 of the raw data on total trade and gross foreign assets and liabilities relative to GDP in order to alleviate the impact of possible outliers.

[18]To improve power and to account for possible measurement error in the construction of the monetary policy shock time series, I consider simultaneously the spillover estimates from the models using the different US monetary policy shock times series. This approach is similar to the multiple indicator multiple cause (MIMIC) model introduced by Goldberger (1972) and used, for example, in Rose and Spiegel (2011). The framework in Equation (71) is less complex than MIMIC, though, as the indicator variable is conceptually identical across measurements: the output spillover from US monetary policy across shock time series. Below it is shown that the results for individual monetary policy shocks are similar to those from the pooled sample considered here.

thereby reflecting contiguity with the US.[19,20] Recall that the estimated spillovers from a US monetary policy tightening are typically negative. Therefore, if higher-order spillovers and spillbacks are better captured by the spillover estimates obtained from the GVAR model one would expect $\widehat{\beta}_1 < 0$ and $\widehat{\beta}_3 < 0$, as the former should be more pronounced for economies which are strongly integrated with the rest of the world overall. At the same time, if direct bilateral spillovers are captured equally well by the two-country VAR models and the GVAR model one would expect $\widehat{\beta}_2 > 0$ and $\widehat{\beta}_4 > 0$, as the former should be more important for economies for which the US accounts for a large share of the spillover-recipients' overall integration with the rest of the world.

Table 4 reports the results from various regressions of Equation (71). The baseline results suggest that the differences between the spillover estimates obtained from the GVAR model and the two-country VAR models are indeed related to differences in economies' integration patterns: In line with the results in Sections 2 and 3, the two-country VAR models produce spillover estimates which are systematically smaller (in absolute terms) than those obtained from the GVAR model for economies which are more integrated with the rest of the world overall, and for economies in which the US accounts for a smaller share in their overall integration with the rest of the world. In particular, when trade and financial integration variables are entered simultaneously the coefficient estimates for overall financial integration and the share of financial integration accounted for by the US are statistically significant (column (1)). In contrast, the coefficient estimates for overall trade integration and the share of trade accounted for by the US are not statistically significant. When the variables reflecting trade and financial integration are entered in separate regressions, the coefficient estimates for multilateral and bilateral integration patterns are all statistically significant and in line with the results from Sections 2 and 3 (columns (2) and (3)). As the lack of statistical significance for individual coefficient estimates when the variables are included jointly is likely to be due to the high correlation between financial and trade integration in the data, in the following I consider the first principal component of economies' multilateral (bilateral) trade and financial integration patterns. Specifically, when I run the regression in Equation (71) with the principal components of economies' multilateral and bilateral integration patterns the coefficient estimates are highly statistically significant and their signs are consistent with the results from Sections 2 and 3 (column (4)).

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[19]Recall that the results in Sections 2 and 3 suggest that the relationship between bilateral and multilateral integration on the one hand and differences between the spillover estimates obtained from the two-country VAR models and the GVAR model may be non-linear. And indeed, as can be seen in Figure 11 economies neighbouring the US exhibit very large values for their bilateral integration with the US that do not appear to be aligned—in particular for trade—with the relationship of the variables for the other economies.

[20]The dependent variable in Equation (71) is generated in a first stage. The consequence of estimation uncertainty in the dependent variable is to magnify the variance of the regression error e_i, and thereby the variance of the estimates $\widehat{\beta}_j$.

4.4 Corroborating Evidence

4.4.1 Robustness

In contrast to the two-country VAR models, the GVAR model considered in this paper accounts for the fact that euro area monetary policy is carried out at the euro area-wide rather than at the individual country level.[21] The differences between the spillover estimates obtained from the GVAR model and the two-country VAR models could be driven by this mis-specification in the two-country VAR models. However, Table 5 suggests that the results are very similar to those from the baseline if euro area economies are dropped from the sample (column (2)). The results are also very similar to those from the baseline when standard errors are clustered at the monetary policy shock time series level j (column (3)), and when I apply robust regression (`rreg` in Stata; column (4)). Moreover, the results are similar to those from the baseline if I consider the trough output spillovers from a US monetary policy shock or those after seven quarters rather than the average spillover over 12 quarters (columns (5) and (6)).

4.4.2 Individual Monetary Policy Shock Time Series

In the baseline I pool the spillover estimates obtained from separate estimations of the two-country VAR models and the GVAR model using monetary policy shock time series from Romer and Romer (2004), Bernanke and Kuttner (2005), Sims and Zha (2006), Barakchian and Crowe (2013), Smets and Wouters (2007), as well as financial market survey data of future monetary policy rates. Table 6 reports the regression results for each individual monetary policy shock time series. While the coefficient estimates are not statistically significant in some cases, they are overall consistent with those from the pooled baseline sample. Most importantly, the baseline results do not seem to be driven by the spillover estimates obtained from using a particular monetary policy shock time series.

4.4.3 Exchange-rate Regime

Direct expenditure-reducing spillovers arising through a drop in US demand for goods of non-US spillover-recipients in response to a US monetary policy tightening are alleviated if economies' exchange rate can depreciate vis-à-vis the US dollar and trigger expenditure-switching. Therefore, other factors held constant, direct trade spillovers from the US should be smaller for economies with a flexible exchange rate vis-à-vis the US dollar relative to economies with a fixed exchange rate. As a result, ceteris paribus, having the US account for a larger share in an economy's overall trade integration should reduce the difference between

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[21]For details see Georgiadis (2015).

the spillover estimates obtained from the GVAR model and the two-country VAR models by less for economies with a flexible exchange rate compared to economies with a fixed exchange rate. Similarly, as flexible exchange rates insulate at least to some extent domestic financial conditions from those in the US (the famous "trilemma"), direct financial spillovers from the US should be smaller in economies with a flexible exchange rate. As a result, other factors held constant, having the US account for a larger share in an economy's overall financial integration should reduce the difference between the spillover estimates obtained from the GVAR model and the two-country VAR models by less for economies with a flexible exchange rate vis-à-vis the US dollar compared to economies with a fixed exchange rate.

The data are consistent with these hypotheses. Specifically, Table 7 reports results from a regression in which an interaction between the bilateral integration with the US and the exchange rate flexibility vis-à-vis the US dollar is included (column (1)).[22] The coefficient estimate for the bilateral integration with the US remains statistically significant with a positive sign. Importantly, the interaction with the exchange rate flexibility vis-à-vis the US dollar is also statistically significant with the expected negative sign.

4.4.4 Trade-network Centrality

Economies which are more central in the global trade network should be subject to larger higher-order spillovers, exacerbating the differences between the spillover estimates obtained from the GVAR and the two-country VAR models. The results from the regression of Equation (71) in which a measure of economies' centrality in the global trade network is included reported in Table 7 are consistent with this hypothesis (column (2)): The coefficient estimate for centrality in the global trade network is negative and statistically significant.[23]

4.4.5 Susceptibility to Higher-order Spillovers

Two economies which are equally strongly integrated with the US may have different links in the global trade network. As a result, they should be differentially susceptible to higher-order spillovers from US shocks. For example, suppose the US accounts for the same share in economies m's and s's total trade, and that they are equally integrated in global trade overall. However, while economy m has many trading partners for which the US is an important partner, the trading partners of economy s trade mostly with non-US economies. In this case, economy m should be more susceptible to higher-order spillovers from US shocks through its trading partners than economy s.

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[22]The data for the exchange rate flexibility vis-à-vis the US dollar are taken from an updated version of the data constructed by Klein and Shambaugh (2006) and are available for the time period from 1960 to 2014.

[23]The data for centrality are taken from the CEPII database. The variable used is the principle component of degree, eigenvector, closeness and strength centrality.

In order to test this hypothesis, I construct an indicator for economies' susceptibility to higher-order spillovers from US shocks based on bilateral trade data.[24] Specifically, I first determine an index of the susceptibility of economy i's trade to US shocks due to second-order spillovers through its trading partners:

$$\chi_i^{(0)} = \sum_{j=2}^{N} w_{ij} \cdot w_{j1} \cdot tradeopenn_j, \quad i = 1, 2, /hdots, N \tag{72}$$

where the components are given by the share of economy i's trade accounted for by economy j (w_{ij}), the share of economy j's total trade accounted for by the US (w_{j1}), and economy j's overall trade integration ($tradeopenn_j$): If economy j accounts for a large share of economy i's total trade, the US accounts for a large share of economy j's total trade and economy j trades much overall, then the potential for second-order spillovers from the US to economy i through economy j is large. Second, I extend the measure in Equation (72) to capture the potential for spillovers of higher than second order using the recursion

$$\chi_i^{(r)} = \chi_i^{(r-1)} + \sum_{j=1}^{N} w_{ij} \cdot tradeopenn_j \cdot \chi_j^{(r-1)}, \quad r = 1, 2, \dots \tag{73}$$

until convergence.[25]

The results for the regression of Equation (71) with the index for an economy's susceptibility to higher-order spillovers $\chi_i^{(\infty)}$ from Equation (73) are reported in column (3) in Table 7. The results are consistent with the hypothesis that the spillover estimates obtained from the two-country VAR models are farther from those obtained from the GVAR model for economies which are more susceptible to higher-order trade spillovers from US shocks. The coefficient estimate for the index of the susceptibility to higher-order spillovers from US shocks is also statistically significant if it is constructed using bilateral trade and financial integration data (column (4)).[26]

4.4.6 Position in Global Value Chains

Economies located further upstream in the global value chain should be more susceptible to higher-order spillovers. Intuitively, being located more upstream can be thought of as being a supplier of intermediate goods to economies which are located further downstream in the global value chain; upstream economies which service demand from downstream economies will experience stronger disruptions of their foreign demand in response to an

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[24]The data are taken from the IMF Direction of Trade Statistics.

[25]I normalise $\chi_i^{(r)}$ to sum to unity across economies in each iteration.

[26]The index of the susceptibility to higher-order spillovers through financial channels is constructed using IMF Coordinated Portfolio Investment Survey data.

external shock than economies which tend to demand rather than supply inputs from/to upstream economies. As a result, the spillover estimates obtained from two-country VAR models should be farther from those obtained from the GVAR model for economies more upstream in the global value chain.

The data are consistent with this hypothesis. Table 7 reports the results from a regression in which a measure of economies' position in global value chains (higher values reflect a more upstream position) is entered as an additional explanatory variable (column (5)).[27] The coefficient estimates for the position in global value chains has a negative sign and is almost statistically significant at the 10% significance level.

4.4.7 Distance to the US

Finally, one could argue that differences between the spillover estimates obtained from the GVAR model and the two-country VAR models could be related to economies' geographic distance to the US. Specifically, other factors held constant, bilateral trade and financial flows between the US and spillover-recipients could be less volatile the farther the latter are from the US, as information asymmetries are larger implying that disrupting and re-establishing trade and financial relationships in response to shocks is costlier. As a result, for a given share of an economy's trade and financial integration with the rest of the world accounted for by the US, direct bilateral spillovers should be less relevant for economies farther away from the US.

The results reported in column (6) in Table 7 are consistent with this hypothesis. While the coefficient estimate for bilateral integration with the US retains its positive sign and statistical significance, the coefficient estimate for the interaction between bilateral integration and the distance to the US is negative and statistically significant. Strong bilateral integration with the US thus reduces the difference between the spillover estimates obtained from the GVAR model and the two-country VAR models, but this effect is weaker the farther away an economy is from the US.

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[27]I calculate the index for the position in global value chains using the World Input-Output Database as

$$gvc_i^{pos} \equiv log\left(1 + iva_i/e_i\right) - log\left(1 + fva_i/e_i\right), \tag{74}$$

where iva_i represents the indirectly exported value added of country i embodied in other economies' gross exports, fva_i the value added from foreign sources embodied in economy i's gross exports, and e_i economy i's gross exports.

5   Conclusion

Due to the dramatic increase of trade and financial integration in the global economy cross-country spillovers have become a major element of economic policy thinking over the last decade. Estimating the magnitude of spillovers is thus an important branch of research in international macroeconomics and finance. The analysis in this paper suggests that spillover estimates obtained from easy-to-implement bilateral, two-country models are significantly less accurate than those obtained from technically more demanding multilateral models. In particular, the accuracy of the spillover estimates obtained from bilateral models depends on spillover-recipients' international integration patterns: Stronger overall sensitivity to developments in the rest of the world renders the spillover estimates obtained from bilateral models inaccurate; and strong bilateral trade and financial integration with the spillover-sender improves their accuracy. The analysis in this paper also suggests that the differences between the spillover estimates obtained from bilateral and multilateral models can be statistically and economically significant in practice. Spillover estimates from bilateral models should thus be treated with caution, and more resources should be devoted to the development of multilateral models for spillover analysis.

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A   Tables

Table 1: Response Surface Regression for the Difference Between the Bias of the Spillover Estimates Obtained from the Multilateral and Bilateral Model

	(1)	(2)	(3)	(4)	(5)
$[\gamma_\ell]_2$	-0.010***	-0.010***	-0.008**	-0.014***	
	(0.00)	(0.00)	(0.03)	(0.00)	
w_{21}	0.023***	0.023***	0.053***	0.045***	
	(0.00)	(0.00)	(0.00)	(0.00)	
$\log(T)$		-0.002***	-0.002***	-0.002***	-0.002***
		(0.00)	(0.00)	(0.00)	(0.00)
$\log(N)$		0.006***	0.006***	0.006***	0.006***
		(0.00)	(0.00)	(0.00)	(0.00)
$[\gamma_\ell]_2$ squared			-0.002	-0.002	
			(0.54)	(0.52)	
w_{21} squared			-0.043***	-0.043***	
			(0.00)	(0.00)	
$[\gamma_\ell]_2$ x w_{21}				0.018***	
				(0.00)	
$\log([\gamma_\ell]_2)$					-0.004***
					(0.00)
$\log(w_{21})$					0.007***
					(0.00)
Constant	-0.012***	-0.025***	-0.030***	-0.027***	-0.017***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Observations	324	324	324	324	324
Adjusted R^2	0.52	0.88	0.91	0.92	0.91

p-values in parentheses

Robust standard errors.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Response Surface Regression for the Difference Between the RMSE of the Spillover Estimate Obtained from the Multilateral and Bilateral Model

	(1)	(2)	(3)	(4)	(5)
$[\gamma_\ell]_2$	-0.011***	-0.011***	-0.001	-0.002	
	(0.00)	(0.00)	(0.88)	(0.81)	
w_{21}	0.030***	0.030***	0.095***	0.094***	
	(0.00)	(0.00)	(0.00)	(0.00)	
$\log(T)$		0.001*	0.001**	0.001**	0.001**
		(0.07)	(0.03)	(0.03)	(0.04)
$\log(N)$		0.006***	0.006***	0.006***	0.006***
		(0.00)	(0.00)	(0.00)	(0.00)
$[\gamma_\ell]_2$ squared			-0.011*	-0.011*	
			(0.09)	(0.09)	
w_{21} squared			-0.093***	-0.093***	
			(0.00)	(0.00)	
$[\gamma_\ell]_2$ x w_{21}				0.002	
				(0.75)	
$\log([\gamma_\ell]_2)$					-0.004***
					(0.00)
$\log(w_{21})$					0.009***
					(0.00)
Constant	-0.014***	-0.039***	-0.050***	-0.050***	-0.026***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Observations	324	324	324	324	324
Adjusted R^2	0.55	0.76	0.86	0.86	0.84

p-values in parentheses

Robust standard errors.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Correlation Between Output Spillover Estimates from US Monetary Policy Obtained from the GVAR and the Two-country VAR Models Using Different Monetary Policy Shock Measures

	(1) B&K	(2) R&R	(3) B&C	(4) S&Z	(5) Consens.	(6) SPF	(7) SW	(8) Pooled
Average two-country VAR IRF	1.02***	0.67***	0.42**	-0.11	0.34**	-0.07	0.62***	0.56***
	(0.00)	(0.00)	(0.02)	(0.79)	(0.02)	(0.84)	(0.00)	(0.00)
Constant	-0.20***	-0.48***	-0.22***	-1.41***	-0.56***	-1.60***	-0.83***	
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	
B&K dummy								-0.25***
								(0.00)
R&R dummy								-0.45***
								(0.00)
B&C dummy								-0.23***
								(0.00)
S&Z dummy								-1.43***
								(0.00)
Consens. dummy								-0.68***
								(0.00)
SPF dummy								-1.89***
								(0.00)
SW dummy								-0.91***
								(0.00)
Observations	54	54	54	54	54	54	54	378
R^2	0.62	0.50	0.14	0.00	0.09	0.00	0.52	0.70

p-values in parentheses

Robust standard errors.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Determinants of Differences Between Estimates of Output Spillovers Obtained from US Monetary Policy obtained from the GVAR and the Two-country VAR Models

	(1) Baseline	(2) Trade int.	(3) Fin. Int.	(4) PCs
Trade rel. to GDP	0.17 (0.41)	-0.38** (0.04)		
Share of trade with US	1.79 (0.13)	3.75*** (0.00)		
GFAL rel. to GDP	-0.29*** (0.00)		-0.27*** (0.00)	
Share of US in overall fin. integration	1.22* (0.10)		1.96*** (0.00)	
Contiguity dummy	-1.31*** (0.00)	-1.77*** (0.00)	-0.73*** (0.00)	-1.25*** (0.00)
Multil. integration				-0.14*** (0.00)
Bil. integration				0.25*** (0.00)
Observations	378	378	378	378
Adjusted R^2	0.12	0.07	0.12	0.11

p-values in parentheses

Robust standard errors.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Determinants of Differences Between Estimates of Output Spillovers Obtained from US Monetary Policy Obtained from the GVAR and the Two-country VAR Models—Robustness

	(1) Baseline	(2) No EA	(3) Cluster	(4) rreg	(5) Trough IRF	(6) Fix h
Multil. integration	-0.14***	-0.14***	-0.14**	-0.17***	-0.15***	-0.11**
	(0.00)	(0.00)	(0.03)	(0.00)	(0.00)	(0.01)
Bil. integration	0.25***	0.21***	0.25***	0.23***	0.24***	0.26***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Contiguity dummy	-1.25***	-1.15***	-1.25***	-1.21***	-1.41***	-1.23***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Observations	378	287	378	378	378	378
Adjusted R^2	0.11	0.08	0.11	0.11	0.11	0.10

p-values in parentheses

Robust standard errors.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Determinants of Differences Between Estimates of Output Spillovers Obtained from US Monetary Policy Obtained from the GVAR and the Two-country VAR Models—Regressions for Individual Monetary Policy Shock Time Series

	(1) Baseline	(2) B&K	(3) R&R	(4) B&C	(5) S&Z	(6) Consens.	(7) SPF	(8) SW
Multil. integration	-0.14***	-0.11	-0.14	-0.24**	-0.21**	-0.06	-0.16*	-0.03
	(0.00)	(0.39)	(0.26)	(0.03)	(0.02)	(0.25)	(0.05)	(0.74)
Bil. integration	0.25***	0.32**	0.25*	0.07	0.32***	0.25***	0.41***	0.13
	(0.00)	(0.05)	(0.09)	(0.53)	(0.01)	(0.00)	(0.00)	(0.35)
Contiguity dummy	-1.25***	-1.58**	-0.82	-0.76	-1.55***	-1.21***	-1.80***	-1.03
	(0.00)	(0.02)	(0.17)	(0.25)	(0.01)	(0.00)	(0.00)	(0.11)
Observations	378	54	54	54	54	54	54	54
Adjusted R^2	0.11	0.10	0.07	0.06	0.17	0.17	0.22	-0.03

p-values in parentheses

Robust standard errors.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 7: Determinants of Differences Between Estimates of Output Spillovers from US Monetary Policy Obtained from the GVAR and the Two-country VAR Models—Exchange Rate Flexibility, Trade Network Position Characteristics, and Distance to the US

	(1)	(2)	(3)	(4)	(5)	(6)
Multil. integration	-0.21***	-0.14***	-0.17***	-0.16***	-0.14**	-0.17***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.03)	(0.00)
Bil. integration	2.17**	0.21***	0.22***	0.23***	0.25***	1.68**
	(0.03)	(0.00)	(0.00)	(0.00)	(0.01)	(0.02)
Contiguity dummy	-0.99***	-1.02***	-1.23***	-1.22***	-1.11**	-1.59***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.04)	(0.00)
ER flexibility to USD	-0.21**					
	(0.03)					
Bil. integration x ER regime	-0.50**					
	(0.05)					
Centrality		-0.13***				
		(0.01)				
Higher-order spillover susceptibility score			-21.27*			
			(0.07)			
Higher-order trade and fin. spillover susceptibility score				-29.85**		
				(0.04)		
GVC position					-6.82	
					(0.11)	
Bil. integration x Distance to US						-0.17**
						(0.04)
Distance to US						0.37**
						(0.02)
Observations	378	364	378	378	231	378
Adjusted R^2	0.12	0.10	0.11	0.12	0.06	0.12

p-values in parentheses

Robust standard errors.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

B   Figures

Figure 1: Differences Between Estimates of the Spillovers from US Monetary Policy Obtained from the GVAR and the Two-country VAR Models
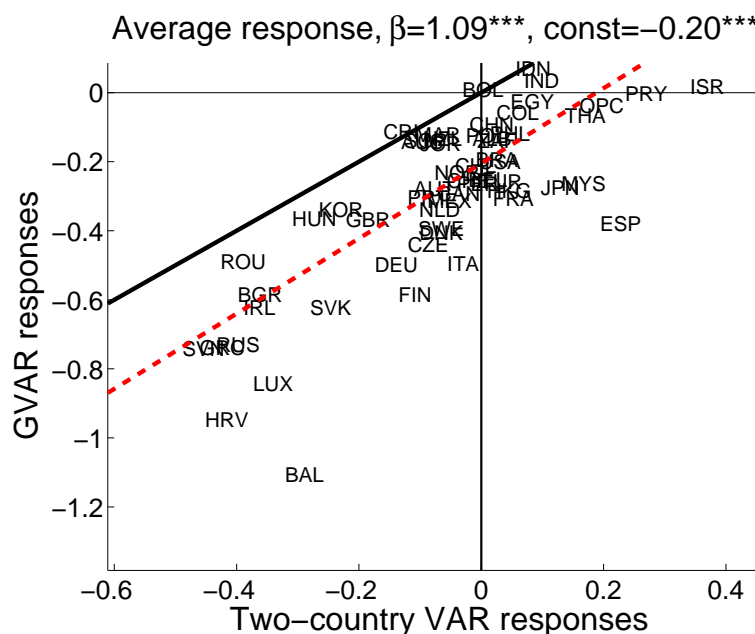
Note: *The figure displays the average of the spillover estimates of real GDP to a 100 basis points contractionary US monetary policy shock over 12 quarters obtained from two-country VAR models (horizontal axis) and a GVAR model (vertical axis). The black solid line represents the 45-degree line and the red dashed line the fit from a regression of the spillover estimates obtained from the GVAR model on those obtained from the two-country models. The slope and intercept estimates from this regression are provided in the figure title.* ***
indicates statistical significance at the 1% significance level. The spillover estimates are based on the monetary policy shocks constructed by Bernanke and Kuttner (2005).

Figure 2: Impulse Response Functions for a Shock s_t in the Spillover-sending Economy
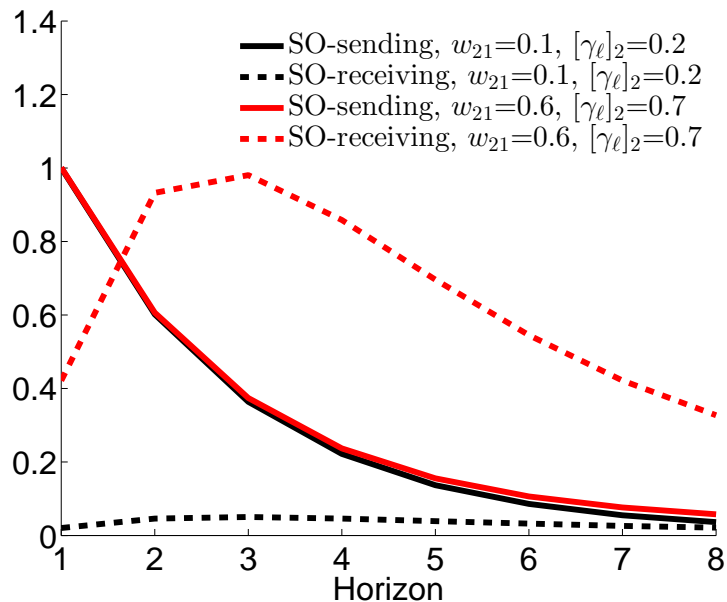
Note: *The figure displays the impulse response functions to a shock s_t in the spillover-sending economy for the spillover-sending and the spillover-receiving economies for different values of w_{21} and $[\gamma_\ell]_2$, $\ell = 0, 1$.*

Figure 3: Asymptotic Bias in the Spillover Estimates Obtained from the Bilateral Model
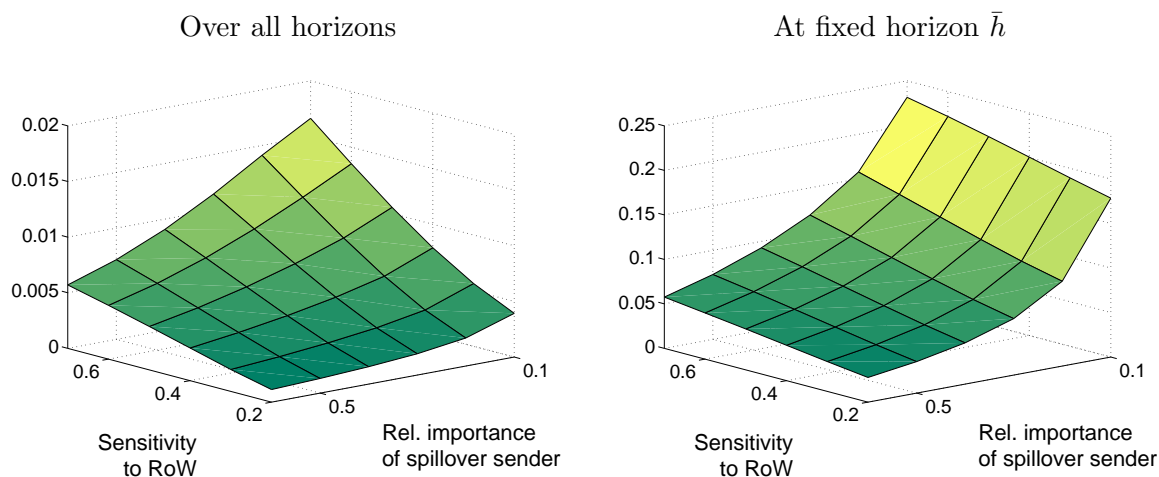
Note: *The panels depict the asymptotic bias in spillovers estimates obtained from the bilateral model for different values of w_{21} and $[\gamma_\ell]_2$.*

Figure 4: Difference Between Bias and RMSE of Spillover Estimates Obtained from the Multilateral and the Bilateral Model

Over all horizons
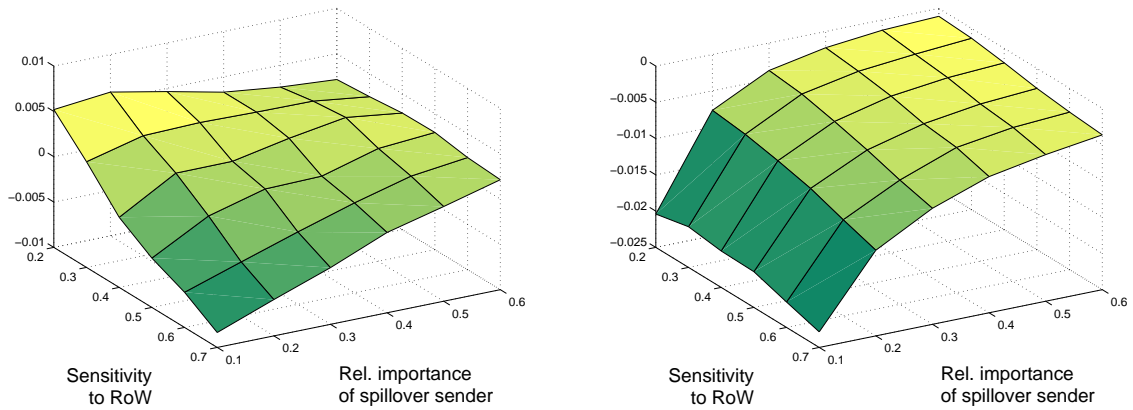
At fixed horizon $\bar{h}$
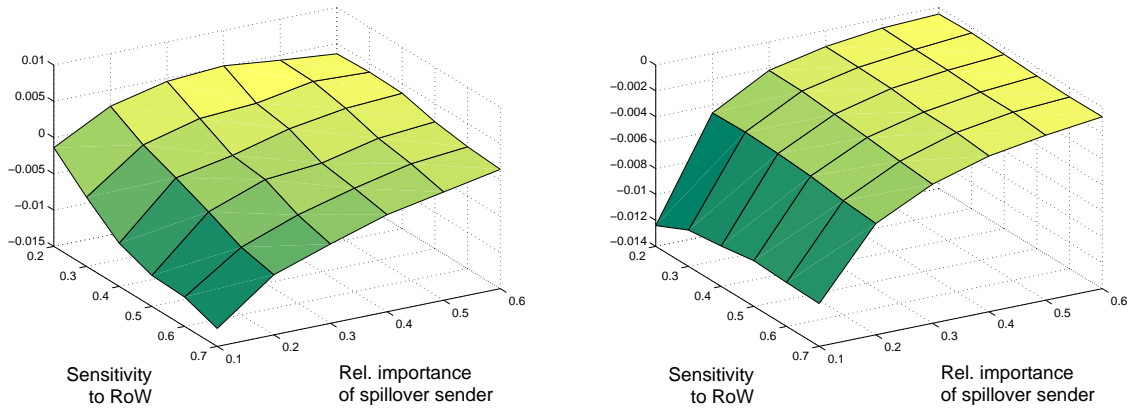
Note: The panels depict the differences between the bias (left-hand side panels) and RMSE (right-hand side panels) in the estimates of the spillovers obtained from the multilateral model and the bilateral model. The differences in the bias and RMSE are plotted for Monte Carlo experiments with different specifications of w_{21} (right-hand side horizontal axes) and $[\boldsymbol{\gamma}_\ell]_2$ (left-hand side horizontal axes).

Figure 5: Difference Between Bias and RMSE of Spillover Estimates Obtained from the Multilateral with Calibrated Weight Matrix $\boldsymbol{C}$ and the Bilateral Model

Over all horizons
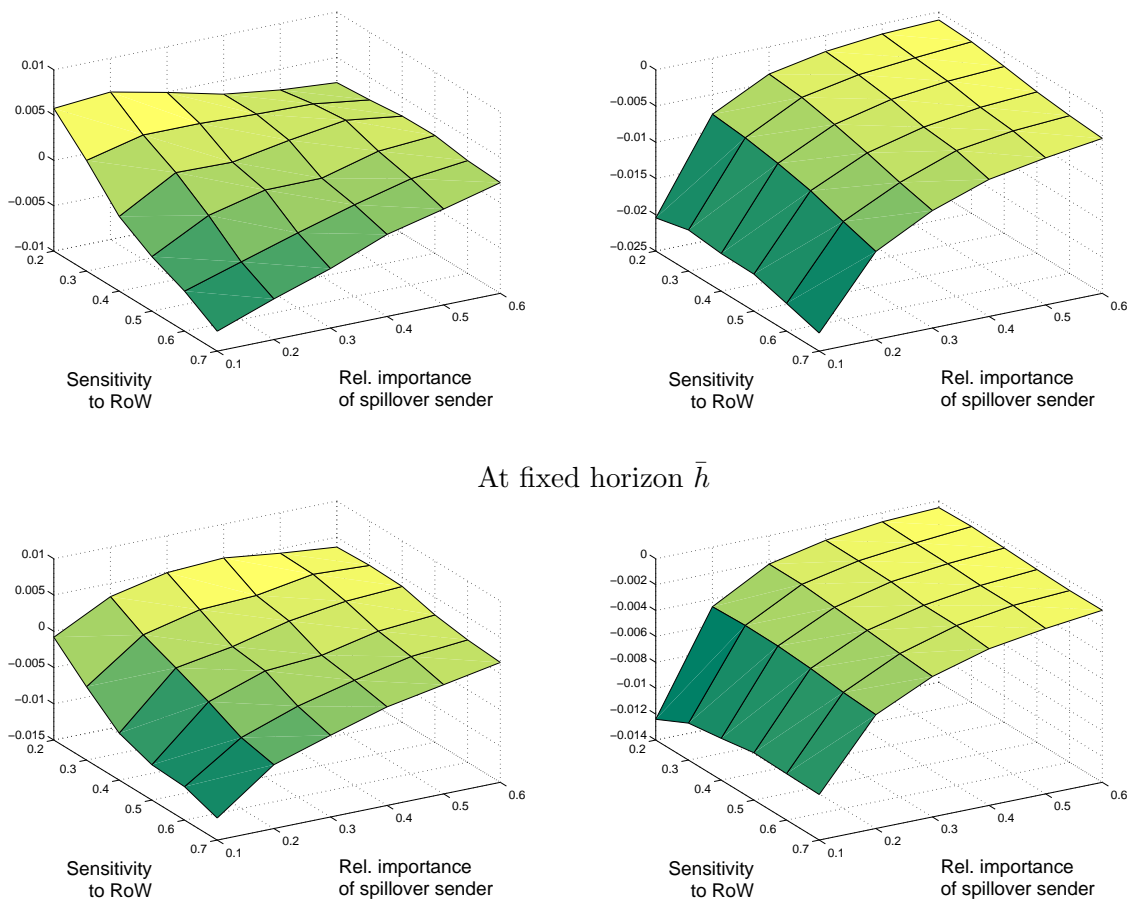
At fixed horizon $\bar{h}$

Note: See the note to Figure 4.

Figure 6: Difference Between Bias and RMSE of Spillover Estimates Obtained from the FAVAR and the Bilateral Model
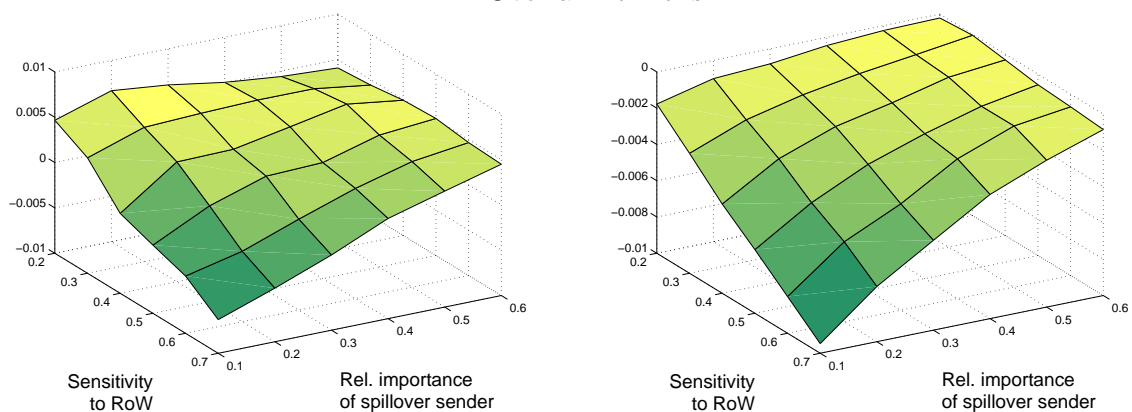
Over all horizons

At fixed horizon $\bar{h}$
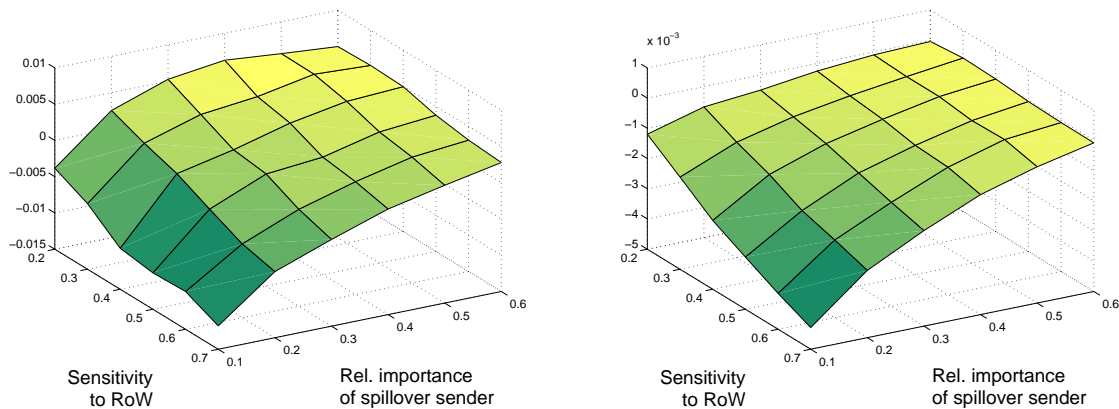
Note: See the note to Figure 4.

Figure 7: Difference Between Bias and RMSE of Spillover Estimates Obtained from the Multilateral Model and the Bilateral Model Augmented by Global Variables

Over all horizons
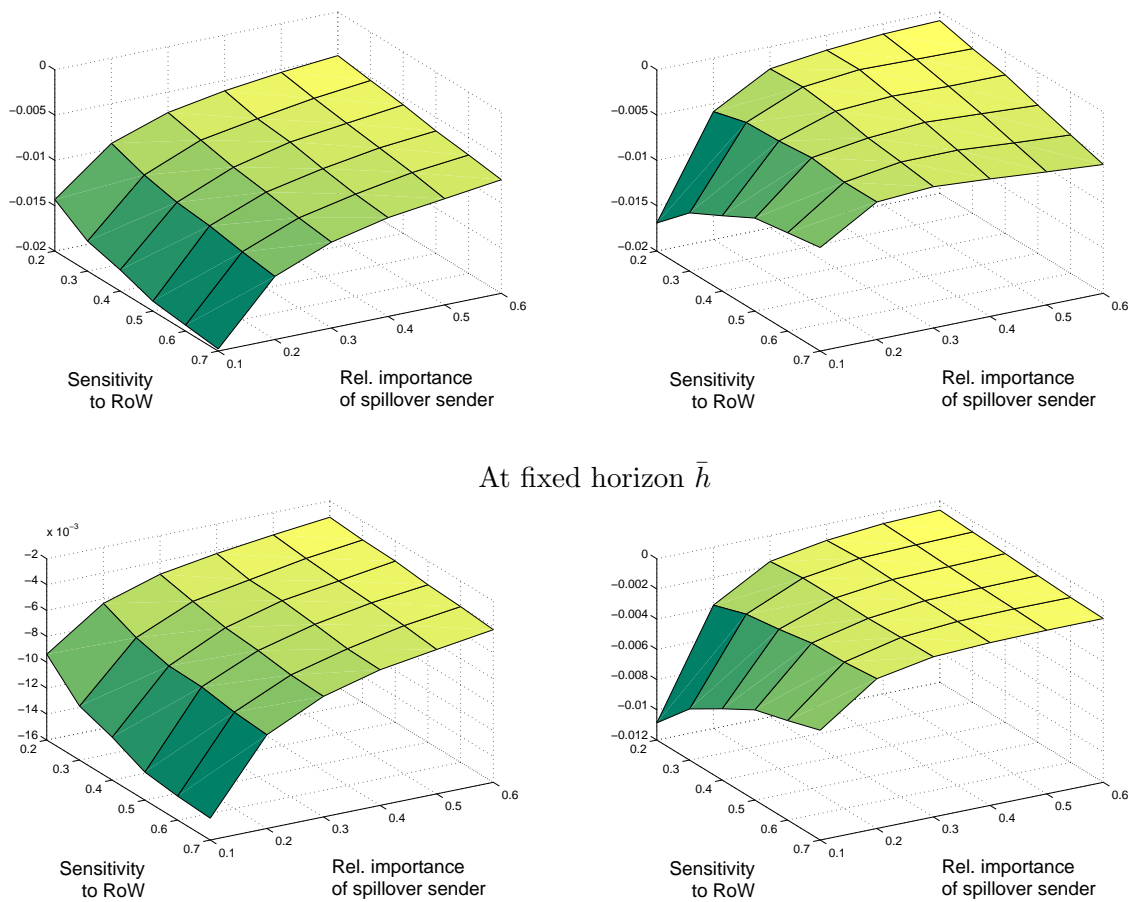
At fixed horizon $\bar{h}$

Note: See the note to Figure 4.

Figure 8: Difference Between Bias and RMSE of Spillover Estimates Obtained from the Multilateral Model and the Bilateral Model with Higher Lag Orders

Over all horizons
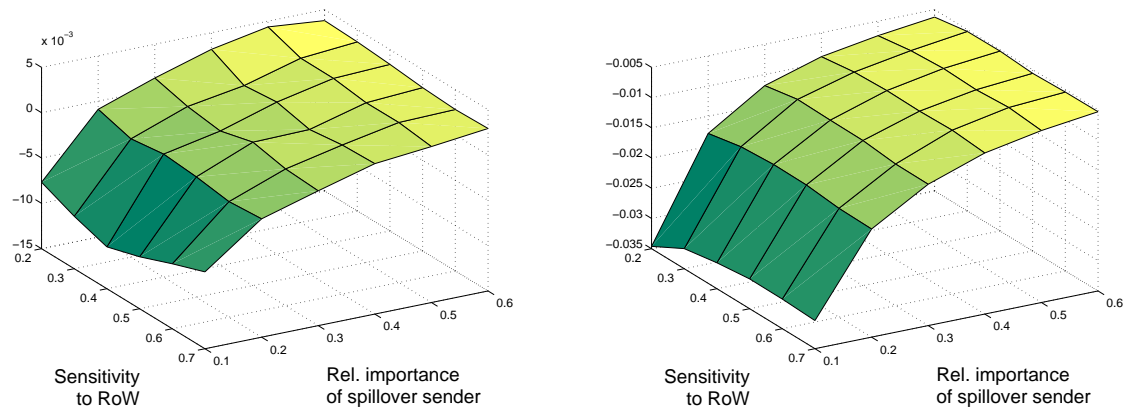
At fixed horizon $\bar{h}$
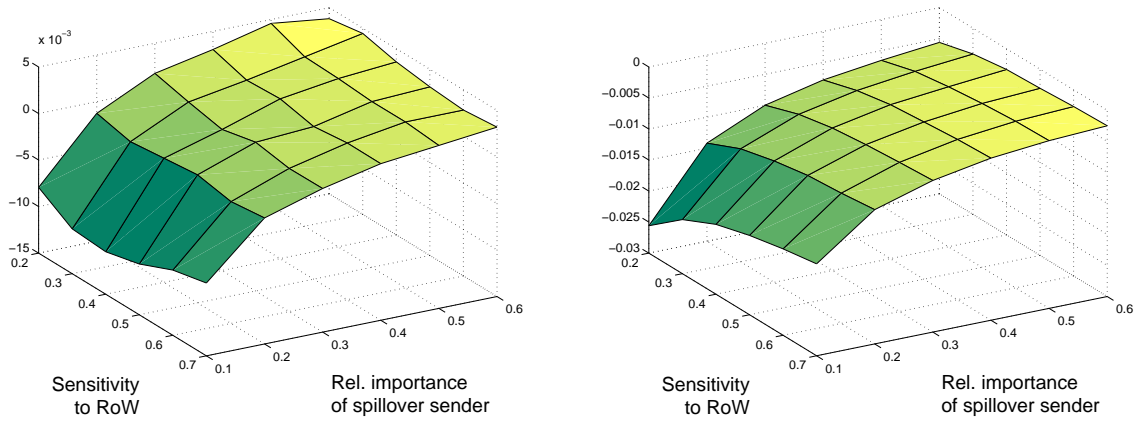
Note: See the note to Figure 4.

Figure 9: Output Gap Responses to A US Contractionary Monetary Policy Shock in the Model of Coenen and Wieland (2002)
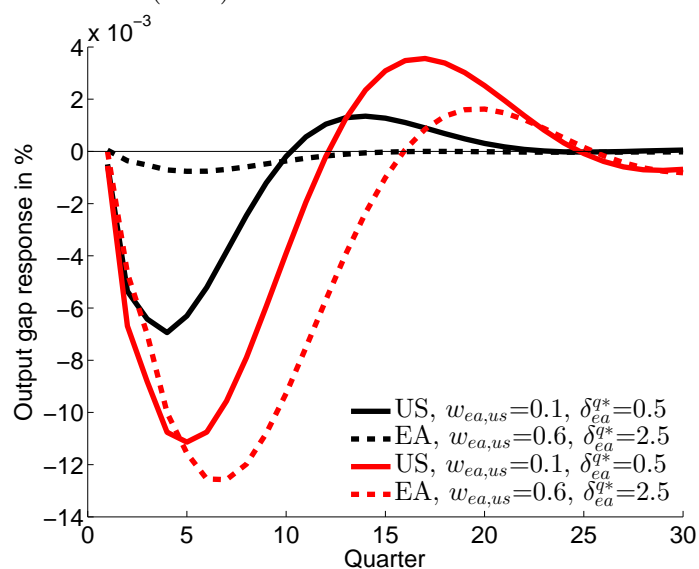
Note: The figure displays the responses of the US and euro area output gap in response to a contractionary US monetary policy shock. The black lines represent impulse response functions that are obtained for a parameterisation with a low overall sensitivity of the euro area to developments in the rest of the world ($\delta^{q^*}_{ea,j}$ in Equation (60)), and a low bilateral share of the US in the euro area's overall integration with the rest of the world ($w_{ea,us}$ in Equation (60)). The red lines represent impulse responses that are obtained from parameterisations with high values of these parameters. The solid lines depicts the output gap response for the US, and the dashed lines those for the euro area.

Figure 10: Difference Between Absolute Bias of Spillover Estimates from the GVAR Model and the Two-Country VAR Model Estimated on Data Simulated Based on the Structural Model of Coenen and Wieland (2002)
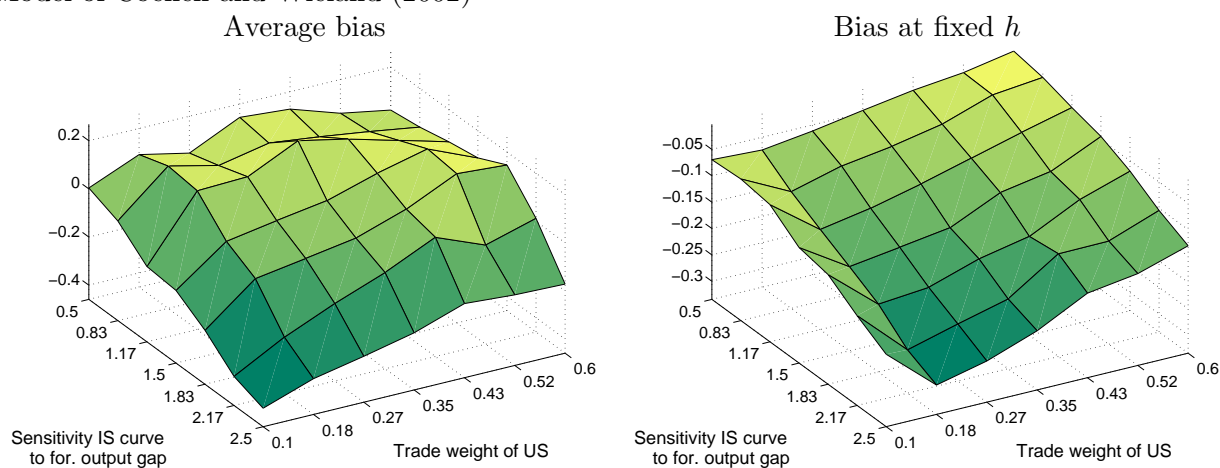

Note: The figure displays the difference between the absolute bias in the spillover estimates obtained from the GVAR model and the two-country VAR model estimated on the data simulated based on the structural model of Coenen and Wieland (2002). In each panel, the right-hand side horizontal axis depicts alternative values of the share of the US in the euro area's overall integration with the rest of the world ($w_{ea,us}$ in Equation (60)), and the left-hand side horizontal axis alternative values of the euro area's overall sensitivity to developments in the rest of the world ($\delta^{q^*}_{ea,j}$ in Equation (60)).
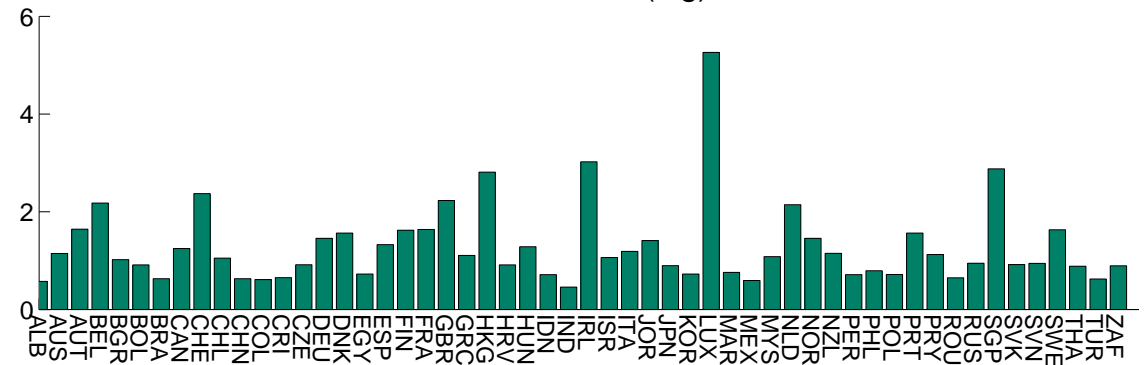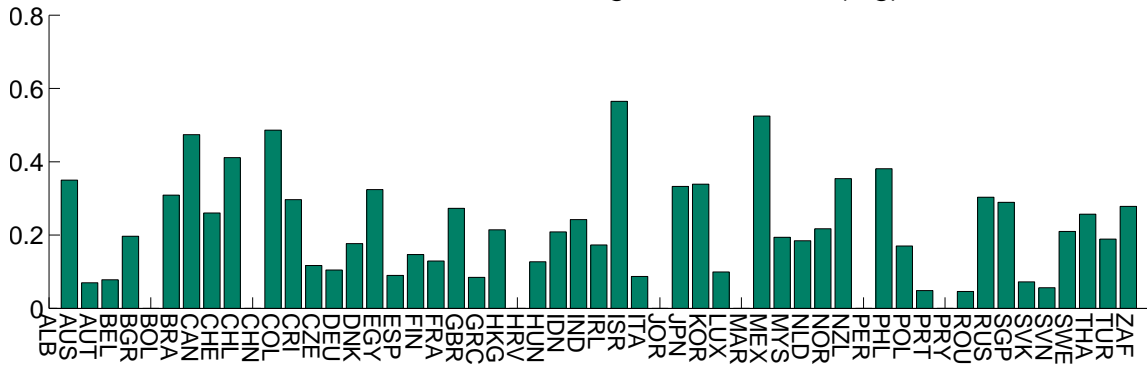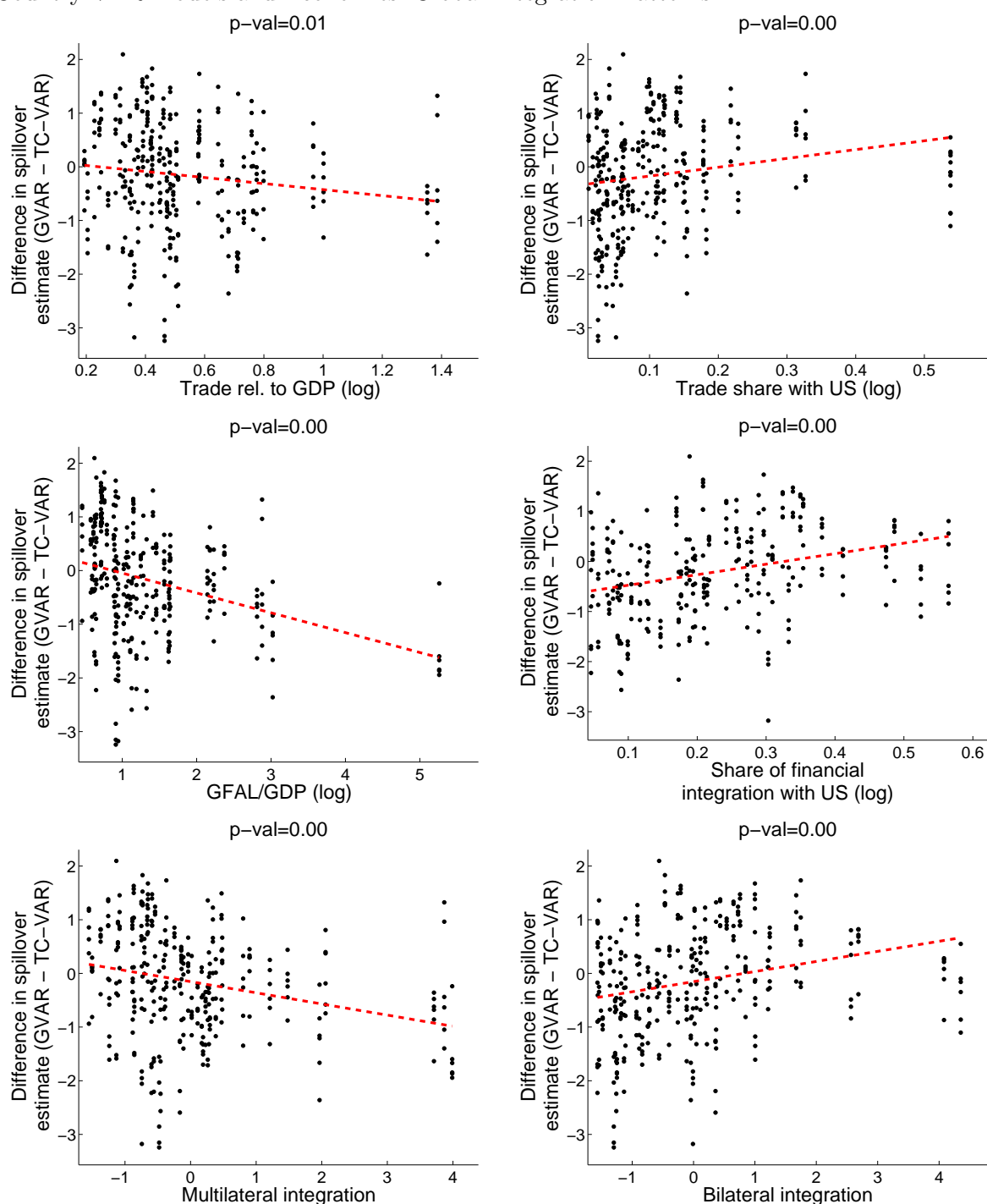
Figure 11: Economies' Integration Patterns
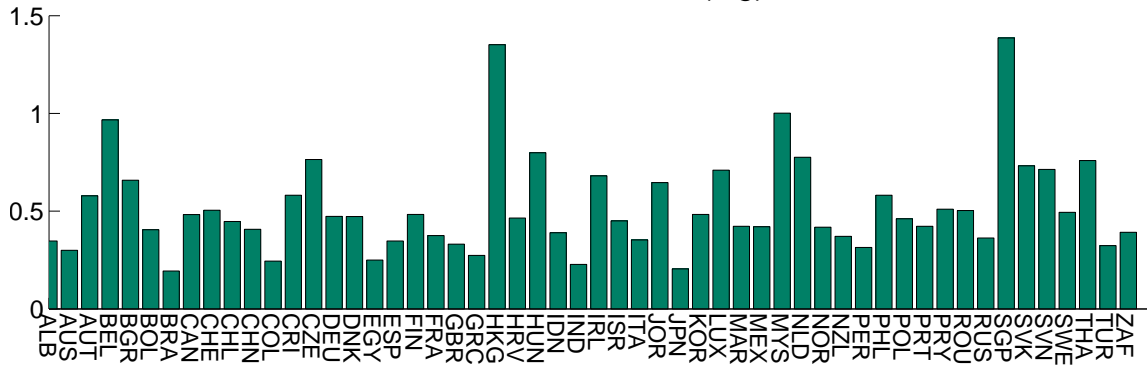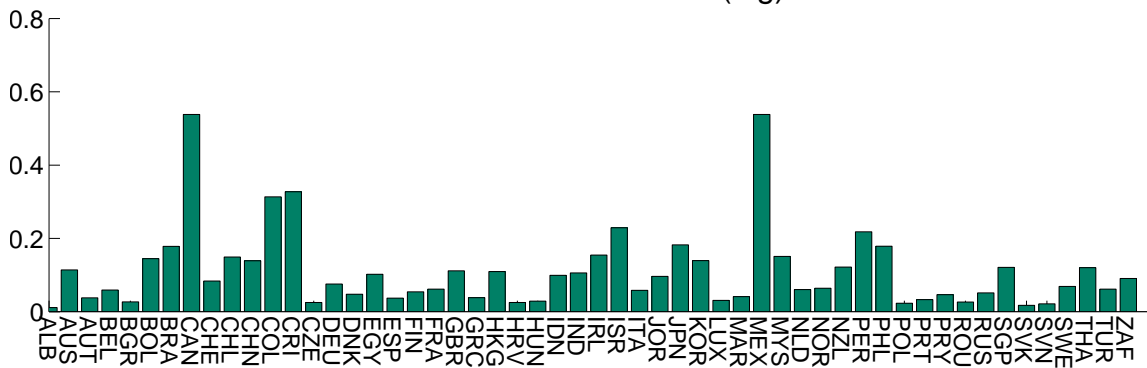
Note: The panels display the logarithm of one plus trade relative to GDP (top panel), the share of imports from and exports to the US in an economy's total trade (second panel), gross foreign assets and liabilities relative to GDP(third panel), and the share of US financial assets held by an economy's domestic residents and economy's foreign liabilities held by US residents in the economy's total foreign assets and liabilities (bottom panel). The data are time averages over 1999 to 2009. See the main text for further details.

Figure 12: Relationship between Differences in Spillover Estimates between GVAR and Two-Country VAR Models and Economies' Global Integration Patterns

Note: The panels show scatter plots of the differences between the spillover estimates obtained from the GVAR model and the two-country VRA models (vertical axes) and economies' multilateral and bilateral integration patterns (horizontal axes). The red dashed lines represent fitted values from linear regressions of the spillover differences on economies' multilateral and bilateral integration patterns. The p-values from these regressions are provided in the panel titles. The differences between the spillover estimates obtained from the GVAR model and the two-country VAR models are standardised for a given monetary policy shock.