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AN ESTIMATED DSGE MODEL OF A SMALL OPEN ECONOMY
WITHIN THE MONETARY UNION:
FORECASTING AND STRUCTURAL ANALYSIS

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An estimated DSGE model of a Small Open Economy within the Monetary Union: Forecasting and Structural Analysis*

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Abstract

In this paper we lay out a two-region DSGE model of an open economy within the European Monetary Union. The model, which is built in the New Keynesian tradition, contains real and nominal rigidities such as habit formation in consumption, price and wage stickiness as well as rich stochastic structure. The framework also incorporates the theory of unemployment as in Gali et al. (2011), small open economy aspects and a nominal interest rate that is set exogenously by the area-wide monetary authority. As an illustration, the model is estimated on Luxembourgish data. We evaluate the properties of the estimated model and assess its forecasting performance relative to reduced form models such as VARs. In addition, we study the empirical validity of the DSGE model restrictions by applying a DSGE-VAR approach. Finally, the estimated model is used to analyze the sources of macroeconomic fluctuations and examine the responses of the economy to structural shocks.

JEL classification: E4, E5, F4
Keywords: DSGE models, DSGE-VAR, open economy, forecasting, VAR.

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1 Introduction

In recent decades a new approach to macroeconomic modeling has involved the development of a generation of real business cycle models (the New Keynesian or New Neoclassical Synthesis models), which propose to extend the general equilibrium framework by introducing imperfect competition and nominal rigidities. An important feature of this class of models-often referred to as DSGE-is that monetary policy has a non-trivial effect on real variables. Therefore, studying the business cycle and macroeconomic implications of alternative government policies has been a natural application of this new generation of models and motivated lots of research. Earlier contributions, including those which extend the framework to open economies, are Clarida, Gali and Gertler (1999) and (2001), Benigno and Benigno (2003), Gali and Monacelli (2005) and many others. Recent developments in numerical and estimation methods enabled the application of advanced econometrics techniques to test the properties of the new generation of DSGE models, which showed a better performance in capturing observed characteristics of real data due to stronger internal persistence mechanisms. Therefore, there is a growing interest from both academia and policymaking institutions in further advancing and using these models for studying macroeconomic fluctuations, assessing economic policy and forecasting. The most influential empirical papers in this area include Smets and Wouters (2003) and (2007), who estimate a DSGE model similar in spirit to Christiano et al. (2005) for the euro area and the US respectively. The authors demonstrate that the estimated model provides a reasonable description of the economy and thus can serve as a useful tool for the analysis of the effects of monetary policy and other structural shocks. Another important conclusion is that the forecasting performance of the DSGE model compares well with reduced form structures such as VAR and BVAR models. Following this seminal work, lots of research has been done to exploit DSGE modeling to study the macroeconomic fluctuations in various countries. In particular, Adolfson et al. (2008) examine the properties of a small open economy model with modified Uncovered Interest Parity condition estimated on Swedish data. Lees et al. (2007) evaluate the performance of a small scale DSGE model applied to New Zealand data. Lubik and Schorfheide (2007) estimate a small-scale DSGE model of a small open economy with a focus on the comparison of the monetary policy conduct in Australia, Canada, New Zealand and the UK. A number of studies employ a two-country framework to analyze the business cycle of European economies within the euro area. In particular, Pytlarczyk (2005) presents a DSGE model for Germany within the monetary union. Burriel et al. (2010) develop a DSGE model for the Spanish economy. There are also similar studies for Austria (Breuss and Rabitsch, 2009), France (Jondeau and Sahuc, 2004), and other countries.

This paper contributes to the fast growing DSGE literature described above and presents a model of a small open economy within the European Monetary Union, combining several of the features in the papers mentioned above. In particular, we develop a medium scale two-region structural model with monopolistic competition in goods and labour markets. The model contains a number of frictions such as habit formation in consumption and price and wage rigidities, which became fairly standard in the recent literature. We adopt a small open economy set up that implies that the rest of the world (euro area) is not affected by domestic dynamics. As a result, the central bank policy instrument - the nominal interest rate - is exogenous from the home economy perspective. We derive a small open economy representation as a limiting case of a two-country framework and, unlike many of the recent DSGE papers, consider a medium rather than small scale specification with an explicit modeling of the labor markets and unemployment. In this respect, we follow an original paper by Gali et al. (2011) that incorporates unemployment into the Smets and Wouters (2007) closed economy model.

From the empirical side, we contribute to the recent DSGE literature by presenting evidence for an additional country on the fit and forecasting performance of DSGE models estimated with a Bayesian approach. More specifically, we analyze the main properties of the estimated model, assessing the importance of various shocks and frictions for explaining the dynamics of the Luxembourgish economy.[1]

We then evaluate the model's point and density forecasting performance by comparing the accuracy of its out-of-sample predictions relative to those from reduced form models such as VARs. In addition, we study the empirical validity of DSGE model restrictions by applying a DSGE-VAR analysis, as developed in Del Negro and Schorfheide (2004) and Del Negro et al. (2007). We include the DSGE-VAR model into the forecasting exercise in order to assess the ability of the DSGE-based versus atheoretical (BVAR) prior to improve the forecasting performance of the unrestricted VAR model.

Finally, the estimated model is used to calculate variance decompositions and impulse responses, in order to evaluate the sources and propagation of macroeconomic fluctuations.

In the process of description of the estimation results we discuss how our work compares to previous studies. Our DSGE model shows a superior out-of-sample forecasting performance (at the one-quarter-ahead horizon) than unrestricted VARs and BVARs. We also demonstrate that the restrictions implied by the DSGE model lead to an improvement of the performance of the standard VAR in predicting the dynamics of the labor market variables such as wages and unemployment.

The paper is organized as follows. In the next two sections we present our small open economy model and its log linear representation. Section 4 describes the data, alternative forecasting models and estimation results. The forecast evaluation and comparison are presented in Section 5. The application of the model to the analysis of business cycle fluctuations is discussed in Section 6. Finally, Section 7 contains some concluding remarks.

[1] As for existing structural models for Luxembourg, Pierrard and Sneessens (2009) have developed an OLG small open economy model. The authors concentrate on modeling the realistic features of the Luxembourg labor market. The "pure" OLG representation allows studying the demographic questions such as the consequences of the ageing of the population and the potential effects of alternative macroeconomic policies. The model is then calibrated on Luxembourg data and simulated. Other studies for Luxembourg based on the DSGE methodology include papers by Deak et al. (2011) and (2012). These papers present an LSM - DSGE small open economy model for Luxembourg, which is built following Blanchard (1985) OLG approach. The model incorporates more realistic goods market structure with monopolistic competition, the distinction between tradable, non-tradable goods and the banking sector The model is calibrated and used to study the reaction of the economy to real and financial shocks.

2 A Small Open Economy Model

In this section we formulate an open economy DSGE model with theoretical foundations closely related to the papers by Gali and Monacelli (2005) and De Paoli (2009). The model contains a number of rigidities typically used in the empirical DSGE literature in order to capture the properties of real data (Christiano, Eichenbaum, Evans (2001), Smets and Wouters (2003) and (2007)). In particular, we introduce habit formation in consumption as well as Calvo price and wage stickiness. Moreover, we explicitly incorporate the theory of unemployment into the model set up following the recent paper by Gali, Smets and Wouters (2011).

The framework is represented by a two-country dynamic general equilibrium model where both sides, Home (the small open economy – H) and Foreign (the rest of the world, the relatively closed economy – F), are explicitly modeled. A continuum of infinitively lived domestic households belongs to the interval $[0, n)$, while foreign agents belong to the segment $(n, 1]$. The small open economy problem is derived as a limiting case ($n \to 0$) of such a framework (as in De Paoli, 2009). Therefore, the home economy due to its small size is assumed to have a negligible impact on the rest of the world. Households receive utility from consumption and disutility from work. The home economy is composed of final and intermediate goods producers, consumers, and labour unions.[2] Agents consume the final consumption good, which includes goods produced by the domestic economy as well as imported goods. The share of imported goods may vary in the consumption basket of each country. Thus, the model allows for the presence of home bias in consumption. Firms, which are monopolistically competitive, hire labor to produce differentiated goods. Prices on the goods market are assumed to be sticky and evolve according to Calvo staggering scheme (1983). In addition, we assume monopolistic competition and Calvo wage setting behavior on the labor market. Furthermore, production subsidies are introduced in order to offset the monopolistic distortions. In this version of the model, we abstract from capital accumulation. The international and domestic asset markets are complete. The law of one price holds for individual goods at all times. The small open economy is assumed to belong to the common currency area with the foreign country. The monetary authority (ECB) sets the interest rate following the Taylor rule based on the economic performance of the whole EMU. Thus, the interest rate is an exogenous variable from the small open economy perspective.

2.1 Representative Households and preferences

The expected life-time utility function maximized by a representative household of country H is given by:

$$U_t^j = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \varepsilon_t^c [U(\widetilde{C}_t^j) - \varepsilon_t^l V(L_t^j)] \right\}, \tag{1}$$

[2] We assume a somewhat simpified structure for the foreing economy. In particular, we abstract from explicit modeling the production side and assumme that housholds are both consumers and producers. Moreover, we assume that there are no labor market frictions and unemployment.

where j is the index specific to the household; $\widetilde{C}_t^j$ denotes the time t per capita consumption of the composite commodity bundle, L_t^j is the labor effort and $0 < \beta < 1$ is the intertemporal discount factor. There exists a continuum h of different labor types, denoted by $l_t^j(h)$ and indexed for home country on the interval $[0, n]$. Then labor effort of the individual j is defined as: $L_t^j = \int_0^n l_t^j(h)dh$. ε_t^c and ε_t^l denote an exogenous preference and labor supply shocks respectively. In our analysis we assume that preferences have the following functional form:

$$U(\widetilde{C}_t^j) = \frac{(C_t^j - \chi C_{t-1})^{1-\sigma_c}}{1-\sigma_c}, \qquad V(L_t^j) = \frac{(L_t^j)^{1+\eta}}{1+\eta},$$

where $\sigma_c > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, and $\eta \geq 0$ is equivalent to the inverse of the elasticity of labour supply. χ is an external habit formation parameter, which determines the dependence of the current individual consumption from the aggregate lagged consumption index. The composite consumption good C is a Dixit-Stiglitz aggregator of goods produced at home and abroad and defined as:

$$C^j = [\upsilon^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-\upsilon)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}. \tag{2}$$

Preferences for the rest of the world (denoted with the asterisk) are specified in a similar fashion:

$$C^{j*} = [(\upsilon^*)^{\frac{1}{\theta}} (C_H^*)^{\frac{\theta-1}{\theta}} + (1-\upsilon^*)^{\frac{1}{\theta}} (C_F^*)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}, \tag{2a}$$

where $\theta > 0$ is the intratemporal elasticity of substitution, υ and υ^* are the parameters that determine the preferences of agents in countries H and F, respectively, for the consumption of goods produced at Home. As in Sutherland (2002) and De Paoli (2009) we assume that $(1-\upsilon)$, the share of imported goods from country F in the consumption basket of country H, increases proportionally to the relative size of the foreign economy $(1-n)$ and the degree of openness α. Therefore, $(1-\upsilon) = (1-n)\cdot\alpha$. Similarly, $\upsilon^* = n\cdot\alpha$. Such a specification allows modeling of home bias in consumption as a consequence of different country size and degree of openness.

The consumption sub-indices of home and foreign-produced differentiated goods are defined as follows:

$$C_H = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c_h(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c_f(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

$$C_{H^*} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c_h^*(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F^* = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c_f^*(z)^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution across the differentiated goods.

The solution to the cost minimization problem yields the following demand equations for differentiated goods produced at home and abroad:

$$c_h(z) = \frac{1}{n}\left(\frac{p_h(z)}{P_H}\right)^{-\sigma} C_H, \qquad c_f(z) = \frac{1}{1-n}\left(\frac{p_f(z)}{P_F}\right)^{-\sigma} C_F, \tag{4}$$

where $p_H(z)$ and $p_F(z)$ are prices (in units of the domestic currency) of the home-produced and foreign-produced intermediate goods. $P_H = \left[\left(\frac{1}{n}\right)\int_0^n p_h(z)^{1-\sigma}d(z)\right]^{\frac{1}{1-\sigma}}$ is the domestic price index and $P_F = \left[\left(\frac{1}{1-n}\right)\int_n^1 p_f(z)^{1-\sigma}d(z)\right]^{\frac{1}{1-\sigma}}$ is a price index for goods imported from country F. The price indices given above represent cost-minimizing prices of a unit of final (home or foreign) good basket.

Furthermore, optimal allocation of expenditures between domestic and imported goods is given by:

$$C_H = \upsilon\left(\frac{P_H}{P}\right)^{-\theta} C; \qquad C_F = (1-\upsilon)\left(\frac{P_F}{P}\right)^{-\theta} C \tag{5}$$

where

$$P = [\upsilon P_H^{1-\theta} + (1-\upsilon)P_F^{1-\theta}]^{\frac{1}{1-\theta}} \tag{6}$$

is the consumer price index for country H.

Similar demand functions can be derived for the foreign country.

2.1.1 The asset market structure and consumer's problem

Similar to Chari et al. (2002) we assume that foreign and domestic households have access to the international financial market, where state-contingent nominal bonds denominated in the home currency are traded. Thus, markets are complete domestically and internationally. The budget constraint of the consumer in the Home country at period t is given by:

$$P_t C_t^j + B_{t+1}^j / R_{t+1} \leq B_t^j + W_t^j L_t^j + TR_t, \tag{7}$$

where B_{t+1}^j is the holding of a nominal state-contingent bond that pays one unit of home currency in period $t+1$, R is the gross nominal interest rate, $W_t^j L_t^j$ represents the total wage income, and TR_t is the dividends and transfers to households. Maximizing the utility function subject to a sequence of budget constraints, households make optimal consumption-saving and labor supply decisions. First order conditions for consumption and bonds holding imply the following Euler equation[3]:

$$\varepsilon_t^c (C_t - \chi C_{t-1})^{-\sigma_c} = \beta\left[\varepsilon_{t+1}^c (C_{t+1} - \chi C_t)^{-\sigma_c} R_t \frac{P_t}{P_{t+1}}\right]. \tag{8}$$

[3]dropping the j index

Similarly for the foreign economy:

$$\varepsilon_t^{c*}(C_t^* - \chi C_{t-1}^*)^{-\sigma_c} = \beta \left[\varepsilon_{t+1}^{c*}(C_{t+1}^* - \chi C_t^*)^{-\sigma_c} R_t^* \frac{P_t^*}{P_{t+1}^*} \right]. \tag{8a}$$

The complete-market assumption implies that the marginal rate of substitution between consumption in the two countries is equalized:

$$\frac{\varepsilon_{t+1}^{c*} U_C(C_{t+1}^*)}{\varepsilon_t^{c*} U_C(C_t^*)} \frac{P_t^*}{P_{t+1}^*} \frac{S_t}{S_{t+1}} = \frac{\varepsilon_{t+1}^{c} U_C(C_{t+1})}{\varepsilon_t^{c} U_C(C_t)} \frac{P_t}{P_{t+1}}. \tag{9}$$

The equation presented above illustrates the equality of nominal wealth in both countries in all states and time periods. Because domestic and foreign agents are identical ex-ante so that agents' marginal utility of income are equal, the international risk sharing condition can be also written as : $\frac{\varepsilon_t^{c*} U_C(C_t^*)}{\varepsilon_t^{c} U_C(C_t)} = k \frac{S_t P_t^*}{P_t}$, where the real exchange rate is defined as $RS_t = \frac{S_t P_t^*}{P_t}$ (where S_t is the nominal exchange rate defined as a unit of foreign currency in terms of the domestic one) and k is a constant that depends on initial conditions ($k \equiv U_C(C_0^*) P_0 / U_C(C_0) P_0^* S_0$). In a model with flexible exchange rate regime, the risk sharing equation determines the endogenous path of the exchange rate. In the monetary union specification (when nominal exchange rate is fixed) this equation can be viewed as a condition restricting the long run divergence of consumption across borders. In particular, in the two-country setting when economies have a comparable size, this equation (together with the domestic Euler equation) can be used to pin down foreign consumption. However, in the small economy framework, foreign consumption should be exogenous from the home economy perspective. Thus, the separate Euler equation for the foreign country or the exogenous process for consumption (output) should be used. In addition, note that completeness of financial markets in the currency union implies the equality of the nominal interest rates across countries at all times, i.e. $R_t = R_t^*, \forall t$.

2.2 Firms

2.2.1 Technology and marginal cost

Each firm, which is a monopolistic producer of a differentiated good, uses the following technology:

$$Y_{h,t}(z) = A_t L_t(z)^{1-\lambda}, \tag{10}$$

where $L_t(z)$ is a composite labour input measured by hours worked; A_t is total factor productivity with $\varepsilon_t^a \equiv \log(A_t)$ and $\varepsilon_t^a = \rho \varepsilon_{t-1}^a + \upsilon_t$, where υ_t is i.i.d shock with zero mean.

The firm's profit is given by:

$$p_{h,t}(z) Y_{h,t}(z) - W_t L_t(z),$$

where W_t is the aggregate nominal wage rate .

The first–order conditions with respect to labor lead to the following condition:

$$(\partial L_t(z)): \quad \Theta_t(z)(1-\lambda)A_t L_t(z)^{-\lambda} = W_t \ ,$$

where $\Theta_t(z) = W_t/MPL_t$ is the Lagrange multiplier associated with the production function and equals marginal cost MC_t.

The nominal marginal cost MC_t is equal to:

$$MC_t = (1-\lambda)^{-1} (A_t)^{-1} W_t L_t(z)^{\lambda}. \tag{11}$$

Then the real marginal cost (expressed in terms of domestic prices) , is given by:

$$MC_t^r = \frac{MC_t}{P_{H,t}} = (1-\lambda)^{-1} (A_t)^{-1} W_t^r \frac{P_t}{P_{H,t}} L_t(z)^{\lambda}, \tag{11a}$$

where $W_t^r = W_t/P_t$ denotes the real wage. The aggregate domestic output index is represented by $Y = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n Y_h(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}$, analogous to the one introduced for consumption.

2.2.2 Optimal Pricing Decisions

The domestic firm sets the price $p_h(z)$ and takes as given P, P_H, P_F, and C. The price-setting behavior is modeled according to Calvo (1983). Each time period a fraction $\gamma^p \in [0,1)$ of randomly picked producers in country H are not allowed to change their prices. Thus the parameter γ^p reflects the level of price stickiness. The remaining fraction $(1-\gamma^p)$ can choose the optimal sector-specific price by maximizing the expected discounted value of profits subject to the demand function derived from the expenditure minimization problem:

$$\max_{\widetilde{p}_{h,t}(z)} E_t \sum_{i=0}^{\infty} (\gamma^p \beta)^i \left[\Lambda_{t,i} \frac{(1-\tau_i)\widetilde{p}_{h,t}(z) - MC_{t+i}}{P_{H,t+i}} Y_{h,t,t+i}(z) \right],$$

$$s.t \quad Y_{h,t,t+i}(z) \quad = \quad \left(\frac{p_h(z)}{P_H}\right)^{-\sigma} Y_H,$$

where $\beta^i \Lambda_{t,i} = \beta^i \frac{U_{C,t+i}}{U_{C,t}}$ is the firm's stochastic discount factor (equal to the discount factor of the households, which are the owners of the firms), $\widetilde{p}_{h,t}(z)$ is the price of the differentiated good z chosen at time t , and $Y_{h,t,t+i}(z)$ is the total demand for good z at time $t+i$, conditional on the fact that the price $\widetilde{p}_{h,t}(z)$ has not been changed; τ_i is a time varying proportional tax rate. All producers who belong to the fraction $(1-\gamma^p)$ choose the same price.

The optimal price $\widetilde{p}_{h,t}(z)$, is derived from the first-order conditions that take the following form:

$$E_t \sum_{i=0}^{\infty} (\gamma^p \beta)^i \Lambda_{t,i} \left(\frac{p_h(z)}{P_H}\right)^{-\sigma} Y_H \left[MC_{t+i}^r - \frac{1}{\mu_i^p} \frac{\widetilde{p}_{h,t}(z)}{P_{H,t}} \right] = 0, \tag{12}$$

where $\mu_i^p = \frac{\sigma}{(1-\tau_i)(\sigma-1)}$ represents the overall degree of monopolistic distortion and leads to a wedge between price and the marginal costs. Benigno and Benigno (2006) and De Paoli (2009) refer to this gap as the mark-up shock, which fluctuates due to time variation of the tax rate. A Calvo-type setting implies the following law of motion for the price indices:

$$P_{H,t} = [\gamma^p (P_{H,t-1})^{1-\sigma} + (1-\gamma^p)\widetilde{p}_{h,t,}(z)^{1-\sigma}]^{\frac{1}{1-\sigma}}. \tag{13}$$

Similar conditions can be derived for the producers in country F.

2.3 Labor decisions and wage setting

The amount of labor used by firm z is given by the following Dixit-Stiglitz aggregator:

$$L_t(z) \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma_{w,t}}} \int_{h=0}^{n} l_t(h,z)^{\frac{\sigma_{w,t}-1}{\sigma_{w,t}}} dh \right]^{\frac{\sigma_{w,t}}{\sigma_{w,t}-1}}, \tag{14}$$

where $l_t(h,z)$ denotes the amount of type h labor used by firm z, $\sigma_w > 1$ is the elasticity of substitution across the differentiated types of labor.

Firm z chooses a sequence of different types of labor $l_t(h,z)$ to minimize the total cost of production given by:

$$\min_{l_t(h,z)} \int_0^n W_t(h) l_t(h,z) dh$$

$$s.t \qquad Y_{h,t}(z) = A_t \left\{ \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma_{w,t}}} \int_{h=0}^{n} l_t(h,z)^{\frac{\sigma_{w,t}-1}{\sigma_{w,t}}} dh \right]^{\frac{\sigma_{w,t}}{\sigma_{w,t}-1}} \right\}^{1-\lambda}.$$

Cost minimization implies the following equation for the demand for labor:

$$l_t(h,z) = \frac{1}{n} \left(\frac{W_t(h)}{W_t}\right)^{-\sigma_{w,t}} L_t(z), \tag{15}$$

where the aggregate wage index (minimizing expenditures needed to purchase one unit of labor L_t) is given by $W_t \equiv \left[\left(\frac{1}{n}\right) \int_{h=0}^{n} W_t(h)^{1-\sigma_{w,t}} dh \right]^{\frac{1}{1-\sigma_{w,t}}}$. Furthermore, note that the relationship between the aggregate labor demand and production is given by:

$$L_t = \int_0^n L_t(z) dz = \int_0^n \left(\frac{Y_{h,t}(z)}{A_t}\right)^{\frac{1}{1-\lambda}} dz = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\lambda}} Z_t, \tag{16}$$

where $Z_t = \int_0^n \left(\frac{Y_{h,t}(z)}{Y_t}\right)^{\frac{1}{1-\lambda}} dz$.

Following Erceg, Henderson and Levin (2000), we introduce staggered wage contracts into the model. In particular, each period the wage rate of a given type h can be reset optimally with the probability $1-\gamma^w$. The fraction γ^w of wage rates that cannot be optimized is set equal to the previous period wages, i.e. $W_t(h) = W_{t-1}(h)$. Thus, the parameter γ^w represents the measure of the nominal wage rigidities. The optimal choice of wage $\widetilde{W}_t(h)$ brings about a maximization of the expected household utility (1) subject to the sequence of budget constraints (7) and a sequence of demand schedules of the form (15). The first order conditions can be written as:

$$E_t \sum_{i=0}^{\infty} (\gamma^w \beta)^i \left\{ \left(\frac{l_{t+i,t}(h)}{(C_{t+i} - \chi C_{t+i-1})^{-\rho}} \right) \left(\frac{\widetilde{W}_{t+i,t}(h)}{P_{t+i}} - \mu^n_{w,t+i} MRS_{t+i,t} \right) \right\} = 0, \tag{17}$$

where $l_{t+i,t}(h)$ denotes period $t+i$ labor inputs of workers whose wage was last reoptimized in period t; $MRS_t = -\frac{U_{L,t}}{U_{C,t}} = \varepsilon^l_t (C_t - \chi C_{t-1})^{\sigma_C} l_t(h)^\eta$ is the marginal rate of substitution between consumption and labor. Finally, $\mu^n_{w,t+i} \equiv \frac{\sigma_{w,t}}{(\sigma_{w,t}-1)}$ is the natural (or desired) wage markup, that would prevail under the flexible wages assumption. Time variation of this parameter leads to changes in worker's market power. The solution $\widetilde{W}_t(h)$ will be the same for all wage-optimizing agents. Thus, the index "h" can be dropped.

Similarly to the price equation, the aggregate wage index can be written as follows:

$$W_t = [\gamma^w (W_{t-1})^{1-\sigma_{w,t}} + (1-\gamma^w) \widetilde{W}_t(h)^{1-\sigma_{w,t}}]^{\frac{1}{1-\sigma_{w,t}}}. \tag{18}$$

2.3.1 Unemployment dynamics

Unemployment is introduced into the model following the approach presented in recent papers by Gali (2011a,b) and Gali, Smets and Wouters (2011). Consider a household j who supplies labor of type h. The condition that determines the participation of the individual in the labor market can be obtained using the welfare optimization criteria (and taking as given wages set on the labor market). More specifically, household will work only if his marginal utility of consumption (per unit of value) will be greater or equal to his marginal disutility of work, i.e :

$$\frac{(C^j_t - \chi C_{t-1})^{-\sigma_c}}{P_t} \geq \frac{\varepsilon^l_t l_t(h)^\eta}{W_t(h)}.$$

In a symmetric equilibrium the supply of type h labor $l^S(h)$ will be determined by a standard intratemporal optimality condition:

$$\frac{W_t(h)}{P_t} = \varepsilon^l_t (C_t - \chi C_{t-1})^{\sigma_c} l^S(h)^\eta. \tag{19}$$

Aggregating over labor types, we can interpret $\widetilde{L}^S$ as the measure of the potential labor force (maximum level of labor employment rate). Then the aggregate unemployment rate at period t is defined as the log difference between the labor force and the actual labor employed:

$$u_t \equiv \ln(\widetilde{L}^S_t) - \ln(L_t). \tag{20}$$

Such a definition of the unemployment rate is taken for practical purposes and, given the low observed unemployment rates, is very close to the conventional level given by $1 - L_t/\widetilde{L}_t^S$.[4] The formulation of unemployment presented here is linked to the concept of involuntary unemployment. In particular, unemployed workers include all the individuals who would like to participate in the labor market (given the current conditions) but are not currently employed.[5]

We would like to note some differences between the modeling approach presented here and the specification in Gali, Smets and Wouters (2011). In particular, the latter one is written in terms of employment rather than hours worked. A reformulation of the model with the different measure of the labor input introduces certain changes in the presentation of consumer preferences but does not affect the functional form of resulting model equations. We did estimate the model totally formulated in terms of employment thus exactly replicating the set up of GSW. However, in our case, using hours as the labor input and introducing the equation linking hours and employees improves the fit of the model. At the same time, our model (implicitly) contains a simplifying assumption that employed and unemployed individuals want to work the same amount of hours. For this reason, equation (20) can be equivalently written in terms of employment as in GSW.

2.4 Real Exchange Rate Decomposition and PPP Violation

The real exchange rate in the model of a currency union is defined as a relative price of foreign and home goods and is equal to $RS_t = P_t^*/P_t$. We assume that the law of one price holds for differentiated goods, i.e., $p_h(z) = p_h^*(z)$ and $p_f(z) = p_f^*(z)$. This in turn implies that $P_H = P_H^*$ and $P_F = P_F^*$. However, our model specification implies violation of the Purchasing Power Parity (PPP) at the aggregate price level, i.e., $P \neq P^*$ and thus $RS \neq 1$. We use the price indexes to express the real exchange rate as a function of relative prices and preference parameters. Then, the real exchange rate can be presented as:

$$RS = \left(\frac{\upsilon^* + (1-\upsilon^*)(P_{FH})^{1-\theta}}{\upsilon + (1-\upsilon)(P_{FH})^{1-\theta}} \right)^{\frac{1}{1-\theta}}, \tag{21}$$

where $P_{FH} = \frac{P_F}{P_H}$ is the terms of trade. Such a decomposition enables to analyze the source of the PPP violation. In particular, under $\upsilon \neq \upsilon^*$, the RS is affected by the terms of trade. For the small open economy model specification, given the assumptions on υ and υ^*, the difference in country sizes necessarily results in different shares of consumption of home-produced goods in countries H and F. This so-called home bias channel of the PPP violation has also been previously analyzed by De Paoli (2009) and Sutherland (2002). The violation of PPP implies

[4] For unemployment rates near zero, the following approximation applies: $1 - L_t/\widetilde{L}_t^S = 1 - \exp\{-u_t\} \simeq u_t$.

[5] Gali, Smets and Wouters (2011) admit that in their model, unemployed individuals will receive a higher utility ex-post, since their consumption will be the same and, in addition, they will not experience a disutility from work. Such a result is an unavoidable consequence of the assumption of full consumption risk-sharing among individuals, which was made in order to preserve the representative household framework and ensure tractability.

that fluctuations in the real exchange rate may result in a divergence in consumption across countries even under optimal risk sharing.

2.5 Market clearing and aggregate demand

The condition for goods market clearing in the small open economy is given by:

$$Y_t(z) = \int_{j=0}^{n} c_h(z)dj + \int_{j^*=n}^{1} c_h^*(z)dj^*, \tag{22}$$

where $c_h(z)$ and $c_h^*(z)$ represent individual domestic and foreign demand for good $z \in (0, n]$ produced at the home economy. Similarly, the total demand in the rest of the world (country F) is given by:

$$Y_t^*(z) = \int_{j=0}^{n} c_f(z)dj + \int_{j^*=n}^{1} c_f^*(z)dj^*, \text{ for } z \in (n, 1]. \tag{23}$$

Plugging in the corresponding demand functions (4 and 5) we obtain the following expression:

$$Y_t(z) = \left(\frac{p_{h,t}(z)}{P_{H,t}}\right)^{-\sigma} \left[\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} \left\{\upsilon C_t + \left(\frac{1}{RS_t}\right)^{-\theta} \upsilon^* C_t^* \frac{1-n}{n}\right\} + G_{H,t}\right] \tag{24}$$

and for goods produced in country F:

$$Y_t^*(z) = \left(\frac{p_{f,t}(z)}{P_{F,t}}\right)^{-\sigma} \left[\left(\frac{P_{F,t}}{P_t}\right)^{-\theta} \left\{(1-\upsilon)C_t \frac{n}{1-n} + \left(\frac{1}{RS_t}\right)^{-\theta} (1-\upsilon^*)C_t^*\right\} + G_{F,t}^*\right] \tag{25}$$

where G and G^* are country-specific exogenous demand (government spending) shocks.

In order to obtain the small open economy version of the general two-country framework, we apply the assumptions $\upsilon^* = n \cdot \alpha$ and $(1-\upsilon) = (1-n) \cdot \alpha$ and take the limit $n \to 0$ similar to De Paoli (2009). Furthermore we use the definition of the aggregate domestic output. As a result, the demand equations can be simplified to:

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} \times \left\{(1-\alpha)C_t + \left(\frac{1}{RS_t}\right)^{-\theta} \alpha C_t^*\right\} + G_{H,t} \tag{26}$$

$$Y_t^* = \left(\frac{P_{F,t}}{P_t}\right)^{-\theta} \times C_t^* + G_{F,t}^*. \tag{27}$$

The demand equations presented above illustrate the small open economy implications. In particular, the demand for goods produced at Home depends on both domestic and foreign consumption as well as the relative prices, whereas the demand for foreign-produced goods is not affected by changes in Home consumption.

2.6 Government policy

We assume that exogenous demand (government spending) in the domestic economy follows a first-order autoregressive process with i.i.d normal error term and (as in Smets and Wouters, 2007) is also affected by the productivity shock:

$$\widehat{g}_t^h = \rho_g \widehat{g}_{t-1}^h + \rho_{ga} \widehat{\varepsilon}_t^a + \epsilon_t^g. \tag{28}$$

where $\widehat{g}_t^h \equiv \log(G_{H,t})$. The assumption $\rho_{ga} > 0$ is empirically motivated by the fact that government spending may include components affected by domestic productivity developments.

Since the small open economy is assumed to belong to the common currency area, the local authority does not conduct an independent monetary policy. Thus the interest rate is common for domestic and foreign economies. It is set by the union-wide monetary authority following the Taylor rule[6] based on the economic performance of the whole EMU. More specifically, the interest rate is gradually adjusted in response to the deviations of area wide CPI inflation and demand (current and past dynamics) from their steady state levels:

$$\widehat{R}_t^* = \omega_r \widehat{R}_{t-1}^* + (1-\omega_r)(\psi_\pi \pi_t^* + \psi_y \widehat{y}_t^* + \psi_{\Delta y}(\widehat{y}_t^* - \widehat{y}_{t-1}^*)) + \widehat{\varepsilon}_t^r \tag{29}$$

and

$$\widehat{R}_t = \widehat{R}_t^*.$$

where $\widehat{R}_t \equiv \log(R_t)$, ω_r is the interest rate smoothing parameter and $\widehat{\varepsilon}_t^r$ is the interest rate shock which follows an AR(1) process with ϵ_t^r i.i.d normal error term.

3 Log-Linear representation

Here, we present a log-linearized version of the model. We define $\widehat{x}_t \equiv \ln \frac{X_t}{X}$ as the log deviation of the equilibrium variable X_t under sticky prices and wages from its steady state value. Moreover, we define the price and wage changes as $\Pi_H = \frac{P_{H,t}}{P_{H,t-1}}$ and $\Pi_W = \frac{W_t}{W_{t-1}}$; consequently, the producer price and wage inflation rates are $\pi_{H,t} \equiv \ln\left(\frac{P_{H,t}}{P_{H,t-1}}\right)$ and $\pi_{W,t} \equiv \ln\left(\frac{W_t}{W_{t-1}}\right)$. We approximate the model around the steady state, in which $\overline{G} = 0$, $\mu^p \geq 1$ and producer prices and wages do not change, i.e., $\Pi_H = 1$ and $\Pi_W = 1$ at all times. In addition, $\overline{RS} = 1$, $\overline{C} = \overline{C}^*, \overline{Y} = \overline{Y}^*$.

The dynamics of consumption follows from the consumption Euler equation (8) and in the log-linearized form is given by:

$$\widehat{c}_t = \frac{1}{(1+\chi)} E_t\left[\widehat{c}_{t+1}\right] + \frac{\chi}{(1+\chi)} \widehat{c}_{t-1} - \frac{(1-\chi)}{\sigma_c(1+\chi)} (\widehat{R}_t - E_t[\widehat{\pi}_{t+1}] + \widehat{\widetilde{\varepsilon}}_t^c) , \tag{30}$$

where $\widehat{\widetilde{\varepsilon}}_t^c = \frac{(1-\chi)}{\sigma_c(1+\chi)}(\widehat{\varepsilon}_t^c - \widehat{\varepsilon}_{t+1}^c)$. The backward looking term arises in the consumption equation due

[6] The specification of the policy rule (29) is standard and widely used in the modern DSGE literature (Smets and Wouters, 2003 and 2007).

to the assumption of external habit formation captured by the parameter χ.Therefore, current consumption ($\widehat{c}_t$) depends on a weighted average of past and expected future consumption. The consumption process is also affected by the ex–ante real interest rate ($\widehat{R}_t - E_t[\widehat{\pi}_{t+1}]$), and a disturbance term $\widehat{\widehat{\varepsilon}}_t^c$, which is assumed to follow a first–order autoregressive process with an iid–Normal error term: $\widehat{\widehat{\varepsilon}}_t^c = \rho_c \widehat{\widehat{\varepsilon}}_{t-1}^c + \epsilon_t^c + \rho_{cf}\epsilon_t^{c*}$. We also assume that the domestic shock is affected by the foreign consumption disturbance[7].

The optimal price–setting condition (12) combined with equation (13) gives rise to the following New–Keynesian Phillips curve, which describes the dynamics of the domestic inflation in terms of the real marginal costs:

$$\widehat{\pi}_{H,t} = \beta E_t\left[\widehat{\pi}_{H,t+1}\right] + \frac{(1-\gamma^p\beta)(1-\gamma^p)}{\gamma^p}(\widehat{mc}_t^r) + \widehat{\mu}_{p,t} \tag{31}$$

The price mark–up disturbance ($\widehat{\mu}_{p,t}$) is assumed to follow an AR(1) process: $\widehat{\mu}_{p,t} = \rho_p \widehat{\mu}_{p,t-1} + \epsilon_t^p$, where ϵ_t^p is an iid–Normal price mark–up shock. The marginal cost is obtained by log-linearizing the equation (11a) and is given by:

$$\widehat{mc}_t^r = \widehat{w}_t^r + \lambda\, \widehat{L}_t - \widehat{p}_{H,t} - \widehat{\varepsilon}_t^a \tag{32}$$

where $p_{H,t} = P_{H,t}/P_t$ denotes domestic relative price. The characterization of real marginal costs in the open economy setting is somewhat different from that of the closed economy due to the impact of relative prices, which reflect the distinction between domestic and consumer prices.

Log-linearizing the optimal wage–setting condition (17) and the law of motion for the wage rate (18), allows us to obtain the following equation for wage inflation:

$$\widehat{\pi}_t^W = \beta E_t\left[\widehat{\pi}_{t+1}^W\right] - \frac{(1-\gamma^w\beta)(1-\gamma^w)}{\gamma^w(1+\sigma_w\eta)}(\widehat{\mu}_{w,t} - \widehat{\mu}_{w,t}^n) \tag{33}$$

where $\widehat{\mu}_{w,t}^n$ is the desired wage markup,

$$\widehat{\mu}_{w,t} = \widehat{w}_t^r - \widehat{mrs}_t \tag{34}$$

and $\widehat{mrs}_t = \widehat{\varepsilon}_t^l + \frac{\sigma_C}{1-\chi}(\widehat{c}_t - \chi\widehat{c}_{t-1}) + \eta\widehat{L}_t$. The wage–mark up disturbance $\widehat{\mu}_{w,t}^n$ is assumed to follow an iid–Normal process: $\widehat{\mu}_{w,t}^n = \widehat{\epsilon}_t^w$. Using the definition of the wage inflation $\widehat{\pi}_t^W = \widehat{w}_t - \widehat{w}_{t-1}$, we can write down the expression for the dynamics of the real wages as follows:

$$\widehat{w}_t^r = \frac{1}{(1+\beta)}\left\{\begin{array}{c} \widehat{w}_{t-1}^r + \beta E_t\left[\widehat{w}_{t+1}^r\right] - \widehat{\pi}_t + \beta E_t\left[\widehat{\pi}_{t+1}\right] \\ +\frac{(1-\gamma^w\beta)(1-\gamma^w)}{\gamma^w(1+\sigma_w\eta)}\left[\frac{\sigma_C}{1-\chi}(\widehat{c}_t - \chi\widehat{c}_{t-1}) + \eta\widehat{L}_t + \widehat{\varepsilon}_t^l - \widehat{w}_t^r\right]\end{array}\right\} + \widehat{\mu}_{w,t}^n \tag{35}$$

where $\widehat{\varepsilon}_t^l = \log(\varepsilon_t^l)$ is labor supply shock which is assumed to follow an ARMA(1,1) process: $\widehat{\varepsilon}_t^l = \rho_l\widehat{\varepsilon}_{t-1}^l - \rho_{ma,l}\epsilon_{l,t-1} + \epsilon_t^l$.

[7] In such a way we introduce "one-way" correlation between domestic and foreign consumption shocks. Such an assumption is however not crucial for the estimation and forecasting results.

Equation (33) demonstrates that the evolution of the wage inflation is determined by fluctuations of the wedge between the actual and desired wage markups. In particular, when the markup charged is higher than the natural level, wages will respond negatively. The dynamics of the markup is driven by fluctuations in the real wage and the marginal rate of substitution. In particular, due to the presence of nominal wage stickiness, the real wages adjust only gradually to the desired wage mark–up. In addition, equation (35) shows that the real wage dynamics is affected by CPI inflation. An increase in the inflation rate will result in a decline of the real wages and a contraction in the wage markup. As a consequence, higher expected inflation rate (translated into lower expected wage markup) will motivate workers to set higher nominal wages today to offset the possible reduction of the real wages in the future.

In order to describe the unemployment dynamics, we log-linearize equations (19) and (20) and obtain the following expressions:

$$\widehat{w}_t^r = \widehat{\varepsilon}_t^l + \frac{\sigma_C}{1-\chi}(\widehat{c}_t - \chi\widehat{c}_{t-1}) + \eta\widehat{\widetilde{L}_t^S} \tag{36}$$

and

$$\widehat{u}_t = \widehat{\widetilde{L}_t^S} - \widehat{L}_t. \tag{37}$$

Furthermore, combining expressions (34), (36) and (37) we can derive the following relationship between the wage markup and the unemployment rate:

$$\widehat{\mu}_{w,t} = \eta\widehat{u}_t. \tag{38}$$

Therefore, the wage inflation equation can be reformulated in terms of the unemployment rate, which can enter the set of observable variables. As Gali, Smets and Wouters (2011) point out, such a representation allows to overcome an important identification problem, which limits the use of the New Keynesian models for policy analysis. In particular, without an explicit measure of unemployment (or alternatively labor supply), the wage markup disturbance and the preference shock that affects the labor disutility cannot be distinguished. Such an identification problem may result in inaccurate policy recommendations, because these shocks call for qualitatively different optimal policy responses.

A common problem with European data is the absence of consistent data on aggregate hours. Therefore, following a number of studies performed for the euro area, we use employment instead of "hours worked" in the estimation procedure. The employment time series is normally more persistent compared to hours. Thus, following Smets and Wouters (2002), we assume hours to be flexible whereas rigidity in employment gives rise to the following Calvo-type auxiliary equation which links these two measures of labor input:

$$\widehat{Em}_t = \beta\widehat{Em}_{t+1} + \frac{(1-\gamma^m\beta)(1-\gamma^m)}{\gamma^m}(\widehat{L}_t - \widehat{Em}_t) + \widehat{\varepsilon}_t^{em}, \tag{39}$$

where $\widehat{Em}_t$ denotes the number of people employed and γ^m denotes the fraction of firms that can adjust employment to the desired level. $\widehat{\varepsilon}_t^{em}$ is an exogenous shock to the employment, which follows an AR(1) process.

The demand for labor is represented by the following expression, based on the first order approximation of the condition (16):

$$(1-\lambda)\widehat{L}_t = \widehat{Y}_t - \widehat{\varepsilon}_t^a. \tag{40}$$

The log-linear representation of equation (26) describes the aggregate demand for domestic goods:

$$\widehat{Y}_t = -\theta\widehat{p}_{H,t} + (1-\alpha)\widehat{c}_t + \alpha\widehat{c}_t^* + \theta\alpha\widehat{RS}_t + \widehat{g}_t^h, \tag{41}$$

where $\widehat{g}_t^h$ is given by (28).

The first order approximation of the optimal risk sharing condition has the following form:

$$\frac{\sigma_C}{1-\chi}(\widehat{c}_t - \chi\widehat{c}_{t-1}) = \frac{\sigma_C}{1-\chi}(\widehat{c}_t^* - \chi\widehat{c}_{t-1}^*) + \widehat{RS}_t - \widehat{\varepsilon}_t^c + \widehat{\varepsilon}_t^{c*} \tag{42}$$

The determinants of the real exchange rate are given by the following expression:

$$\widehat{RS}_t = (1-\alpha)\widehat{p}_{FH,t} + \widehat{\varepsilon}_t^{rs}, \tag{43}$$

where $\widehat{p}_{FH,t}$ denotes the terms of trade, $\widehat{\varepsilon}_t^{rs}$ is an exogenous real exchange rate shock, which captures the developments in other types of relative prices at home and abroad that affect the evolution of the real exchange rate but not modeled here explicitly[8]. $\widehat{\varepsilon}_t^{rs}$ is assumed to follow a first–order autoregressive process with an iid–Normal error term: $\widehat{\varepsilon}_t^{rs} = \rho_{rs}\widehat{\varepsilon}_{t-1}^{rs} + \epsilon_t^{rs}$.Moreover, from the price index relation it follows that:

$$\widehat{p}_{H,t} = -\alpha\widehat{p}_{FH,t}.$$

Log-linearization of price indices around a symmetric steady state satisfying the PPP condition $P_H = P_F$ yields: $\widehat{P}_t = (1-\alpha)\widehat{P}_{H,t} + \alpha\widehat{P}_{F,t}$. Applying the definition of inflation $\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \widehat{P}_t - \widehat{P}_{t-1}$, we obtain the expressions for CPI inflation as a function of domestic and foreign inflation:

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t}. \tag{44a}$$

Moreover, the definition of the terms of trade implies that $\Delta\widehat{p}_{FH,t} = \pi_{F,t} + \pi_{H,t}$. The combination of the equations presented above results in the following relationship between CPI, domestic inflation and the terms of trade:

$$\pi_t = \pi_{H,t} + \alpha\Delta\widehat{p}_{FH,t}. \tag{45}$$

Under the assumption of the common currency area, the dynamic expression for the terms of trade can be written as follows: $(1-\alpha)\Delta\widehat{p}_{FH,t} = \pi_t^* - \pi_t$. Finally, the evolution of the real

[8]For example, relative price of non-tradable goods.

exchange rate takes the form:

$$\widehat{RS}_t - \widehat{RS}_{t-1} = \pi_t^* - \pi_t + \widetilde{\widehat{\varepsilon}}_t^{rs}, \tag{46}$$

where π_t^* is CPI inflation in the foreign country[9] and $\widetilde{\widehat{\varepsilon}}_t^{rs} = \widehat{\varepsilon}_t^{rs} - \widehat{\varepsilon}_{t-1}^{rs}$.

In this version of the paper we consider a simplified (three-equation) structure for the foreign economy, associated with the euro area. We also do not focus on asymmetries between the domestic economy and the rest of the world. Thus, we assume the same values of such parameters as habit formation and preferences for home and foreign economies. Calvo price rigidities and exogenous processes are country specific. Foreign inflation is governed by the following Phillips curve relation:

$$\widehat{\pi}_t^* = \beta E_t\left[\widehat{\pi}_{t+1}^*\right] + \frac{(1-\gamma^{p*}\beta)(1-\gamma^{p*})}{\gamma^{p*}}(\sigma_C\widehat{c}_t^* + \eta\widehat{y}_t^* + \widehat{\mu}_{p,t}^* - \eta\widehat{\varepsilon}_t^{a*}). \tag{47}$$

The dynamics of foreign consumption is derived from log-linearization of equation (8a):

$$\widehat{c}_t^* = \frac{1}{(1+\chi)}E_t\left[\widehat{c}_{t+1}^*\right] + \frac{\chi}{(1+\chi)}\widehat{c}_{t-1}^* - \frac{(1-\chi)}{\sigma_c(1+\chi)}\left(\widehat{R}_t^* - E_t[\widehat{\pi}_{t+1}^*] + \widehat{\widehat{\varepsilon}}_t^{C*}\right) \tag{48}$$

where $\widehat{\widehat{\varepsilon}}_t^{C*}$ denotes foreign preference consumption shock which is assumed to follow an AR(1) process: $\widehat{\widehat{\varepsilon}}_t^{C*} = \rho_{c*}\widehat{\widehat{\varepsilon}}_{t-1}^{C*} + \epsilon_t^{c*}$. . Foreign demand is obtained by log-linearization of equation (27):

$$\widehat{y}_t^* = \widehat{c}_t^* + \widehat{g}_t^* \tag{49}$$

Finally, the nominal interest rate dynamics is given by equation (29). Note that foreign dynamics is completely exogenous from the small open economy perspective. In the estimation procedure we include only 3 time series related to the foreign economy (inflation, output, and interest rate). Therefore, certain shocks can be poorly identified. For this reason, we assume no foreign government spending shock, $\widehat{g}_t^* = 0$. Moreover, foreign productivity and price markup shocks are not identified separately. Thus, we consider their aggregated impact on the foreign inflation.

4 Estimation strategy and results

4.1 Data

We use quarterly time series for Luxembourg for the following macro–economic variables: real GDP, employment (residents and non-residents employed by resident producer units), compensation per employee (working in a resident production unit), consumer price index, unemployment rate and real effective exchange rate (CPI deflated). The first two variables are expressed

[9]In the small open economy specification presented here, $\pi^* = \pi^F$.

in per capita terms. The foreign variables are real GDP, Euro area short-term nominal interest rate and CPI inflation. All variables (except the nominal interest rate) are seasonally adjusted and log differenced. The sample is from 1995Q1 to 2011Q3 since quarterly data are not available before 1995. The time series of real wages is constructed as compensation per employee divided by consumer prices. The nominal rate time series is divided by 4 to obtain quarterly data. All variables have been demeaned prior to estimation. The DSGE model presented in the previous section is augmented by the following measurement equations:

$$\begin{bmatrix} \Delta \ln RGdp_t \\ \Delta \ln P_t \\ \Delta \ln REER_t \\ \Delta \ln RWage_t \\ \Delta \ln Empl_t \\ \Delta \ln Unempl_t \\ STN_t \\ \Delta \ln P_t^* \\ \Delta \ln RGDP_t^* \end{bmatrix} = \begin{bmatrix} \widehat{y}_t - \widehat{y}_{t-1} \\ \widehat{\pi}_t \\ \widehat{RS}_t - \widehat{RS}_{t-1} \\ \widehat{w}_t^r - \widehat{w}_{t-1}^r \\ \widehat{Em}_t - \widehat{Em}_{t-1} \\ \widehat{u}_t - \widehat{u}_{t-1} \\ \widehat{R}_t^* \\ \widehat{\pi}_t^* \\ \widehat{y}_t^* - \widehat{y}_{t-1}^* \end{bmatrix} \tag{50}$$

Data on the real exchange rate is taken from the IMF International Financial Statistics. The source for unemployment rate is the OECD Statistics. The rest of the data is taken from STATEC national accounts.

Using the data set described above, we estimate and compare the forecasting performance for the following model specifications:

- DSGE
- Unrestricted VAR
- Univariate AR(2)
- Bayesian VAR(2)
- DSGE-VAR(2)

4.2 DSGE model. Estimation results

In this subsection we describe the estimation results of the DSGE structural model presented in the previous section. The model is estimated using Bayesian techniques. On a theoretical level, the Bayesian approach to estimation takes the observed data as given, and treats the parameters of the model as random variables. In general terms, the estimation procedure involves solving the linear rational expectations model described in the sections 2 and 3. The solution can be written in a state space form, i.e. as a reduced form state equation augmented by the observation (measurement) equations. At the next step, the Kalman filter is applied to construct the likelihood function. Posterior distribution of the structural parameters is formed by combining the likelihood function of the data with a prior density, which contains information about the model parameters obtained from the other sources (microeconometric, calibration, and cross-country evidence), thus allowing to extend the relevant data beyond the time series that are used as observables. An additional benefit of using prior information is that it allows to steer parameter estimates towards values that are considered to be 'reasonable' by the literature and to regularize highly nonlinear and often multi–modal posterior distribution. The second advantage is very important when comparing Bayesian methods to alternative estimation strategies such as maximum likelihood. Finally, numerical methods such as Monte-Carlo Markov-Chain (MCMC) are used to characterize the posterior with respect to the model parameters. See Smets and Wouters (2003,2007), Dynare Manual and An and Schorfheide (2005) for more details on Bayesian estimation of DGSE models.

4.2.1 Calibration and priors

Following the recent DSGE and New Open Macroeconomy literature, we calibrate a number of parameters. In particular, the discount factor β is fixed at 0.99, which implies an annual steady state real interest rate of 4%. The elasticity of substitution across the differentiated types of labor σ_w is set to 6, which implies a steady state wage markup of about 20%. The elasticity of substitution between foreign and home goods θ is assumed to be unitary. The policy rule parameter which determines the interest rate response to inflation is set to 1.5. In addition, we fix the standard deviation of the exogenous demand (government spending) shock at 0.1 and the autoregressive coefficient of the productivity shock at 0.9. The latter two parameters have been calibrated because the government spending shock is not separately identified and the productivity shock is imprecisely estimated. In our case, the reason for a weak identification of these stochastic processes can be related to the short data sample that turns out to be not informative enough and fails to introduce "sufficient" curvature in the likelihood function in certain directions. In addition, we have to use employment data rather than hours worked (since the latter is not available) and link these two measures of the labor input via equation (39). Such an ad hoc relation can also distort the estimated productivity process. The calibrated values for the shocks have been chosen to approximate the standard deviation of the output growth from 1995 to 2011. Parameter identification is an important problem facing current generation of DSGE models that feature complex structure and, as a consequence, highly non-linear relationship between the structural and reduced form parameters. Thus, the mapping between the two might be unknown and only an approximation can be obtained. In practice, lack of identification is a complex issue that can be related to the model specification, dimensionality of the problem, assumptions regarding the shock processes as well as the sample size.[10]

In the choice of priors, we mainly follow the original papers by Smets and Wouters (2003 and 2007) as well as Gali, Smets and Wouters (2011). The first two papers present a careful

[10] Canova and Sala (2006) investigate identification issues in DSGE models and their consequences for parameter estimation. They point out that small samples exacerbate the consequences of identification problems for estimation and inference.

description of the estimation methodology as well as the justification for the choice of priors. The estimation procedure starts with the estimation of the mode of the posterior distribution by maximizing the log posterior function. Secondly, the Metropolis–Hastings algorithm was used to compute the posterior distribution and to evaluate the marginal likelihood of the model. 100 000 MCMC draws have been performed using three chains.

4.2.2 Parameters estimates

A visual diagnostic of the estimation results can be found in Figures 1A in the Appendix, where we plot prior versus posterior distributions. Most of the parameters are identified as their posterior is significantly different from prior. For the majority of the parameters, the variance of the posterior is lower compared to the prior distribution, indicating that data is quite informative. In case of no identification for a particular parameter, the likelihood function would be flat in the corresponding direction and the posterior distribution would be prior-driven. Figures 1A illustrate that a policy rule parameter which determines the impact of output changes suffers from the lack of identification. All the marginal posterior distributions are unimodal which is one of the criteria for assessment of MCMC's convergence. Metropolis-Hastings convergence graphs (not presented here) indicate that convergence for all parameters is efficient and fast.

Tables 1a and 1b report the estimates of the DSGE model parameters. The tables show the mode, which maximizes the posterior distribution, along with the approximate standard deviation computed from the inverse Hessian at the posterior mode. Furthermore, the tables present a posterior statistics from MCMC - posterior means and the 95% probability intervals of the model parameters. Our estimate of the utility function parameter σ_c implies the value of intertemporal elasticity of substitution is less than one. Such an estimate is generally in line with the calibration made in the majority of the RBC literature, which sets an elasticity of substitution between 0.5 and 1. Another parameter that determines the impact of the interest rate changes on consumption is habit formation, which is estimated to be 0.77. Such a relatively high value implies initially lower but more persistent response of consumption following changes in the short term interest rate or consumption preference shock. The posterior mean of the habit parameter is somewhat higher than the estimates obtained in Smets and Wouters (2003), who report the value of 0.55, but is close to numbers from other studies performed on European data. In particular, Pytlarczyk (2005) finds habit persistence estimate 0.68 for Germany and 0.8 for the rest of the euro area. Jondeau and Sahic (2004) estimate the multi-country euro area model and report values of 0.73 for France and 0.84 for Italy. The inverse of the elasticity of labour supply has the posterior mean equal to 3.45 which implies that the response of labor supply to changes in the wage rate is relatively small. The estimate of this parameter is close to the value of 4.0 reported in Gali, Smets and Wouters (2011). Together with the calibrated steady state wage markup, the estimated value of the inverse Frisch elasticity is consistent with the average unemployment rate of about 5.8%.

Table 1a. Prior and posterior distribution of structural parameters for the baseline DSGE model

Parameters		Prior distribution			Posterior distribution				
		Type	Mean	St.dev	Mode	St.dev	Mean	5%	95%
Production function	λ	Beta	0.3	0.1	0.202	0.077	0.215	0.096	0.332
Degree of openness	α	Beta	0.3	0.15	0.102	0.034	0.106	0.051	0.161
Consumption utility	σ_c	Norm	1	0.375	1.256	0.292	1.283	0.816	1.75
Labor utility	η	Norm	2	1.5	2.873	0.804	3.45	2.065	4.883
Consumption habit	χ	Beta	0.5	0.15	0.776	0.062	0.777	0.677	0.875
Calvo prices	γ^p	Beta	0.75	0.15	0.923	0.022	0.919	0.884	0.957
Calvo wages	γ^w	Beta	0.75	0.15	0.929	0.019	0.933	0.899	0.967
Calvo employment	γ^m	Beta	0.75	0.15	0.918	0.021	0.914	0.875	0.951
Calvo foreign prices	γ^{p*}	Beta	0.75	0.15	0.977	0.01	0.977	0.962	0.992
Pol.rule: lagged int.rate	ω_r	Beta	0.5	0.2	0.973	0.010	0.97	0.958	0.985
Pol.rule: output	ψ_y	Gam	0.25	0.125	0.201	0.101	0.25	0.075	0.414
Pol.rule: lagged output	$\psi_{\Delta y}$	Gam	0.25	0.125	0.151	0.034	0.155	0.094	0.212
DSGE prior weight	$\widetilde{w}$	Unif	0	10	1.880	0.442			

[11]

The degree of openness parameter is estimated at about 10% which is somewhat lower than could be expected for such an open economy. When we add terms of trade series to the set of observables, this parameter drops to 5%. The reason for such a result is extra volatile dynamics of terms of trade time series which implies a degree of openness of about 150% . Obviously such a value cannot be reasonably fitted into a theoretical model framework. Calibrating this parameter at relatively high level would result in much higher implied volatility of other real variables compared to actual data and thus lead to a deterioration of the model fit.

Structural rigidities parameters, which are found to play a crucial role in capturing the business cycle fluctuations, are well identified. The estimates of the Calvo parameters at 0.91 for prices and 0.93 for wages imply an average duration of contracts of two and half years. These values are higher compared to microevidence for some European countries like Germany and also greater than estimates obtained by Smets and Wouters (2003) and (2007) for the euro area and the US respectively or Adolfson et al. (2008) for Sweden. At the same time, Burriel et al. (2010) report a similar estimate for Calvo price parameter for the Spanish economy. One factor that could explain the high degree of the price stickiness is the assumption of i.i.d price and wage markup shocks. Smets and Wouters (2007) assume ARMA structure for these stochastic processes. However, in our case such an assumption is not supported by the data and reduces the marginal likelihood of the model. The absence of such factors as sluggish capital adjustment, which affect the process driving marginal costs, can bias upward the estimate of Calvo price stickiness. In our estimation exercise, we also tried to evaluate indexation parameters, which measure the proportion of prices/wages that cannot adjust in the current period but instead

[11] DSGE prior weight parameter is estimated in DSGE-VAR(2) model specification

are indexed to the lagged inflation rates. Price indexation parameter is estimated at the low value, which is in line with the European evidence, and does not significantly affect the model likelihood. The wage indexation parameter is not separately identified from the parameter measuring the slope of the wage Phillips curve. Thus we have decided to abstract from modeling the indexation process.

Table 1b. Prior and posterior distribution of shock processes for the baseline DSGE model

Parameters		Prior distribution			Posterior distribution				
		Type	Mean	St.dev	Mode	St.dev	Mean	5%	95%
Standard deviations									
Consumption preference	υ_c	Inv.G	0.1	2	0.037	0.01	0.05	0.027	0.071
Productivity	υ_a	Inv.G	0.1	2	1.296	0.306	1.389	0.887	1.885
Price markup	υ_p	Inv.G	0.1	2	0.212	0.038	0.223	0.155	0.284
Wage markup	υ_w	Inv.G	0.1	2	0.54	0.049	0.553	0.47	0.636
Relative price	υ_{rs}	Inv.G	0.1	2	0.985	0.088	1.01	0.855	1.155
Labor supply	υ_l	Inv.G	0.1	2	0.108	0.033	0.135	0.073	0.193
Exogenous employment	υ_{em}	Inv.G	0.1	2	0.142	0.042	0.16	0.087	0.231
Foreign demand	υ_{c*}	Inv.G	0.1	2	0.071	0.017	0.081	0.052	0.11
Foreign prices	υ_{p*}	Inv.G	0.1	2	0.463	0.042	0.475	0.403	0.546
Interest rate	υ_r	Inv.G	0.1	2	0.08	0.011	0.086	0.065	0.106
Persistence and correlat.									
Consumption	ρ_c	Beta	0.5	0.2	0.909	0.024	0.886	0.836	0.939
Price markup	ρ_p	Beta	0.5	0.2	0.368	0.122	0.364	0.171	0.566
Relative price	ρ_{rs}	Beta	0.5	0.2	0.184	0.087	0.201	0.062	0.33
Labor supply - AR	ρ_l	Beta	0.5	0.2	0.85	0.055	0.826	0.733	0.924
Labor supply - MA	$\rho_{ma,l}$	Beta	0.5	0.1	0.631	0.079	0.63	0.501	0.763
Exogen.employment	ρ_{em}	Beta	0.5	0.2	0.635	0.134	0.587	0.362	0.817
Interest rate	ρ_r	Beta	0.5	0.2	0.438	0.101	0.444	0.283	0.61
Foreign demand	ρ_{c*}	Beta	0.5	0.2	0.789	0.068	0.759	0.652	0.873
Demand-Productivity	ρ_{ag}	Norm	0.5	0.25	0.785	0.173	0.786	0.521	1.049
Consum.-Foreign demand	ρ_{cf}	Norm	0.5	0.25	0.468	0.160	0.515	0.247	0.772

Overall, the data is quite informative about the persistence and volatility of exogenous disturbances. The preference and labour supply shocks appear to be the most persistent with AR(1) coefficients of 0.89 and 0.83 respectively. In general, the level of persistence of stochastic processes is not very high. Such a result indicates that the model contains sufficient endogenous propagation mechanism. Regarding the estimates of the volatility of shocks, various studies do not seem to reach a consensus. The values of the parameters of stochastic processes is highly model dependent. In addition, many authors normalize structural shocks, which reduces their volatility. Our results suggest that productivity, relative price and wage markup shocks have the highest estimated standard deviations. As in Gali, Smets and Wouters (2011) adding unemployment as an observable variable allows us to separately identify labour supply and wage markup shocks, which appear to have quite different stochastic properties. Such a result will translate into the differentiated impact of these shocks on the forecast error variance of real variables when explaining the business cycle fluctuations.

Finally, turning to the parameters of the Taylor rule, there is a high degree of interest rate smoothing which is generally supported by the literature[12]. The monetary policy appears to respond relatively strongly to changes in output, with the posterior mean of the corresponding coefficient being equal to 0.15. The estimates of the inflation and output level reaction coefficients are driven by a prior. This can be partially explained by the relatively short data sample which implies a higher weight on the prior information. In addition, we assume a highly simplified model of the foreign economy. However, such a lack of identification does not affect the overall results. Finally, we would like to note that our estimation sample ends at 2011q3 and thus includes the recent financial crisis observations. Thus our estimates can be to some extent affected by the unconventional measures implemented by the monetary authority but not captured in this modeling framework. In particular, the estimated persistence of the economy can be biased upward. As a robustness check, we compare the parameters of the model estimated on a sample that ends in 2007 q4 and on the full sample. The tables 1A and 2A in the Appendix demonstrate that the parameters, especially those that determine the model persistence, do not differ significantly and thus our results are not driven by specific dynamics caused by inclusion of financial crisis observations.

4.3 Alternative forecasting models. Description and comparison

In additon to the DSGE model, we estimate and compare the forecasting performance of the following model specifications:

- Unrestricted VAR. The model can be written in the following general form:

$$Y_t = \Phi_x X_t + \Phi_1 Y_{t-1} + ... + \Phi_p Y_{t-p} + u_t, \qquad u_t \sim i.i.d.N(0; \Sigma_u), \tag{51}$$

  where $p = 2$ to allow for sufficient dynamics without exhausting degrees of freedom, due to the rather small sample available. The vector of endogenous variables is the same as in DSGE estimation, i.e. $Y_t = [\Delta \ln(\text{Real GDP}),\ \Delta \ln(\text{CPI}), \Delta \ln(\text{Real.Effect.Exch.Rate}),$ $\Delta \ln(\text{Real wages}), \Delta \ln(\text{Employment}), \Delta \ln(\text{Unemployment})]$. In order to make the models comparable, in VAR estimation we impose the small open economy restriction, which implies that foreign variables are considered as exogenous, i.e the vector of exogenous variables is $X_t = [\text{Nomin.Inter.rate},\ \Delta \ln(\text{Foreign GDP}),\ \Delta \ln(\text{Foreign CPI})]$. If we write the VAR in a matrix form as $Y = Z\Phi + U$, where Y is a $T \times n$ matrix and Z is $T \times k$ matrix (with $k = np + n_x$), the likelihood function takes the form:

$$p(Y \mid \Phi, \Sigma_u) \propto \mid \Sigma_u \mid^{-T/2} \exp\left(-\frac{1}{2} tr\left[\Sigma_u^{-1}(Y - Z\Phi)'(Y - Z\Phi)\right]\right) \tag{52}$$

[12] Estimates vary depending on the estimation sample.

- Univariate AR(2). Such a specification implies that the matrices of parameters Φ and variance-covariance matrix Σ_u in the VAR specification are diagonal.

The solution of the linearized DSGE model generates a restricted (and possibly misspecified) moving average representation for the vector of observed data Y_t. The MA representation can be approximated by a constrained VAR with p-lags and coefficient restrictions given by nonlinear functions of the DSGE parameter vector ϑ:

$$Y_t = \Phi_x^*(\vartheta)X_t + \Phi_1^*(\vartheta)Y_{t-1} + ... + \Phi_p^*(\vartheta)Y_{t-p} + u_t. \tag{53}$$

Because of this close relationship between structural and reduced form models, unconstrained VARs are widely used in the literature as a benchmark for evaluating the empirical validity of cross equation restrictions imposed by the DSGE structure. On the one hand, VAR represents a flexible and unrestricted framework. At the same time, coefficient estimates can be very imprecise and forecasts have large standard errors due to the large number of parameters and short time series. The current literature addresses this problem by the use of Bayesian estimation techniques. In this paper we consider two types of priors on VAR coefficients, one is non-theoretical and another one is based on the DSGE model. The corresponding model specifications are described below.

- Bayesian VAR(2). The model combines the VAR Likelihood function (52) with the prior information summarized by the prior density $p_0(\Phi;\Sigma)$. This approach represents a flexible way to reduce the dimensionality of the parameter space, incorporate additional information and thus decrease the parameter uncertainty. As a result, the forecasting performance can be improved over the standard VAR methods. In this paper we choose Sims-Zha Normal-Wishart priors (described in Sims and Zha, 1998), which proved to be the best practice in recent empirical studies. This BVAR specification combines a Minnesota-style prior (see Litterman, 1984) with priors that take into account the degree of persistence in the variables. Since we work with stationary data, the original Sims and Zha prior is adapted by setting the prior mean on the first own lag to zero for all the variables. In general terms, the prior consists of 3 components. The first one is Jeffrey's improper prior. The second component can be described as the likelihood of the form (52) of the VAR model estimated on the basis of T_1 dummy observations Y_1 and Z_1, which are constructed to reproduce desirable dynamic properties governed by a set of hyperparameters. We assume the standard values of hyperparameters found to work well in most forecasting applications: "overall tightness" and the “decay” parameter, which determine the rate at which prior coefficients decline as lag increases, are set to 1. The AR(1) tightness is set to 0.5. And the "sum of coefficient prior weight" is set to 0.1. The third component of the prior is equal to the likelihood of the form (52) of the VAR model estimated on the basis of T_2 observations Y_2 and Z_2 from a training sample. Due to the short time series we do not include this part of the prior.

- DSGE-VAR(2), a sort of Bayesian approach to VAR that uses DSGE model restrictions to construct a micro-founded prior about VAR parameters and thus may improve VAR estimates by incorporating extra information. Alternatively, this method can be viewed as a way to improve the empirical properties of the DSGE model by relaxing tight cross-equation restrictions that might be at odds with real data. The idea of the approach is to simulate data from the model, append simulated to actual data and estimate a VAR on extended data. The optimal proportion (can be estimated) of simulated to actual data measures the weight on DSGE restrictions. Del Negro and Schorfheide (2004) describe the procedure of constructing a hierarchical DSGE-VAR prior using the notion of "dummy observations" and show that the model has the following prior structure:

$$p_0(\Phi, \Sigma_u, \vartheta, \widetilde{w}) = p_0(\vartheta) \times p_0(\widetilde{w}) \times p_0(\Phi, \Sigma_u \mid \vartheta, \widetilde{w}). \tag{54}$$

First we formulate a prior on the DSGE model structural parameters $p_0(\vartheta)$, which is a standard procedure in estimation of DSGE models[13]. We also define a prior distribution for the hyperparameter $\widetilde{w}$, which is assumed to be uniform over the interval [0,10]. Conditional on this prior, we form a prior view for VAR parameters $p_0(\Phi, \Sigma_u \mid \vartheta, \widetilde{w})$. To obtain this one, the DSGE model is used to simulate $\widetilde{w}T$ artificial ("dummy") observations, which are added to the sample of actual data. The VAR is estimated on the augmented sample. The relative size of the simulated and actual data, which is proportional to $\widetilde{w}$, determines the impact of DSGE restrictions on the estimates. The quasy-likelihood function for artificial observations (sample size $T^* = \widetilde{w}T$) generated from the DSGE model takes the form:

$$p(Y^*(\vartheta) \mid \Phi, \Sigma_u) \propto \mid \Sigma_u \mid^{-\widetilde{w}T/2} \exp\left(-\frac{1}{2} tr\left[\Sigma_u^{-1}(Y^* - Z^*\Phi)'(Y^* - Z^*\Phi)\right]\right). \tag{55}$$

Then the joint likelihood of the sample of actual and artificial observations is given by:

$$p(Y^*(\vartheta), Y \mid \Phi, \Sigma_u) \propto p(Y \mid \Phi, \Sigma_u) p(Y^*(\vartheta) \mid \Phi, \Sigma_u). \tag{56}$$

Such a decomposition suggests that the term $p(Y^*(\vartheta) \mid \Phi, \Sigma_u)$ can be interpreted as a prior density for Φ and Σ_u. It summarizes the information about the VAR parameters contained in the sample of artificial observations. To simplify the computation of the prior density, (artificial) sample moments $Y^{*'}Y^*, Y^{*'}Z^*$, and $Z^{*'}Z^*$ are replaced by their expected values equal to (scaled) population moments $E_\vartheta^D[Y^{*'}Y^*] = \widetilde{w}T\ \Gamma_{yy}^*(\vartheta)$, etc. , where autocovariance matrices are defined as $\Gamma_{yy}^*(\vartheta) = E_\vartheta^D[y_t y_t']$, $\Gamma_{zz}^*(\vartheta) = E_\vartheta^D[z_t z_t']$, $\Gamma_{zy}^*(\vartheta) = E_\vartheta^D[z_t y_t']$, and $E_\vartheta^D[.]$ denotes the expectation under the DSGE model. Population moments can be analytically computed given the solution to the log-linearized DSGE model. The use of

[13] Prior distributions are presented in tables 1a and 1b.

population moments implies that we replace (55) with

$$p_0(\Phi, \Sigma_u) \mid \vartheta, \widetilde{w}) = c^{-1}(\vartheta) \mid \Sigma_u \mid^{-\frac{\widetilde{w}T+n+1}{2}} \times \tag{57}$$
$$\exp\left(-\frac{1}{2}tr\left[\widetilde{w}T\Sigma_u^{-1}\left(\Gamma_{yy}^*(\vartheta) - \Phi'\Gamma_{zy}^*(\vartheta) - \Gamma_{yz}^*(\vartheta)\Phi + \Phi'\Gamma_{zz}^*(\vartheta)\Phi\right)\right]\right) \quad ,$$

where the probability in (55) has been multiplied by the normalization factor and improper (non-informative) prior $p_0(\Phi, \Sigma_u) \propto \mid \Sigma_u \mid^{-\frac{n+1}{2}}$. In addition, the $p-th$ order VAR approximation of the DSGE provides the first moment of the prior distributions through the population least-square regression:

$$\begin{aligned} \Phi^*(\vartheta) &= \Gamma_{zz}^{*^{-1}}(\vartheta)\Gamma_{zy}^*(\vartheta) \\ \Sigma_u^*(\vartheta) &= \Gamma_{yy}^*(\vartheta) - \Gamma_{yz}^*(\vartheta)\Gamma_{zz}^{*^{-1}}(\vartheta)\Gamma_{zy}^*(\vartheta). \end{aligned} \tag{58}$$

In other words, implied coefficient matrices $\Phi^*(\theta)$ and $\Sigma_u^*(\theta)$ are defined as the OLS (or maximum likelihood) estimates of Φ and Σ_u for a VAR(p) on an infinitely large sample of the artificial observations. Conditional on the vector of DSGE parameters ϑ and $\widetilde{w}$, the prior distribution of VAR parameters (57) is of the conjugate, Inverted-Wishart-Normal form:

$$\begin{aligned} \Sigma_u \mid & \vartheta, \widetilde{w} \sim IW(\widetilde{w}T\Sigma_u^*(\vartheta), \widetilde{w}T - k - n) \\ \Phi \mid & \Sigma_u, \vartheta, \widetilde{w} \sim N(\Phi^*(\vartheta), \Sigma_u \otimes (\widetilde{w}T'\Gamma_{xx}^*(\vartheta))^{-1}. \end{aligned} \tag{59}$$

The hyperparameter $\widetilde{w}$ reflects the "tightness" of the DSGE model prior. Large $\widetilde{w}$ means that the estimates of Φ and Σ_u will concentrate on the restrictions implied by the DSGE model - $\Phi^*(\vartheta)$ and $\Sigma_u^*(\vartheta)$. The domain of $\widetilde{w}$ is restricted to the interval $[np + n/T, \infty]$ for the prior distribution to be proper. The posterior distribution is composed of the posterior density of the VAR parameters Φ and Σ_u given DSGE model parameters and the marginal posterior density of the DSGE model parameters:

$$p(\Phi, \Sigma_u, \vartheta, \widetilde{w} \mid Y) = p(\Phi, \Sigma_u \mid Y, \vartheta, \widetilde{w}) \times p(\vartheta, \widetilde{w} \mid Y). \tag{60}$$

The first density function in (60) is obtained by combining likelihood function (52) with the hierarchical prior (54) and has a closed form expression. Because of the choice of a conjugate prior for the VAR parameters given ϑ, the posterior of Φ and Σ_u is of the same form as the prior. The posterior of Φ and Σ_u is centered at the MLE on *both* actual and artificial data. The joint posterior probability of DSGE model parameters and $\widetilde{w}$, $p(\vartheta, \widetilde{w} \mid Y)$, typically has no closed form expression. Therefore, it is recovered from the MCMC algorithm.

4.3.1 Comparing the fit of the DSGE and DSGE-VAR models

The fit of a model estimated using Bayesian methods can be ascertained using marginal data density, defined as

$$p\left(Y|\mathcal{M}\right) = \int \mathcal{L}\left(\vartheta|Y\right) p_0(\vartheta) d\vartheta,$$

where $\mathcal{L}\left(\vartheta|Y\right)$ is the likelihood function of the data Y given parameters of the model ϑ, and $p_0(\vartheta)$ is the prior density. In other words, the marginal data density is simply an integral over the posterior density, where posterior is understood as likelihood times prior. This measure allows a straightforward comparison of several models estimated on the same data with respect to a reference model. To evaluate a marginal density of the data we can use a Gaussian approximation of the posterior function (so called Laplace approximation), which takes the following form:

$$\widehat{p}\left(Y|\mathcal{M}\right) = (2\pi)^{\frac{k}{2}} \left|\Sigma_{\vartheta^m}\right|^{1/2} \mathcal{L}\left(\vartheta^m|Y\right) p_0(\vartheta^m),$$

where ϑ^m is the posterior mode. This technique is computationally efficient since only numerically calculated posterior mode and covariance of the estimated parameters are required. Another option to compute the marginal density is to use information from the MCMC runs and is typically referred to as the Modified Harmonic mean estimator. The idea is to simulate the marginal density and to simply take average of these simulated values. In our estimation exercise, both measures of marginal density are very close, which indicates that the posterior function is close to being symmetric and does not possess features such as fat tails and therefore can be reasonably approximated by a multivariate normal distribution. Table 2 reports logarithms of marginal data densities for several DSGE model specifications we have estimated. In particular, we estimate a baseline model specification, summarized by equations (30)-(50). In addition, we estimate a version of the model without the unemployment rate as an observable variable. We would like to test whether the unemployment rate contains relevant information for estimation and forecasting. Finally, we assess the fit of the small scale DSGE model (nested into the baseline specification) which is similar in spirit to the set up presented in Lees at al (2007) and Lubik and Schorfheide (2005). In all cases we compare the performance of the DSGE model with the more flexible DSGE-VAR specification.

Recent literature reports a rather mixed evidence on the comparative performance of structural, reduced form models and mixed specification such as DSGE-VARs. An important finding of studies by Smets and Wouters (2003) and (2007) performed for European and US data respectively is that large-scale new-Keynesian DSGE model fits better than unrestricted VAR. Smets and Wouters (2007) demonstrate that only BVAR(4) with Sims and Zha prior can do as well as the DSGE model. Sims (2003) draws attention to a number of shortcomings in Smets and Wouters (2003) analysis, which can potentially lead to over-evaluation of DSGE advantages in terms of the data fit. One of the critical points is related to the use of linearly detrended instead of raw data. The author claims that the data transformation method can distort in- and out-of-sample comparisons. Del Negro, Schorfheide, Smets, and Wouters (2005) address the criticism of Sims, performing a more consistent evaluation exercise based on the original data. More importantly, they apply a new tool for model evaluation, namely the DSGE-VAR approach. Their findings are less favorable for the DSGE model, pointing to a certain degree of model misspecification since the optimal DSGE prior weight is positive but relatively small. Thus relaxing DSGE restrictions significantly improves the model fit. A number of studies evaluate the performance of open economy DSGE model specifications. In particular, Adolfson et al. (2008) test empirical properties and forecasting outcomes of a small open economy DSGE model with modified UIP condition estimated on Swedish and EA data. The authors also evaluate the degree of model misspecification combining a VAR(VECM) with a DSGE prior. More specifically, they compare cross correlation functions for optimal $\widetilde{w}$ and $\widetilde{w} = \infty$ along with the standard deviations of the variables taken from the VECM covariance matrix. Their results suggest that there are significant differences for real exchange rate autocorrelations and standard deviations, indicating that the model remains misspecified in this direction even with more empirically relevant specification of UIP condition. In addition, they demonstrate that the DSGE-VAR correction does not support the cointegration restrictions in the DSGE model. At the same time, their results suggest that micro-based economic prior is still informative and thus improves marginal likelihood of unrestricted VAR. Lees et al.(2007) apply DSGE-VAR methodology to a small open economy model of New Zealand with explicit inflation target. They assess the DSGE-VAR forecasting performance and use the estimated hybrid structure to identify optimal policy rules. This paper shows that the weight placed on the DSGE prior is significant, both the DSGE and DSGE-VAR model outperform the official forecasts of the Reserve Bank of New Zealand.

Table 2. Model Comparison in Terms of Log Data Density (LDD)

Model specification	*DSGE*	*DSGE-VAR(2)*	
	LDD	LDD	DSGE weight
Baseline - medium scale DSGE	-577.54	-597.69	1.880
Baseline - medium scale DSGE w/o unemployment	-395.44	-404.68	1.868
Small scale DSGE w/o labor market block	-279.29	-280.37	1.142

Now lets turn to the analysis of the results presented in Table 2 and see how do they contrast with the previous studies. LDD for DSGE model is higher compared to DSGE-VAR(2) with the optimal DSGE prior weight being equal to 1.88. This result implies that relaxation of DSGE restrictions via VAR(2) correction does not improve the empirical properties of the model. It should be noted that the value of $\widetilde{w}$ cannot be directly compared across different studies. The interpretation of the value of the DSGE-VAR hyperparameter depends on the model size and the size of the data set. In particular, part of artificial DSGE observations are "consumed" in the process of construction of the proper prior distribution[14] and therefore do not count in the actual model evaluation. For example, in our case $\widetilde{w}_{\min} \approx 0.42$ whereas the model of Adolfson et al. implies $\widetilde{w}_{\min} \approx 2.7$. Thus, it is reasonable to consider the "effective" value of the hyperparameter ($\widehat{\widetilde{w}} - \widetilde{w}_{\min}$) which will measure the number of post-training artificial

[14]Recall that $\widetilde{w}_{\min} = (k+n)/T$.

observations relative to the actual data. Our results imply the weight of 60% on DSGE model and 40% on VAR(2). This measure is comparable with previous papers.[15] The analysis of Table 2 and Figure 1 also gives an idea about how well the VAR(2) approximates the DSGE model. Figure 1 shows the marginal likelihood as a function of the DSGE prior weight. The

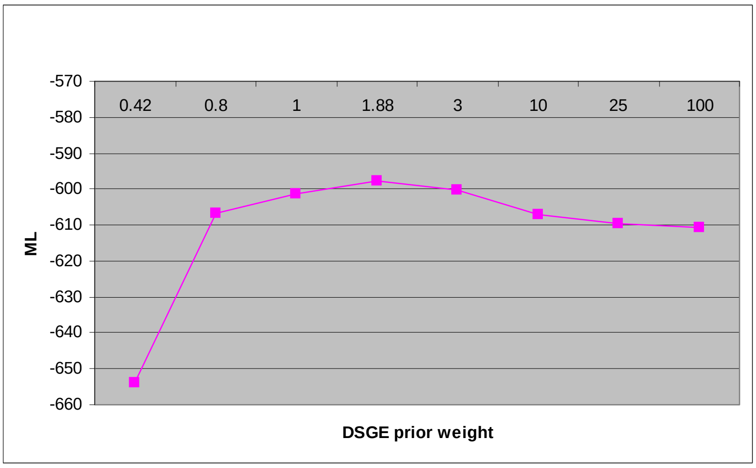

Figure 1: Marginal data density as a function of DSGE prior weight

graph demonstrates that the LDD of DSGE-VAR with $\widetilde{w} = \infty$ is different from the DSGE LDD. This result implies that the DSGE model can be approximated by a VAR(2) process only to a limited degree. In other words, the DSGE model embeds a transmission mechanism with greater internal persistence.

An approximation error present in our analysis makes it difficult to assess the dimensions in which the DSGE model can be misspecified. In this paper we would like to focus more on the forecast comparison and leave the analysis of the potential model misspecification for further research. However, we believe that the results in Table 2 support the validity of the DSGE modeling assumptions. Table 2 also demonstrates that the VAR(2) approximation of the small scale DSGE model without the labor market block is satisfactory. However, the weight on the DSGE restrictions is lower compared to the baseline specification, at about 45%. Thus, the part of DSGE restrictions associated with the labor market seems to be supported by the data. Modeling labor market dynamics (and rigid wages in particular) substantially adds to the internal propagation mechanism thus making the DSGE model more in line with actual dynamics.

5 Forecast evaluation and comparison

5.1 Point forecasts

Forecasting performance is an important criterion in the assessment of a model's credibility and usefulness for policy analysis. In this section, we compare the out-of-sample forecast accuracy

[15]Del Negro et al. and Lees et al. report the optimal weight on DSGE of about 50% , Adolfson et al. - 70%.

of the estimated DSGE model and various VARs estimated on the same data set. In particular, we would like to test whether predictions based on the theoretically-grounded DSGE model are competitive with those of reduced form approaches. Furthermore, by evaluating the outcomes obtained from the models which utilize the prior beliefs, we check whether the prior information plays a role in improving the forecast density and which prior, atheoretical or implied by the DSGE restrictions, has more relevant content for predicting the future dynamics. We calculate forecasts for 6 macroeconomic time series: output, inflation, real wages, real effective exchange rate, employment and unemployment rate. All the variables except the inflation are in growth rates. The accuracy of the predictions is assessed by using a standard recursive forecast procedure, which implies that the model is estimated up to a certain time period where the forecast distribution from one to eight quarters is computed. Then the estimation sample is extended by one more data point. The forecasts are computed for the period from 2006Q1 to 2011Q3, which gives 23 observations (roughly 1/3 of the full sample). All the models are re-estimated every quarter. As a criterion of the forecast accuracy we use a traditional measure - RMSE which is computed for one, four and eight step ahead predictions. As a robustness check, we compare 1Q ahead forecasts across different models when a dimension of the observable data set is reduced. In particular, we check whether our conclusions continue to hold if labor market data is not used in the analysis. The results are presented in Tables 3a and 3b. By numbers "in bold" we highlight the first and the second best performing model in terms of the RMSE. Table 3 allows drawing the following conclusions. First of all, the DSGE model shows a superior one step ahead predictive performance for all the variables except employment. The greatest improvement over the unrestricted VAR is observed for output, REER, unemployment and especially real wages. Over the period up to two years the DSGE model forecasting error for output is comparable to that of VAR, whose prediction accuracy improves for the medium-run (4 to 8 quarters) horizons. Table 3a also demonstrates that reduced form models outperform the DSGE in terms of precision of 4Q and 8Q inflation and employment forecasts. At the same time, the DSGE does considerably better in predicting REER, unemployment and real wages over the longer term. For this data sample, the forecasting performance of the DSGE is not improved by the VAR correction. The BVAR model performs worse in forecasting output but produces more accurate 1Q and 4Q inflation predictions compared to both VAR and DSGE. Moreover, the BVAR model outperforms both AR and VAR in forecasting unemployment and wages for short and medium term horizons. Finally, augmenting the VAR with a theoretical prior based on the DSGE model restrictions significantly improves short term forecast accuracy for output and delivers a superior exchange rate, unemployment and wages predictions over all the forecast horizons considered here. In addition, a DSGE prior appears to be more informative compared to a Minnesota-style prior when forecasting output and REER, whereas the opposite is true for employment. In predicting wages, the models deliver similar results. As for the robustness check, the DSGE compares to the VAR equally well in smaller scale specifications. Table 3b also indicates that using unemployment as an observable variable brings an improvement in output and wage forecasts.

Table 3a. Point forecast accuracy[16]

RMSE	Models				
	AR(2)	VAR(2)	BVAR(2)	DSGE	DSGE-VAR(2)
			Output		
1Q	1.6572	1.9784	1.866	**1.5185**	**1.6412**
4Q	1.7595	**1.6482**	1.8726	**1.6726**	1.676
8Q	1.6824	**1.621**	1.8524	**1.6613**	1.6782
			Inflation		
1Q	0.4130	0.4259	**0.3986**	**0.4102**	0.408
4Q	**0.3986**	0.4403	**0.4151**	0.4696	0.4834
8Q	**0.3976**	**0.4148**	0.4285	0.5013	0.4985
			REER		
1Q	1.1730	1.2542	1.0466	**0.9212**	**0.9059**
4Q	1.2283	1.271	1.081	**0.9692**	**0.9565**
8Q	1.2721	1.177	1.0317	**0.9339**	**0.9404**
			Employment		
1Q	**0.2236**	0.2947	0.2573	0.2537	**0.2411**
4Q	0.2893	**0.2795**	**0.2786**	0.480	0.4806
8Q	**0.2851**	**0.2207**	0.347	0.5143	0.4932
			Unemployment		
1Q	3.8411	4.3869	**3.6546**	**3.539**	3.9935
4Q	5.2867	6.2366	**3.6665**	**3.933**	4.1593
8Q	4.7127	6.3764	**3.3753**	**4.185**	4.1766
			Real wages		
1Q	1.0549	1.2292	0.801	**0.7475**	**0.7753**
4Q	0.9364	0.959	**0.8405**	**0.8382**	0.843
8Q	1.0342	1.075	0.8606	**0.8251**	**0.8342**

[16] All models are estimated on the same data set, which includes 6 endogenous and 3 exogenous variables. The estimation sample starts in 1995q2. The forecast evaluation sample is 2006 q1-2011q3. Bold numbers indicate the first and second best forecasting model.

Table 3b. Comparing the forecasting performance. Robustness analysis

1Q, RMSE	Models		
	VAR(2)	BVAR(2)	DSGE
	w/o unemployment data		
Output	1.978	1.889	1.65 ↑
Inflation	0.452	0.432	0.418
REER	1.29	1.072	0.934
Employment	0.33	0.277	0.239
Real wages	1.052	0.755	0.821 ↑
	w/o labor market data		
Output	1.931	1.895	1.567
Inflation	0.435	0.43	0.419
REER	1.222	1.027	0.921

[17]

The visual demonstration of the forecasting performance is shown in Figures 2 and 3, which present 1Q forecast comparison across alternative models. These plots are useful because they enable us to evaluate which models did a better job in predicting the most recent financial crisis event. The graphs show that VAR predictions are generally more volatile. In particular, this model predicts a sharp decline in the output growth around Q1 of 2009 followed by a quick recovery. The VAR overpredicts the decrease in inflation, employment and wages and also overestimates the growth of the unemployment rate after the financial distress. DSGE predictions show more persistent evolution of real variables followed by a slower recovery. Thus, qualitative characteristics of DSGE-produced forecasts better comply with the observed dynamics. BVAR models generate most accurate predictions (in terms of magnitude and persistence) for inflation and employment decline during this period. At the same time, BVAR fails to forecast a pronounced drop in the output growth. BVAR's predictions for real wages and unemployment are close to that of the DSGE.

Overall, the analysis presented here demonstrates that DSGE forecasts can compete well with more empirical models. The results of this section agree well with the conclusions from other recent studies that evaluate the ability of structural models to represent a viable alternative to reduced form specifications in forecasting experiments. In particular, Adolfson et al. (2008) report that a DSGE small open economy model developed for Sweden appears to be the best forecasting tool out of different (including VARs) models they compare. Smets and Wouters (2003) and (2007) confirm the good forecast performance of the DSGE model relative to the VAR and BVAR. Lees et al. (2007) also emphasize a competitive performance of DSGE and DSGE-VAR in forecasting the dynamics of the New Zealand economy. For their sample, the BVAR with Minnesota prior shows the best predictive accuracy.

[17] Up arrows indicate an increase of RMSE comparing to the same measure of the forecasting performance obtained under the baseline model specification which includes unemployment as an observable variable.

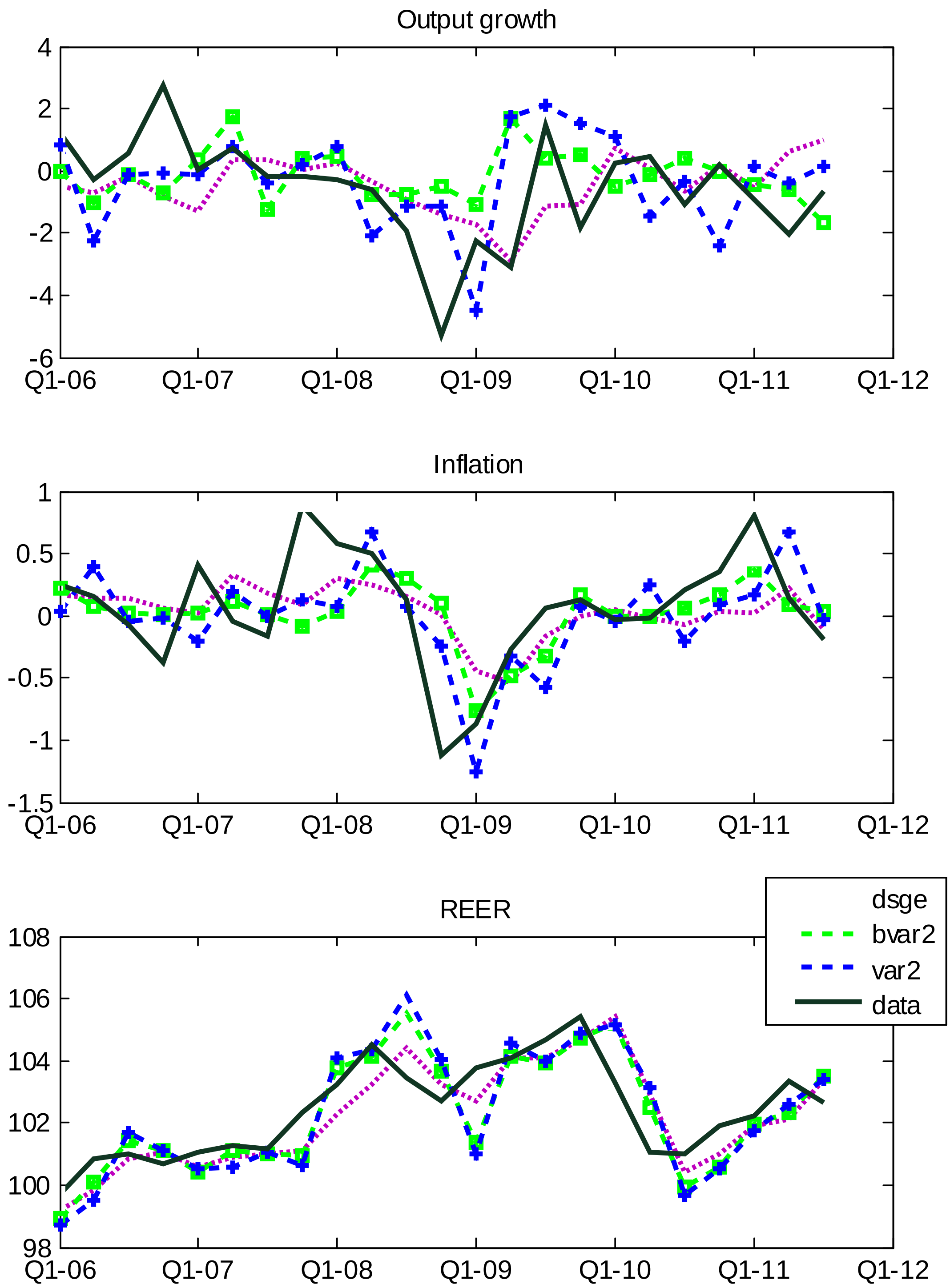

Figure 2: 1Q forecast comparison

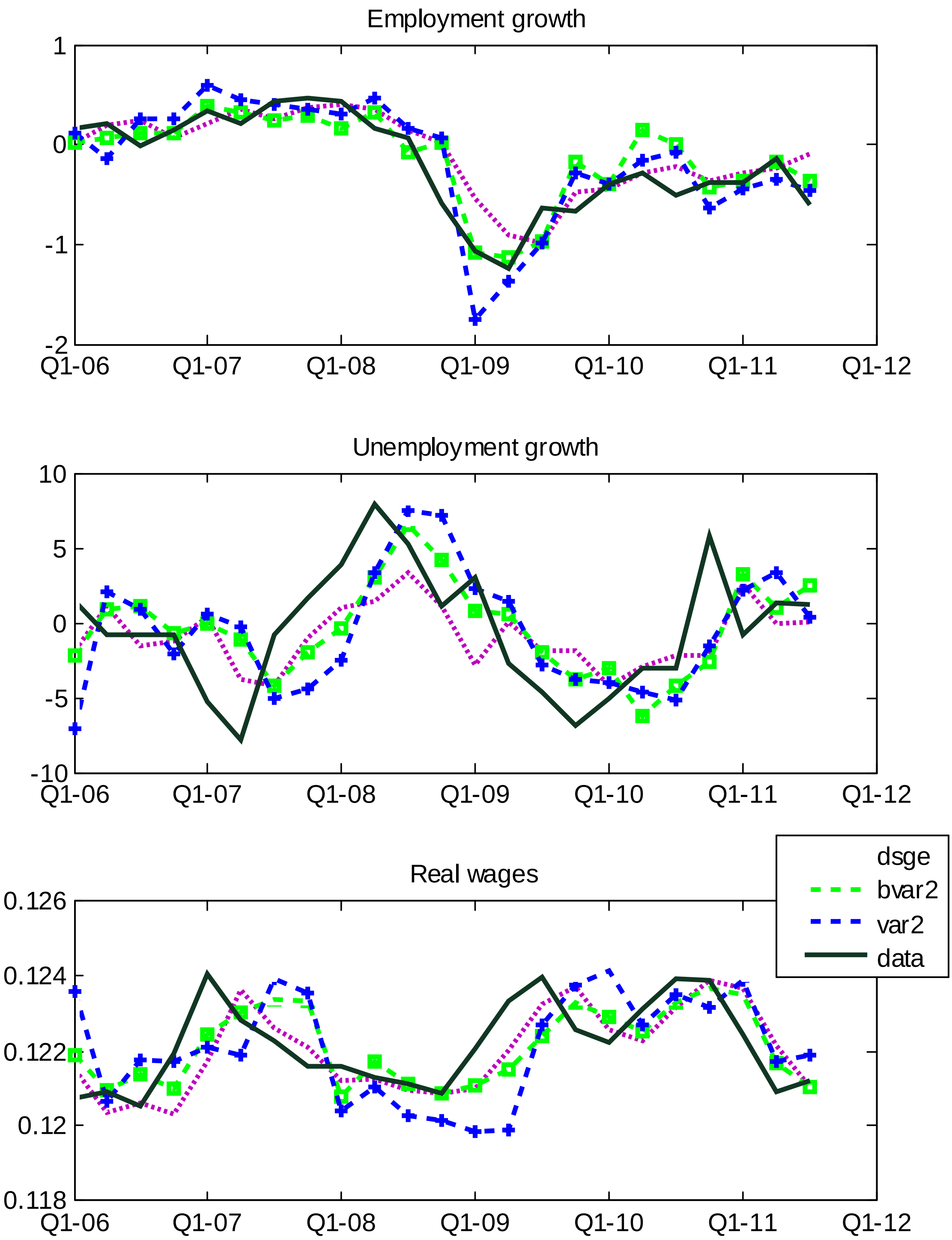

Figure 3: 1Q forecast comparison

5.2 Density forecasts

In the previous subsection, we compared the alternative models in terms of their point forecast ability. Another important measure of the forecasting performance is the comparison of predictive densities, which enables evaluating the accuracy of forecasts by taking into account the forecast uncertainty. The evaluation and ranking of the density forecasts can be done by comparing the log predictive density scores (LPDS), as described in Adolfson et al (2007) and Christoffel et al (2010). Under the assumption that $h-$step ahead predictive density is normally distributed, the LPDS for variable i can be written as:

$$s_t\left(y^i_{t+h}\right) = -0.5\left[\log(2\pi) + \log(V^i_{t+h/t}) + \left(y^i_{t+h} - \overline{y}^i_{t+h/t}\right)^2 / V^i_{t+h/t}\right],$$

where $\overline{y}^i_{t+h/t}$ and $V^i_{t+h/t}$ are the posterior mean and variance of $h-$step ahead simulated forecast distribution for variable i. The average score in forecasting variable i with the model m is given by:

$$Score^m_{i,h} = T_h^{-1}\sum_{t=T}^{T+T_h-1} s_t\left(y^i_{t+h}\right),$$

where T_h denotes the number of $h-$step ahead forecasts. It should be noted that the predictive density of the DSGE model estimated with Bayesian methods does not have a known analytical form. Following Adolfson et al (2007) we will use the multivariate normal approximation of the DSGE predictive density. This assumption is convenient because of the property of the multivariate normal density that the distribution of any subset of variables is also normal. Christoffel et al (2010) point out that, for models estimated with Bayesian methods, the only source of non-normality of the predictive density is the parameter uncertainty. Since only a small fraction of the forecast error variance is attributed to the parameter uncertainty, the normality assumption does not involve significant misspecification in computation of the log predictive score. Table 3c reports the average log predictive scores in forecasting the endogenous variables from 1 to 8-step ahead. Analyzing this measure of the accuracy of the predictions, we can see that DSGE (-based) models have significantly better forecast density for output and inflation at shorter horizon. At longer horizons, the reduced form (VAR) and structural models deliver similar predictive score for output, while for inflation and employment VAR model outperforms the DSGE. The LPDS also suggests a superior performance of the DSGE model in terms of the forecast density for REER, unemployment and real wages at all considered forecast horizons. BVAR is particularly successful in terms of the Score in predicting employment, unemployment and real wages.

Table 3c. Density forecast accuracy

SCORE	Models			
	VAR(2)	BVAR(2)	DSGE	DSGE-VAR(2)
		Output		
1Q	-2.3096	-2.3455	**-1.8574**	**-1.9377**
4Q	**-1.9425**	-2.1455	-1.951	**-1.9437**
8Q	-1.9476	-2.1227	**-1.9388**	**-1.9304**
		Inflation		
1Q	-1.6526	-1.028	**-0.6937**	**-0.9341**
4Q	-1.0384	**-0.8669**	**-0.8928**	-1.2207
8Q	**-0.6647**	**-0.9126**	-0.9523	-1.2941
		REER		
1Q	-3.0939	-1.8552	**-1.3365**	**-1.3882**
4Q	-2.1563	-1.6497	**-1.3912**	**-1.4817**
8Q	-1.7591	-1.5579	**-1.3655**	**-1.4734**
		Employment		
1Q	-0.2	**-0.0767**	-0.2	**-0.1322**
4Q	**-0.2952**	**-0.2456**	-0.7317	-0.7455
8Q	**-0.3087**	**-0.3945**	-0.79	-0.7615
		Unemployment		
1Q	-2.9121	**-2.722**	**-2.7929**	-2.8178
4Q	-3.2762	**-2.803**	-2.8656	**-2.8500**
8Q	-3.2734	**-2.7626**	-2.9188	**-2.8682**
		Real wages		
1Q	-1.8865	**-1.2025**	-1.2540	**-1.1778**
4Q	-1.4052	**-1.2601**	-1.3115	**-1.2495**
8Q	-1.5155	**-1.2801**	-1.3095	**-1.2355**

6 Contribution of structural shocks to business cycle fluctuations

6.1 Variance decomposition

In this section we study the contribution of structural shocks to the forecast error variance of the main endogenous variables at various horizons (1 quarter, 1 year, 25 years). We examine the relative importance of domestic and foreign shocks. Domestic shocks are in turn categorized into "demand" (consumption preference, exogenous demand), "supply" (productivity, price markup) and "labor market" shocks (wage markup, labor supply, exogenous employment). Foreign disturbances, associated with the openness of the domestic economy to external trade, include euro area interest rate, consumption and inflation shocks as well as a shock to the real exchange rate (terms of trade).

Table 4 demonstrates that short run fluctuations in domestic output are primarily explained by productivity shock whereas, on the longer horizons, the contribution of the consumption preference shock, which affects the intertemporal consumer choice, and foreign shocks become more important. The latter ones account for about 45 % of the total variation. Thus, in the long run domestic output is mainly driven by demand (domestic and foreign) and relative price shocks. The price markup shock is the most significant determinant of the domestic consumer price inflation. This "cost-push" shock can be interpreted as a collection of various shocks which are not explicitly modeled such as oil price changes, tax variations, etc. Productivity and demand shocks account for only 10% of inflation volatility. Such a small relative contribution can be explained by the estimated high level of price rigidities, which makes the slope of the Phillips curve very small. This implies that developments in marginal costs will have only limited impact on inflation unless these developments are very large and extremely persistent. The real effective exchange rate is mainly driven by the terms of trade as well as the foreign price shock. Domestic factors account for about 35% of the variation in this variable with a dominant role of labor market and consumption shocks. Among domestic factors that explain the employment dynamics are wage markup, labor supply and consumption shocks. Spillover effects from the euro area shocks accounts for over 45% of employment fluctuations. A similar result is reported by Pytlarczyk (2005) who finds a significant impact of the foreign factors on the business cycle of the German economy. A significant portion of unemployment rate variations are driven by labor supply and domestic consumption preference shocks, while foreign consumption and terms of trade affect domestic unemployment to a lesser extent. Real wages are mainly determined by domestic factors with the most significant impact of the price and wage markups and labor supply shocks.

In our work, a dependence of real variables on external disturbances is found to be greater compared to the majority of other studies. At the same time, for such an open and extremely small economy as Luxembourg it is not a surprising result. Another conclusion which differentiates our results from some of the DSGE papers is a small contribution of the productivity shock to the long run business cycle fluctuations. However, our estimates are in line with the

VAR-based analysis of Gali (1999) and (2010) who finds that euro area fluctuations in employment and GDP driven by technology shocks account for a small fraction of the variance of those variables (5% of employment and 9% of GDP). Clearly, the Luxembourg economy is quite specific and results reported for the Euro Area in general do not necessarily apply. Among the factors which could potentially generate a stronger role of the technology shock is a different stochastic process for the productivity shock. In particular, modeling the unit root technological process is quite common in the recent DSGE literature.

Table 4. Forecast Error Variance Decomposition

	Domestic shocks							Foreign shocks			
Variables	e_c^D	e_g^D	e_a^S	e_p^S	e_w^L	e_l^L	e_em^L	e_r	e_c*	e_p*	e_rs
t=1											
Output	21.19	0.53	57.04	0.51	0.02	0.00	0.00	2.62	10.72	0.76	6.61
Inflation	1.33	0.00	2.83	67.66	2.39	0.51	0.00	1.45	0.83	3.33	19.68
REER	0.14	0.00	0.30	7.13	0.25	0.05	0.00	0.00	0.02	16.88	75.22
Employment	8.73	0.00	1.64	0.20	0.22	0.10	79.77	1.92	4.45	0.21	2.75
Unemployment	21.34	0.07	0.46	0.07	0.67	62.16	0.00	2.25	9.50	0.27	3.22
Real wages	0.04	0.00	0.15	6.61	91.87	0.41	0.00	0.00	0.02	0.17	0.72
t=4											
Output	44.38	0.09	13.66	1.04	0.13	0.03	0.00	6.54	23.21	0.81	10.12
Inflation	3.61	0.00	5.36	55.32	5.09	1.46	0.00	4.25	2.23	2.75	19.94
REER	0.81	0.00	1.27	8.65	1.18	0.32	0.00	0.02	0.13	12.46	75.15
Employment	16.55	0.00	2.68	0.38	0.49	0.24	61.64	3.84	8.46	0.40	5.33
Unemployment	15.86	0.01	2.74	0.10	0.40	69.15	0.00	1.93	7.08	0.19	2.53
Real wages	0.26	0.00	0.84	11.50	83.59	2.71	0.00	0.03	0.11	0.15	0.80
t=100											
Output	36.76	0.02	7.14	0.93	3.70	2.48	0.00	11.19	20.97	1.18	15.63
Inflation	5.28	0.00	4.97	40.48	5.59	2.52	0.00	11.61	3.30	2.58	23.66
REER	13.19	0.00	4.20	3.32	10.69	7.11	0.00	0.10	5.61	7.23	48.56
Employment	21.20	0.00	1.22	0.59	8.02	5.75	13.09	15.02	12.18	1.62	21.30
Unemployment	18.53	0.00	1.70	0.18	2.13	57.21	0.00	4.47	9.21	0.46	6.10
Real wages	2.23	0.00	6.39	11.27	62.64	15.23	0.00	0.55	1.11	0.10	0.50

6.2 Impulse response analysis

Table 5 summarizes the responses of the main endogenous variables to 1% temporary structural shocks to price and wage markups, short term interest rate and the domestic productivity. The responses are computed on the basis of the estimated (at the posterior mode) parameters. Table 5a shows that a decrease in the price markup, which can be associated with the reduction of monopolistic competition on the goods' market, lowers prices and inflation on impact. As a result, real wages and consumption rise. The presence of nominal rigidities results in a more gradual adjustment of prices compared to the economy with flexible price dynamics. Thus, the negative impact of the shock on firm's profit (caused by the price reduction) is more than offset by higher consumer demand which stimulates production and consequently employment[18]. In addition, a fall in the domestic prices implies a superior relative price competitiveness thus improving the terms of trade. The overall impact of this shock on the economy is positive.

Table 5b presents the effects of a temporary decline in the wage markup. This shock is the dominant factor behind the wage dynamics. Thus, not surprisingly, real wages fall significantly. As a result, marginal costs and inflation decline. Employment rises in line with output. Unemployment shows a persistent decrease.

The responses to a 1% temporary increase in the euro area interest rate are shown in Table 5c. The monetary contraction leads to a hump-shaped fall in output and consumption. Lower aggregate demand and production reduces labor demand which brings about reduction in employment and real wages. The area-wide interest rate shock has also a non-zero negative effect on relative prices, which deteriorates domestic competitiveness. We would like to point out an important difference with respect to the response of the Luxembourg economy to the monetary policy shock described in Deak et al. (2012). The model presented in this paper explicitly incorporates the financial services sector and thus can take into account a (potentially different) response of the banking segment to the shock. In particular, the authors show that a higher policy rate can translate into higher foreign (non euro area) deposits in the international banking sector, which leads to an expansion in this segment and has a positive stimulating effect on the whole economy.

Finally, Table 5d demonstrates that, following a positive productivity shock, aggregate demand, output and real wages increase, which is accompanied by an immediate reduction in hours worked and, consequently, employment[19]. The rise in the productivity leads to a fall in marginal costs. Because of the assumption of small open economy, the euro area monetary policy rate does not respond and the negative output gap emerges. Due to the presence of nominal rigidities, prices and inflation respond only gradually. Thus, firms react by adjusting hour and employment. Compared to the flexible-price-and-wage responses, the immediate impact of the productivity shock on output is significantly lower but, at the same time, more persistent with the pick of the response achieved in about two years.

Overall, our impulse response results are in line with the analytics presented in Deak et al. (2011) and (2012) for the structural model of Luxembourg, except for the response to the monetary policy and, partially, price markup shocks.

[18]Deak et al. (2012) show that in the economy without New Keynesian features the negative markup shock has welfare improving consequences in a form of higher real wages, income and consumption. Thus, in this respect our two papers reach the similar conclusions. At the same time, under flexible prices lower markups decrease firm's profit to a greater extent compated to our model which translates into the employment reduction and unemployment increase.

[19]Gali (1999), Gali and Rabanal (2004) and Smets and Wouters (2002) and (2007) also describe the negative impact of productivity on hours.

Table 5.

a) 1% decrease in the price markup

Variable	1y	2y	3y	4y	5y	10y
Output	++	++	++	+	+	+
Consump-n	++	++	++	+	+	+
Inflation	- -	-	-	+	+	+
REER	++	++	++	+	+	+
Empl-nt	+	+	+	+	+	+
Unempl-nt	- - -	- - -	- -	- -	-	+
Wages	++	++	++	++	+	+

[20]

b) 1 % decrease in the wage markup

Variable	1y	2y	3y	4y	5y	10y
Output	+	+	+	+	++	+
Consump-n	+	+	+	+	+	+
Inflation	-	-	-	+	+	+
REER	+	+	+	+	+	+
Empl-nt	+	+	+	+	+	+
Unempl-nt	- -	- -	- - -	- -	- -	- -
Wages	- - -	- - -	- -	- -	-	-

c) 1% increase in the interest rate

Variable	1y	2y	3y	4y	5y	10y
Output	- - -	- - -	- - -	- - -	- -	- -
Consump-n	- - -	- - -	- - -	- - -	- -	- -
Inflation	-	-	-	-	-	-
REER	-	-	-	-	-	-
Empl-nt	- - -	- - -	- - -	- -	- -	- -
Unempl-nt	+++	+++	+++	+++	++	++
Wages	-	-	- -	- -	-	-

d) 1 % increase in the productivity

Variable	1y	2y	3y	4y	5y	10y
Output	+	+	+	+	+	+
Consump-n	+	+	+	+	+	+
Inflation	-	-	-	+	+	+
REER	+	+	+	+	+	+
Empl-nt	-	-	-	-	-	+
Unempl-nt	++	+	+	+	+	+
Wages	+	+	+	+	+	+

7 Conclusions

In this paper we develop and estimate a DSGE model for Luxembourg, as an example of a small open economy within the single currency area. We allow for a sufficiently rich specification which enables us to include unemployment as well as open economy variables such as the real exchange rate into the estimation procedure, along with the standard macroeconomic and labor market indicators. The model contains a set of frictions and structural shocks typically used in the DSGE literature. We demonstrate that the estimated DSGE model is relatively well identified, has good data fit and reasonably estimated parameters. In addition, the model shows a competitive forecasting performance (in terms of both point and density) compared to reduced form models such as VARs. In this respect, our results are in line with the conclusions reached in previous studies that the new generation of DSGE models no longer faces the tension between rigor and fit. In particular, we illustrate that the DSGE model produces sizable (one-step-ahead) forecasting gains in terms of $RMSE$ and $Score$ over the unrestricted VAR, especially for such variables as GDP, real exchange rate, unemployment and real wages. The predictions stay competitive at longer forecasting horizons.

As a result of a sufficiently rich specification, the solution to the model implies rather tight cross equation restrictions on the estimated structure. On the one hand, this can be considered

[20] +, ++, +++ denote an increase in the range of 0-0.5%, 0.5-1% or larger than 1% respectively.

as a limitation of the approach. On the other hand, micro-founded restrictions that have a realistic content can bring useful additional information into the estimation procedure and thus improve the model fit. In particular, the DSGE-VAR analysis demonstrates that the optimal weight on the DSGE restrictions is significant and the VAR(2) correction is not helpful in improving the DSGE model fit. At the same time, the DSGE-based prior significantly improves the short term forecast accuracy of the unrestricted VAR for output, and also determines a superior performance of the DSGE-VAR model in predicting exchange rate, unemployment and wages over all the forecast horizons considered here. When compared to an atheoretical Minnesota-style prior, the DSGE restrictions appears to be more useful in forecasting output and REER, whereas the opposite is true for employment. The results of this analysis do not imply of course the absence of model misspecification but at the same time they show that a DSGE structure provides a reasonable approximation of the reality. In addition, we admit that the evaluation of the model on the relatively short data sample available for Luxembourg (66 observations) can lead to overestimation of the performance of the prior-based specifications.

Application of the model to the analysis of the business cycle fluctuations demonstrates that "open economy" disturbances such as relative price, foreign demand and interest rate shocks explain a significant portion of the variation of output growth, inflation, real exchange rate and employment. Price and wage markup shocks are important determinants of inflation and real wages dynamics respectively.

Finally, we would like to discuss possible extensions. First of all, it would be useful to extend the model by considering a more disaggregated structure and, in particular, incorporate the financial services sector, which constitutes a significant portion of the Luxembourg economy and can be a driving force of the economy as a whole. Since the responses of this sector to monetary and other shocks might be quite specific, the overall characteristics and model predictions can be affected. Secondly, the properties of the Luxembourg economy differ significantly from the rest of the EMU. Therefore, it would make sense to improve the existing specification by modeling heterogeneous features of both regions other than the size and degree of openness (for example, we could allow for different growth rates and provide more elaborate modeling of the EMU with individual parameterization and better identification of area-wide shocks).

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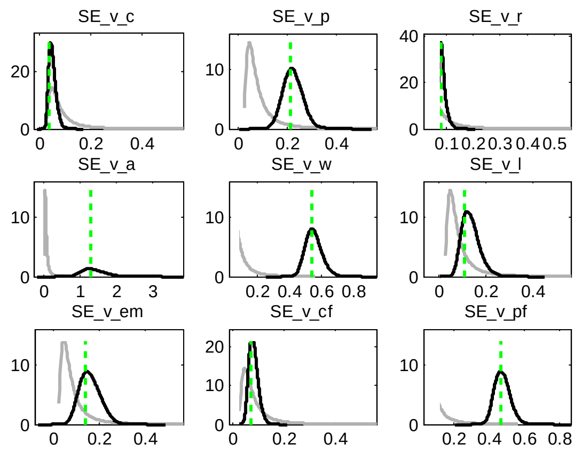
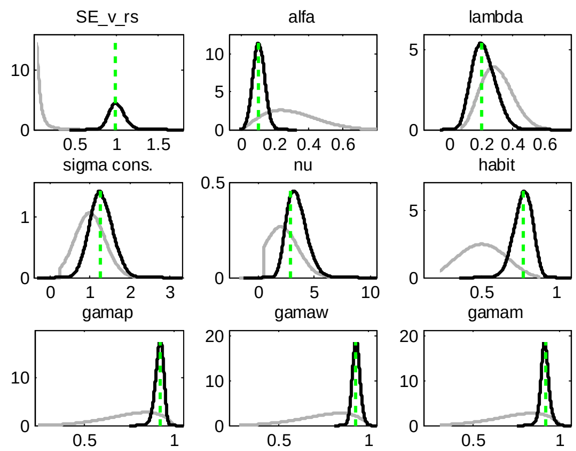
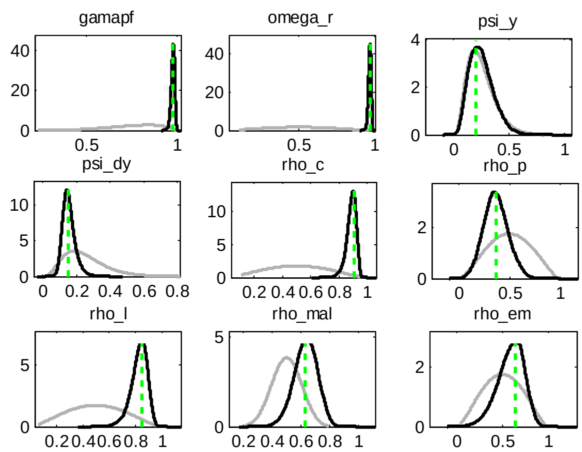
8 Appendix

Figures. 1A. Priors and posteriors







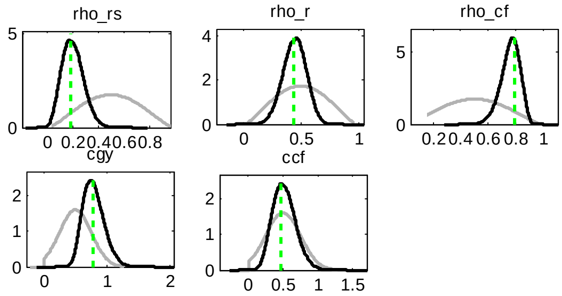


Table 1A. Comparison of the posterior distribution of DSGE structural parameters for alternative estimation samples

Parameters		Prior distribution			Posterior distribution			
					1995q1-2007q4		1995q1-2011q3	
		Type	Mean	St.dev	Mode	St.dev	Mode	St.dev
Production function	λ	Beta	0.3	0.1	0.223	0.087	0.202	0.077
Degree of openness	α	Beta	0.3	0.15	0.098	0.039	0.102	0.034
Consumption utility	σ_c	Norm	1	0.375	1.370	0.297	1.256	0.292
Labor utility	η	Norm	2	1.5	2.303	0.737	2.873	0.804
Consumption habit	χ	Beta	0.5	0.15	0.701	0.095	0.776	0.062
Calvo prices	γ^p	Beta	0.75	0.15	0.929	0.023	0.923	0.022
Calvo wages	γ^w	Beta	0.75	0.15	0.939	0.023	0.929	0.019
Calvo employment	γ^m	Beta	0.75	0.15	0.929	0.028	0.918	0.021
Calvo foreign prices	γ^{p*}	Beta	0.75	0.15	0.986	0.009	0.977	0.01
Pol.rule: lagged int.rate	ω_r	Beta	0.5	0.2	0.975	0.010	0.973	0.010
Pol.rule: output	ψ_y	Gam	0.25	0.125	0.220	0.111	0.201	0.101
Pol.rule: lagged output	$\psi_{\Delta y}$	Gam	0.25	0.125	0.183	0.051	0.151	0.034

Table 2A. Comparison of the posterior distribution of DSGE shock processes for alternative estimation samples

Parameters		Prior distribution			Posterior distribution			
					1995q1-2007q4		1995q1-2011q3	
		Type	Mean	St.dev	Mode	St.dev	Mode	St.dev
Standard deviations								
Consumption preference	υ_c	Inv.G	0.1	2	0.043	0.014	0.037	0.01
Productivity	υ_a	Inv.G	0.1	2	1.197	0.315	1.296	0.306
Price markup	υ_p	Inv.G	0.1	2	0.225	0.036	0.212	0.038
Wage markup	υ_w	Inv.G	0.1	2	0.583	0.059	0.54	0.049
Relative price	υ_{rs}	Inv.G	0.1	2	0.905	0.092	0.985	0.088
Labor supply	υ_l	Inv.G	0.1	2	0.089	0.032	0.108	0.033
Exogenous employment	υ_{em}	Inv.G	0.1	2	0.135	0.038	0.142	0.042
Foreign demand	υ_{c*}	Inv.G	0.1	2	0.054	0.015	0.071	0.017
Foreign prices	υ_{p*}	Inv.G	0.1	2	0.374	0.038	0.463	0.042
Interest rate	υ_r	Inv.G	0.1	2	0.075	0.013	0.08	0.011
Persistence and correlat.								
Consumption	ρ_c	Beta	0.5	0.2	0.910	0.031	0.909	0.024
Price markup	ρ_p	Beta	0.5	0.2	0.235	0.133	0.368	0.122
Relative price	ρ_{rs}	Beta	0.5	0.2	0.173	0.094	0.184	0.087
Labor supply - AR	ρ_l	Beta	0.5	0.2	0.876	0.051	0.85	0.055
Labor supply - MA	$\rho_{ma,l}$	Beta	0.5	0.1	0.629	0.082	0.631	0.079
Exogen.employment	ρ_{em}	Beta	0.5	0.2	0.670	0.113	0.635	0.134
Interest rate	ρ_r	Beta	0.5	0.2	0.465	0.101	0.438	0.101
Foreign demand	ρ_{c*}	Beta	0.5	0.2	0.785	0.089	0.789	0.068
Demand-Productivity	ρ_{ag}	Norm	0.5	0.25	0.834	0.198	0.785	0.173
Consum.-Foreign demand	ρ_{cf}	Norm	0.5	0.25	0.430	0.194	0.468	0.160

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