The DELPHI model
(**D**ynamic **E**conometric **L**arge-scale **P**rognosticator of **H**ungarian **I**nflation)

Ágnes Horváth – Áron Horváth – Balázs Krusper –
Benedek Nobilis – Tímea Várnai – Viktor Várpalotai[1]
Magyar Nemzeti Bank

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[1]Corresponding author. Email address: varpalotaiv@mnb.hu

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Introduction

The macroeconometric modelling started more than a decade ago at the Magyar Nemzeti Bank. The development of the first macroeconometric model has begun in 1999, and it become the Hungarian block of the NIGEM multi-country model. After the refinement of the model structure it was known as N.E.M. model (Quarterly Forecasting Model, see Benk et al, 2006) and it served as a macro forecasting model of the Magyar Nemzeti Bank for several year. The N.E.M. model was used not only for producing regular forecasts but also for running policy simulations (see e.g. Hornok– Jakab–Tóth, 2006, Horváth et al, 2006, Jakab–Várpalotai–Vonnák, 2006).

The N.E.M. model proved to be very useful in economic analyses, however, it turned out that further developments were needed. The Magyar Nemzeti Bank needed model that fulfills stock-flow consistency, incorporates knowledge accumulated by the bank's experts about the Hungarian economy, has detailed fiscal block, delivers consistent view about convergence path and long run projections free from cyclical movements.

Considering the above mentioned needs, development of a completely new model has been started. However the new model has similarity with respect to its predecessor. Both are middle-sized macroeconometric model, however some of the blocks of the new model is more detailed (new model has around 150 equations those of which 38 are behavioral equations, the N.E.M. model has around 100 equations and 29 behavioral equations). In both models, the long run equilibrium path is determined by neoclassical growth theorem, however, in the short run, frictions and nominal rigidities delay the adjustment towards long run equilibrium.

Besides similarities, there are differences between the new and N.E.M. model. The new model relies on national accounts identities, i.e. both intra and inter temporal identities are fulfilled. In the new model government sector links other sectors via several types of taxes and expenditures. Detailed fiscal sector helps to conduct more reliable fiscal analysis with the new model. In the new model the consistency of fiscal account and national accounts is ensured. A new quarterly database is compiled in which data series, as the model does, fulfill the national account identities. This new database is a valuable contribution in itself, because, to our knowledge there was no previosly available database suitable for macromodelling in Hungary.[1] The neoclassical growth framework employed in the model ensures dynamics free from overhangs, and delivers reasonable convergence path and steady state ratios. A further feature of the new model is that it is heavily relies on knowledge accumulated by expert analysts of the Magyar Nemzeti Bank, thus, it is not a competitor to the bank's analysts, but contrary, it compiles partial relations used by the analysts into a consistent simultaneous system.[2]

[1] The actual databaes can be downloaded from http://english.mnb.hu/Monetaris_politika/mnben_monpol_rendszere/report-on-inflation.

[2] We compiled the expert knowlegde into the model in two ways. (1) Either we matched the partial impulse response functions of the DELPHI model with experts' partial models impulse response functions. We used this technique to calibrate the labor market and consumer price blocks of the model. (2) Or we reformulated and reestimated equations used by experts, who applied new estimates in their own forecasting tools. This approach was used for export and import equation for example.

This paper presents the new model named DELPHI (Dynamic Econometric Large Scale Prognosticator of Hungarian Inflation). In the first section we describe the main features of the DELPHI model in a nutshell. In the second section model equations are presented block by block. In the third section we discuss the main channels of the monetary transmission in the DELPHI model. In the fifth section we review some potential direction of future development.

1 Review of the model structure

The DELPHI model delivers a neoclassical balanced growth path in the long run, however, in the short run it shows Keynesian features due to nominal and real rigidities. The potential output is determined by the capital and labor inputs and technology via a Cobb-Douglas production function. Actual demand may differ from the potential output level in the short run, however excess or insufficient demand starts off nominal and real adjustment processes.

The DELPHI model is able to capture not only the long run balanced growth path and the short run forecasting horizon but convergence path (the middle run) also. The behavioral equations of the model are written in error correction form that is a convenient way to link long run constraints derived from theory with Keynesian frictions and rigidities in the short run.

1.1 Sectors and their balance sheets

There are four sectors in the DELPHI model in accordance with the Hungarian national account breakdown: households, corporations, government and rest of the world. Household sector incorporates nonprofit sector, corporations incorporate both financial and non-financial corporations. It is the corporations and the government sectors of the four above mentioned sectors that produce value added. Distinguishing four sectors in the model enables that the total value added (GDP) produced in the country is the sum of the value added produced by the corporations and the government sectors, the total value added is completely divided into factor incomes, moreover, income flows across sectors are identifiable.

Next we sketch out income-expenditure balances of each sector.

The households decide on consumption depending on their income and accumulated financial wealth. Households' consumption is subject to taxes and fees. Households' income has four sources: (1) after tax labor income from corporations and government sectors, (2) financial transfer from government, (3) interest and dividend payments from corporations sector and (4) transfers from abroad. Households spend their income on consumption and housing investment. Income-expenditure balance of the households (i.e. net financial savings of the households) augments the financial wealth of the households.

The government sector incomes are (1) social contribution on gross labor income, (2) income tax on gross labor income, (3) consumption tax (VAT and excise duties), (4) fees proportional to household consumption, (5) profit tax, (6) net balance of subsidies and other taxes of corporations and (7) transfers from abroad (mainly from EU funds).

The expenditures of the government are: (1) wage bills, (2) goods and services purchased from the market, (3) financial transfers to households, (4) transfers to households in kind, and (5) government investment.

Balance of the previously enumerated items is the primary balance of the government. Adding the interest payment to the primary balance yields total balance of the government that changes the net financial wealth (debt) of the government. In the long run financial transfers to household is determined so that the deficit doesn't exceed the 3% of GDP and the government debt to GDP ratio remains below 60% (Maastrich criteria consistent budget rule).

The corporations produce output according to actual demand by properly changing its capacity utilization. In the long run, the factors (i.e. labor and private capital) demand is adjusted so that the production of private output can meet the actual demand at full capacity utilization level. In the short run, output gap is the difference of actual demand and supply determined by technology, factors available for production and full capacity utilization.

The corporations pay gross wages for labor supplied by households and social contribution to government. The rest of corporations' income is subject to profit tax and other corporate tax. The corporations have financial intermediator role as well: it reallocates financial resources (net savings) and incomes among the sectors. It also redistributes interest payments after net financial wealth of the sectors

The foreign sector is linked to domestic economy via (1) financial transfers (mainly pay offs from EU funds), (2) trade and (3) providing financing opportunity to the domestic economy (foreign debt).

1.2 Relationship among the main variables

The supply side of the model is as follows. Private output is pinned down by the production function combining labor and capital. In the long run the labor supply is exogenous determined by demography and NAIRU. Exogenously set yields pin down the equilibrium amount of capital (and hence investment) via corporations' profit maximization. The output of the government sector is determined by government capital stock and the amount of labor employed in government sector.

The demand side of the economy consists of standard demand components of national accounts. Households' consumption is determined is affected by income, net financial wealth and net credit flow.[3] Government consumption is pinned down by identity among certain government expenditure items.[4] Corporate investment is determined by profit maximization. Housing investment, government investment and change in inventories are fix proportions to GDP in the

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long run. Export is governed by foreign demand and real exchange rate. Import is set by import share of other demand components and the real exchange rate.

In the long run, consumer prices are determined by their cost (unit labor cost, exchange rate, foreign prices, oil prices), in the short run it is also influenced by expectation, output gap and actual GDP growth (Phillips curve). In the long run, wage in private sector equals to the marginal product of the labor, in the short run it is also affected by expectation and labor market tightness.

The current version of the DELHPI model has no forward looking variables and it treats monetary variables (exchange rate and interest rate) exogenous.

2 Model structure in detail

In this section we present the model equation in detail. We apply the following notation. The time indices of model variables are neglected except for lags (or leads). It means that if there is no time subscript in a particular equation then each variable applies to same time period. In general, model parameters can vary over time. Constant parameters are replaced by their actual value. In most cases, behavioral equations are written in error correction form, where X^* denotes the long run value of variable X.

2.1 Supply side [A]

Value added is produced in corporate and government sectors. The private potential output (YP) is the potential output of the corporate sector (YP). It is pinned down by a Cobb-Douglas type production function:

$$YP = TFP \cdot KG_{-1}^{\alpha_G} \cdot KC_{-1}^{\alpha_E} \cdot EPTR^{1-\alpha_E}, \tag{A.1}$$

where KG_{-1} and KC_{-1} are end of previous period government and private capital stocks, respectively. TFP is the level of the technology. $EPTR$ is the equilibrium number of employed in the private sector set by the number of trend of active population determined by demography ($LFTR$), by the equilibrium unemployment rate (UTR) and by the equilibrium ratio of employed in private sector to total employment ($EPRATIOTR$):

$$EPTR = (1 - UTR) \cdot LFTR \cdot EPRATIOTR. \tag{A.2}$$

Potential output of the government sector is set by accounting identity (analogous to one describing the actual output of government sector):

$$YG^* = \frac{G\_COMP^* + INC\_KG^*}{PYG}, \tag{A.3}$$

where G_COMP^* and INC_KG^* are compensation of employee in the government sector and amortization of government capital stock at current prices. PYG is the deflator of value added

of the government sector. The equilibrium value of compensation of employees in government sector is an exogenously given fix proportion to private potential output:[5]

$$G\_COMP^* = \gamma_{COMP} \cdot \frac{PYP \cdot YP}{\frac{\overline{PYP \cdot YP}}{\overline{PY \cdot Y}}}, \tag{A.4}$$

where $\frac{\overline{PYP \cdot YP}}{\overline{PY \cdot Y}}$ is the private value added to total value added ratio in equilibrium, PYP is the deflator of the private value added, thus expression $\frac{PYP \cdot YP}{\frac{\overline{PYP \cdot YP}}{\overline{PY \cdot Y}}}$ is the potential GDP at current prices. γ_{COMP} is the proportion of the nominal GDP spent on compensation of employees.

The equilibrium amount of amortization of government capital stock is:

$$INC\_KG^* = \delta_{INC\_KG^*} \cdot KG_{-1}, \tag{A.5}$$

where δ_{INC_KG} is the amortization rate of government capital for accounting purpose.

The total potential output at factor prices (YP_TOT) is the chain-weighted average of the potential outputs of the two sectors:

$$YP\_TOT = \frac{PYP_{CHAIN} \cdot YP + PYG_{CHAIN} \cdot YG^*}{PY_{CHAIN}}, \tag{A.6}$$

where PYP_{CHAIN}, PYG_{CHAIN} and PY_{CHAIN} are the chain index of private, government and total value added (GDP), respectively.

Using the potential and actual outputs we can calculate the private, government and total output gap measures as it follows:

$$GAP\_PRIV = \frac{YPD}{YP} - 1 \tag{A.7}$$

$$GAP\_GOV = \frac{YG}{YG^*} - 1 \tag{A.8}$$

$$GAP = \frac{YP\_TOT}{YD} - 1. \tag{A.9}$$

2.2 Factor demand [B]

2.2.1 Labor demand (employment)

Equation (A.1) determines the potential output of the private sector. However actual output is pinned down by actual aggregate demand (see equation (N.11)). Therefore actual demand for factors is affected by actual output. The equilibrium private employment (EP^*) is labor input needed to match actual output with given technology and accumulated capital stock (inverse

[5] The reason behind assuming fix proportion in the long run is that the private economy is able to pay only a certain part of its income as tax. Furthermore, only a part of the collected taxes can be spent on compensation of employees.

production function). Actual employment may differ from its equilibrium level:[6]

$$EP^* = \left[\frac{YPD}{TFP \cdot KG_{-1}^{\alpha_G} \cdot KC_{-1}^{\alpha_E}}\right]^{\frac{1}{1-\alpha_E}} \tag{B.1}$$

$$\begin{aligned} \mathrm{dlog}(EP) &= 0.050 \cdot \mathrm{dlog}(EP_{-1}) + 0.050 \cdot \mathrm{dlog}(YPD_{-1}) + 0.050 \cdot \mathrm{dlog}(YPD_{-2}) - \\ &\quad -0.200 \cdot (\log(EP_{-1}) - \log(EP^*_{-1})) - 0.033 \cdot (\log(WP_{-1}) - \log(WP^*_{-1})), \end{aligned} \tag{B.2}$$

where WP and WP^* are the private actual and equilibrium wage, respectively.

The equilibrium employment in the government sector is the difference of number of active people and the equilibrium employment in the private sector:

$$EG^* = LFTR \cdot (1 - UTR) \cdot (1 - EPRATIOTR) \tag{B.3}$$

$$\mathrm{dlog}(EG) = 0.390 \cdot \mathrm{dlog}(EG_{-1}) - 0.150 \cdot (\log(EG_{-1}) - \log(EG^*_{-1})). \tag{B.4}$$

Unemployment rate (U) is the ratio of total number of employed in private and government sectors to number of active population (LF):

$$U = 1 - \frac{EP + EG}{LF}. \tag{B.5}$$

2.2.2 Capital demand (corporate investment)

Corporate investment is pinned down by profit maximization of corporations. Investments are subject to adjustment cost, thus the profit maximizing investment (CI) depends on the excess rate of return on investment (QE)[7], adjustment costs (λ_{KC}), amortization rate (δ_{KC}) and the growth rate of potential output ($g_P \equiv \frac{YP}{YP_{-1}} - 1$):

$$C\_I^* = \left(\frac{QE}{\lambda_{KC}} + \delta_{KC} + g_P\right) \cdot KC_{-1}, \tag{B.6}$$

$$\begin{aligned} \mathrm{dlog}(C\_I) &= (1 - 0.200 - 0.800) \cdot \mathrm{dlog}(C\_I^*) + 0.200 \cdot \mathrm{dlog}(C\_I_{-1}) + \\ &\quad +0.800 \cdot \mathrm{dlog}(X) - 0.250 \cdot (\log(C\_I_{-1}) - \log(C\_I^*_{-1})) \end{aligned} \tag{B.7}$$

where X is the export volume.

We assume that the price index of private capital equals to that of private investment. Thus, the excess rate of return (QE) is a function of after tax marginal product of capital, amortization

[6] It means that we implicitly assume time varying capacity utilization in the model.

[7] The particular form of QE is analogous to expressions one can get if assumes present value maximizing firms subject to quadratic adjustment costs. See equation (B.8).

(δ_{KC}), long run interest rate (RL) and the investment risk premium ($kprem\_hp$):

$$QE = (1 - \tau_{PROF}) \cdot \alpha_E \cdot \frac{YP}{KC_{-1}} \cdot \frac{PYP}{PCI} - (\delta_{KC} + RL + kprem\_hp) \tag{B.8}$$

where PCI is the deflator of corporate investment, RL is the two-year moving average of the long term interest rate (R).

2.3 Price setting [C]

2.3.1 Consumer prices

The dependent variable of the Phillips curve is the core inflation filtered from changes in indirect taxes ($COREVAI$). In the long run, core price level is determined by exchange rate ($NEER$), foreign prices (PF) and oil price (P_OIL) both expressed in domestic currency, price level of a basket of agricultural goods (P_MG)and unit labor cost (ULC). As a consequence of this cost based approach, the Phillips curve is vertical in the long run. In the short run, however, there exists nominal rigidities, thus core inflation is affected by the output gap, the actual growth and the inflation expectation (INF_EXP):

$$\begin{aligned} \log(COREVAI^*) &= -1.437 + 0.6800 \cdot \log(ULC) + 0.0628 \cdot \log(P\_MG) + \\ &\quad +0.3216 \cdot \log(NEER) + 0.2226 \cdot \log(PF) + 0.0346 \cdot \log(P\_OIL) \end{aligned} \tag{C.1}$$

$$\begin{aligned} \text{dlog}(COREVAI) &= 0.5434 \cdot \text{dlog}(COREVAI_{-1}) + 0.3898 \cdot INF\_EXP + \\ &\quad +0.0432 \cdot \text{dlog}(ULC) + 0.0233 \cdot \text{dlog}(P\_MG) + 0.0365 \cdot \text{dlog}(NEER) + \\ &\quad +0.0283 \cdot \text{dlog}(PF) + 0.0044 \cdot \text{dlog}(P\_OIL) + \\ &\quad +0.084 \cdot GAP + 0.06 \cdot \text{dlog}(YD) - \\ &\quad -0.1084 \cdot (\log(COREVAI_{-1}) - \log(COREVAI^*_{-1})). \end{aligned} \tag{C.2}$$

Unit labor cost (ULC) is set by the wage cost, employment and output:

$$ULC = \frac{(1 + \tau_{SSCP}) \cdot WP \cdot EP \cdot \frac{3}{1000000}}{YPD} \tag{C.3}$$

Inflation expectation is a combination of past inflation and the weighted average of actual and target inflation:

$$\begin{aligned} \log(INF\_EXP) &= 0.800 \cdot \log(INF\_EXP_{-1}) + \\ &\quad + (1 - 0.800) \cdot \left(0.333 \cdot \text{dlog}(CPI) + 0.667 \cdot \text{dlog}((1 + TARGET)^{0.25})\right) \end{aligned} \tag{C.4}$$

In the long run, price level filtered from indirect tax changes of out-of-core item ($N\_COREVAI$) is pinned down by exchange rate, oil prices, agricultural prices and a residual term (P_NCMISC).

The short run dynamics of is affected by dynamics of the previously enumerated variables:

$$\begin{aligned}\log\left(NCOREVAI^{*}\right) &= 0.2495\cdot\log(NEER)+0.1896\cdot\log(P\_OIL)+ \\ &\quad +0.1414\cdot\log(P\_MG)+0.669\cdot\log(P\_NCMISC)\end{aligned} \tag{C.5}$$

$$\begin{aligned}\mathrm{dlog}(NCOREVAI) &= 0.4404\cdot\mathrm{dlog}(NCOREVAI_{-1})+0.0956\cdot\mathrm{dlog}(NEER)+ \\ &\quad +0.0718\cdot\mathrm{dlog}(P\_OIL)+0.0784\cdot\mathrm{dlog}(P\_MG)+ \\ &\quad +0.4094\cdot\mathrm{dlog}(P\_NCMISC)- \\ &\quad -0.1071\cdot\left(\log(NCOREVAI_{-1})-\log(NCOREVAI^{*}_{-1})\right)\end{aligned} \tag{C.6}$$

Consumer price index filtered from changes in indirect taxes is the weighted average of the core and the non-core index filtered from changes in indirect taxes:

$$\log(CPIVAI)=0.66973\cdot\log(COREVAI)+(1-0.66973)\cdot\log(NCOREVAI) \tag{C.7}$$

Adding the indirect tax change effect (VAI_CORE and VAI_NCORE) we get the core ($CORE$) and non-core (N_CORE) indices:

$$CORE = VAI\_CORE\cdot COREVAI \tag{C.8}$$

$$NCORE = VAI\_NCORE\cdot NCOREVAI \tag{C.9}$$

Consumer price index is the weighted average of the core and the non-core index. Analogously, we can calculate the effect of indirect tax changes on consumer price (VAI_CPI) as follows:

$$\log(CPI) = 0.66973\cdot\log(CORE)+(1-0.66973)\cdot\log(NCORE) \tag{C.10}$$

$$\log(VAI\_CPI) = 0.66973\cdot\log(VAI\_CORE)+(1-0.66973)\cdot\log(VAI\_NCORE) \tag{C.11}$$

For simulation and forecasting purposes we assume that in indirect taxes rates remain unchanged:

$$VAI\_CORE = VAI\_CORE_{-1} \tag{C.12}$$

$$VAI\_NCORE = VAI\_NCORE_{-1}. \tag{C.13}$$

Moreover, we forecast the following price indices by a simple rule:

$$P\_OIL = P\_OIL_{-1}\cdot(1+\pi_F) \tag{C.14}$$

$$PF = PF_{-1}\cdot(1+\pi_F) \tag{C.15}$$

$$P\_MG = P\_MG_{-1}\cdot(1+\pi) \tag{C.16}$$

$$P\_NCMISC = P\_NCMISC_{-1}\cdot(1+\pi), \tag{C.17}$$

where π_F and π are the foreign and domestic equilibrium inflation rate, respectively. (Both parameters are set to annual 2%).

2.3.2 Wages

Real wages cost of employed in the private sector is pinned down by the marginal product of labor in the long run. Due to the employed Cobb-Douglas production function, the wage cost share is a fix share of the output. The gross wage (WP) is the wage cost less the social contribution proportion to gross wages (tax rate is τ_{SSCP}). In the short run gross wages are affected by output growth (g_P), inflation expectation (INF_EXP). Dummy variables ($D0101$, $D0201$) capture the effect of minimum wage increase:

$$\frac{WP^*}{PYP} \cdot EP = \frac{1-\alpha_E}{1+\tau_{SSCP}} \cdot YPD \cdot \frac{1000000}{3}. \tag{C.18}$$

$$\begin{aligned} \mathrm{dlog}(WP) &= (1-0.481)\cdot(\log(1+g_P)+INF\_EXP)+0.481\cdot\mathrm{dlog}(WP_{-1})+ \\ &+0.0234\cdot D0101+0.0284\cdot D0201-0.064\cdot(\log(WP_{-1})-\log(WP^*_{-1})) \end{aligned} \tag{C.19}$$

The wage bill of employed in the government sector is a fixed proportion of the GDP in the long run:

$$\frac{WG^*}{PYP} \cdot EG = \frac{\gamma_{COMP}}{1+\tau_{SSCP}} \cdot YD \cdot PY \cdot \frac{1000000}{3} \tag{C.20}$$

$$\mathrm{dlog}(WG) = (1-0.700)\cdot\mathrm{dlog}(WG^*)+0.700\cdot\mathrm{dlog}(WG_{-1})-0.075\cdot(\log(WG_{-1}-WG^*_{-1})) \tag{C.21}$$

2.4 Monetary policy [D]

In accordance with the currently applied forecasts assumptions of the Magyar Nemzeti Bank we assume that nominal exchange rate ($NEER$) and interest rate ($RNOM$) remain unchanged:

$$NEER = NEER_{-1} \tag{D.1}$$

$$RNOM = RNOM_{-1}, \tag{D.2}$$

where RNOM is the 3 month interbank interest rate.

The real interest rate (R) is:

$$R = \frac{(1+RNOM)}{INFD} - 1, \tag{D.3}$$

where $INFD$ is the inflation of the households' consumption deflator:

$$INF = \frac{PC}{PC_{-1}} - 1. \tag{D.4}$$

The long term interest rate (RL) is a two-year moving average of the short term interest rate:

$$RL = \sum_{i=-7}^{0} R. \tag{D.5}$$

The foreign 3 month interest rate ($RFNOM$) is:

$$RFNOM = RFNOM_{-1}. \tag{D.6}$$

In the model $RNOM\_GFA$ and $RNOM\_FFA$ denote the yield of government and foreign debt, respectively:

$$\begin{aligned} RNOM\_GFA &= RNOM + GFA\_PREM & \text{(D.7)} \\ RNOM\_FFA &= RFNOM + FFA\_PREM, & \text{(D.8)} \end{aligned}$$

where GFA_PREM and FFA_PREM are the yield premium of government and foreign debt, respectively.

2.5 Domestic demand [E]

2.5.1 Households' consumption

Households' consumption (H_C) depends on disposable income and end of previous period net financial wealth (HFA). Consumption reacts almost one to one after a change in net labor income ($INCLAB - TAXPRIV$) or financial transfer (G_FTRAN), however, the reaction is much more restrained after a change in other personal income (OPI) or foreign transfer (H_FORTR). In the short run consumption is also affected by net credit flow ($CRED\_CYC$):

$$\begin{aligned} \log(H\_C^*) &= 0.268 + 0.817 \cdot \log\left(\frac{INC\_LAB - TAX\_PRIV + G\_FTRAN}{PC}\right) + & \text{(E.1)} \\ &\quad +0.117 \cdot \log\left(\frac{OPI + H\_FORTR}{PC}\right) + (1 - 0.817 - 0.117) \cdot \log\left(\frac{HFA_{-1}}{PC}\right) \end{aligned}$$

$$\begin{aligned} \text{dlog}(H\_C) &= 0.180 \cdot \text{dlog}(H\_C_{-1}) - 0.150 \cdot (\log(H\_C_{-1}) - \log(H\_C^*_{-1})) + & \text{(E.2)} \\ &\quad +(1 - 0.180) \cdot \text{dlog}\left(\frac{INC\_LAB - TAX\_PRIV + G\_FTRAN}{PC}\right) + \\ &\quad +0.350 \cdot (CRED\_CYC - (1 + 0.180 - 0.150) \cdot CRED\_CYC_{-1}) \end{aligned}$$

where constant parameter in equation (E.1) ensures the consistency of equilibrium net financial wealth and consumption of the households.

2.5.2 Housing investment

Households spend their income on housing investment (H_I) as well. Housing investment augments the housing stock (KH). In the long run, housing investment is a fix share (γ_{H_I}) of the GDP:

$$H\_I^* = \frac{\gamma_{H\_I}}{PHI} \cdot \frac{PYP \cdot YP}{\frac{\overline{PYP \cdot YP}}{\overline{PY \cdot Y}}}, \tag{E.3}$$

$$\text{dlog}(H\_I) = 0.970 \cdot \text{dlog}(H\_I_{-1}) + (1 - 0.970) \cdot \text{dlog}(H\_I^*) - 0.049 \cdot \big(\log(H\_I_{-1}) - \log(H\_I^*_{-1})\big) \tag{E.4}$$

where PHI denotes the housing investment deflator.

2.5.3 Government consumption

Government consumption (G_C) both in real and nominal term is calculated by national account identities. Thus, the value of government consumption is ultimately determined by its component:

$$G\_C = \frac{YG\_NOM + PC \cdot G\_MAT + G\_NAT - TAX\_CPAY}{PG}, \tag{E.5}$$

where PC and PG are the households' and government consumption deflators, respectively, G_MAT denotes the expenditures on goods and services, G_NAT is government transfers to households in kind, YG_NOM is the value added of the government sector at current prices and TAX_CPAY denotes copayments.

2.5.4 Changes in inventories

Changes in inventories (DS) are a fixed share of the nominal GDP:

$$DS = \phi_{DS} \cdot \frac{PY}{PDS} \cdot YD \tag{E.6}$$

2.6 International trade [F]

The export volume (X) in pinned down by foreign demand and competitiveness:

$$\log(X) = \beta_X + 1.196 \cdot \sum_{i=-1}^{1} \log(YF_i) + 0.285 \cdot RULC\_SMOOTH \tag{F.1}$$

whereβ_X is a time-varying parameter (quadratic trend), YF is the foreign demand (in the equation above we use a centered 3 quarter moving average of the logarithm of foreign demand), $RULC\_SMOOTH$ measures competitiveness which is the 8 quarter moving average

of $RULC$. Competiveness ($RULC$) is measured as a ratio of foreign price level converted into forint ($NEER \cdot PF$) to private value added deflator (PYP):

$$RULC = \frac{NEER \cdot PF}{PYP} \tag{F.2}$$

Import (M) is the sum of import content of each demand component:

$$M = \beta_M + 0.631 \cdot H\_C + 0.450 \cdot G\_C + 0.763 \cdot (H\_I + G\_I + C\_I + DS) + 0.815 \cdot X, \tag{F.3}$$

where β_M is a time-varying parameter (quadratic trend).

Trade balance is the gap between export and import:

$$NX = X - M \tag{F.4}$$

2.7 Balance sheet of government sector [G]

The DELPHI model has detailed fiscal balance sheet. It ensures that financial accounts and national account are compatible with each other. It also delivers a great variety of potential fiscal simulation that can be investigated with this model.

2.7.1 Fiscal policy rule

The fiscal balance (G_BAL) is determined by a fiscal policy rule ensuring the fulfillment of Maastricht criteria (i.e. fiscal deficit and debt must not exceed 3% and 60% of the GDP respectively).[8]

The next equation calculates the government budget balance which is consistent with Maastricht criteria:

$$\frac{G\_BAL^*}{PY \cdot YD} = \max \left\{ \overline{D}_{\max}; \left[\begin{array}{c} \left(\frac{PY_{-1} \cdot YD_{-1}}{PY \cdot YD} - 1 \right) \frac{GFA_{-1}}{PY_{-1} \cdot YD_{-1}} - \frac{GFA\_REVAL}{PY \cdot YD} - \\ -\lambda_{GFA} \left(\frac{GFA_{-1}}{PY_{-1} \cdot YD_{-1}} - \frac{\overline{\mathbf{GFA}}}{\overline{\mathbf{PY}} \cdot \overline{\mathbf{YD}}} \right) \end{array} \right] \right\}, \tag{G.1}$$

where $\overline{D}_{\max}$ is the maximal value of budget deficit as a percentage of GDP[9], GFA denotes the net financial asset of the government (government debt times -1), $\lambda_{GFA}$is the speed of adjustment toward equilibrium government debt (e.g. if government debt to GDP ratio (GFA/Y) is greater (less) than equilibrium $\overline{\mathbf{GFA/Y}}$ ratio, then government spends less (more) on financial transfers).

[8] The two criteria are not fully consistent as for example assuming a 2% annual growth and 2% annual inflation a 3% deficit generate a 77.3% debt to GDP ratio in the long run.

[9] $D_{\max}$ is calculated so that in the equilibrium it holds $D_{\max} = 60\% \frac{(1+g)(1+\pi)}{(1+g)(1+\pi)-1}$, where g is the growth rate, π is the inflation rate in the balance growth path.

2.7.2 Government incomes

Government collect the following (tax)incomes:

1. Private income tax (TAX_PRIV)
2. Social contribution (TAX_SSC)
3. Value added tax and excise duties (TAX_VAT);
4. Profit tax (TAX_PROF)
5. Net other income from corporations (TAX_CREST);
6. Copayments (TAX_CPAY);
7. Transfers from EU (G_FORTR).

Government levies private income tax on the total gross wage bill (INC_LAB) which is earned in the private sector (INC_LABP) and in government sector (INC_LABG). The private income tax rate is denoted by τ_{PRIV}:

$$TAX\_PRIV = \tau_{PRIV} \cdot INC\_LAB. \tag{G.2}$$

Employers have to pay social contribution proportion to gross wage bill. The tax rate of social contribution is τ_{SSCP} and τ_{SSCG} in the private and the government sector, respectively:

$$TAX\_SSC = \tau_{SSCP} \cdot INC\_LABP + \tau_{SSCG} \cdot INC\_LABG. \tag{G.3}$$

The consumption expenditures of the households ($PC \cdot HC$), where PC and H_C are the deflator of and the volume of consumption expenditure, and the government expenditure on goods and services are subject to so called consumption tax including both value added tax and excise duties:[10]

$$TAX\_VAT = \frac{\tau_{VAT}}{1 + \tau_{VAT}} \cdot PC \cdot (H\_C + G\_MAT). \tag{G.4}$$

Firms' income after paying off wage bills (INC_KC) subject to profit tax. τ_{PROF} denotes the effective profit tax rate:

$$TAX\_PROF = \frac{\tau_{PROF}}{1 - \tau_{PROF}} \cdot INC\_KC. \tag{G.5}$$

[10] In theory housing and government investment are also subject to value added taxes. For simplification we ignore this fact.

Income flows between firms and government is captured by variables (TAX_CREST). The value of TAX_CREST is proportional to GDP:

$$TAX\_CREST = \tau_{CREST} \cdot PYP \cdot YPD. \tag{G.6}$$

Households pay copayments (TAX_CPAY) for use of common goods. The value of TAX_CPAY is proportional to GDP:

$$TAX\_CPAY = \frac{\tau_{CPAY}}{1+\tau_{VAT}} \cdot PC \cdot H\_C \tag{G.7}$$

The $1-\varphi_{FORTR}$ ratio of foreign transfer ($FORTR$) denominated in Euro goes to government expressed in Forint:

$$G\_FORTR = (1-\varphi_{FORTR}) \cdot NEER \cdot FORTR. \tag{G.8}$$

Total income of the government is the sum of the above seven items:

$$\begin{aligned} G\_INC \;\; = \;\; & TAX\_PRIV + TAX\_SSC + TAX\_VAT + TAX\_PROF + \\ & + TAX\_CREST + TAX\_CPAY + G\_FORTR. \end{aligned} \tag{G.9}$$

Though, amortization of government capital stock is not listed in financial account of the government, however, it is a part of government value added according to national accounts:

$$INC\_KG = \delta_{INC\_KG} \cdot PGI \cdot KG_{-1} \tag{G.10}$$

2.7.3 Government expenditures

The expenditure items of the government are:[11]

1. Compensation of employees (G_COMP)
2. Expenditures on goods and services ($PC \cdot G\_MAT$)
3. Transfers to households in kind (G_NAT)
4. Government investment (G_I);
5. Financial transfers to households (G_FTRAN);
6. Interest payment after government debt (INC_GFA).

[11] In the long run, one item out of six is determined by end of previous period financial wealth of the government (INC_GFA), four items are pinned down by behavioral equations (G_COMP, G_MAT, G_NAT és G_I) and finally one item is determined by the fiscal rule (G_FTRAN).

Compensation of employees in the government sector is determined by the number of employed(EG) and gross wage (WG):

$$G\_COMP = (1 + \tau_{SSCG}) \cdot WG \cdot EG, \tag{G.11}$$

where τ_{SSCG} is the social contribution tax rate.

The amount of purchased goods and services is fix share of the GDP in the long run, short run dynamics gradually adjust to long run equilibrium:

$$G\_MAT^* = \gamma_{GMAT} \cdot \frac{PY \cdot YD}{PC} \tag{G.12}$$

$$\begin{aligned} \mathrm{dlog}(G\_MAT) &= 0.300 \cdot \mathrm{dlog}(G\_MAT_{-1}) + (1 - 0.300) \cdot \mathrm{dlog}(G\_MAT^*) - \\ &\quad -0.150 \cdot (\log(G\_MAT_{-1}) - \log(G\_MAT^*_{-1})) \end{aligned} \tag{G.13}$$

The amount of transfers to households in kind is a fixed proportion of the GDP in the long run, short run dynamics gradually adjust to long run equilibrium:

$$G\_NAT^* = \gamma_{GNAT} \cdot PY \cdot YD \tag{G.14}$$

$$\begin{aligned} \mathrm{dlog}(G\_NAT) &= 0.500 \cdot \mathrm{dlog}(G\_NAT_{-1}) + (1 - 0.500) \cdot \mathrm{dlog}(G\_NAT^*) - \\ &\quad -0.075 \cdot (\log(G\_NAT_{-1}) - \log(G\_NAT^*_{-1})) \end{aligned} \tag{G.15}$$

Government investment is a fixed share of GDP in the long run (γ_{G_I}). Assuming fixed share also means that the government capital stock will also be a fixed share of the GDP ($\frac{\overline{\mathbf{KG}} \cdot \overline{\mathbf{PGI}}}{\overline{\mathbf{YD}} \cdot \overline{\mathbf{PY}}}$). In the long run equation of government investment, parameter (λ_{KG} controls the speed of adjustment:

$$G\_I^* = \gamma_{G\_I} \cdot \frac{PY \cdot YD}{PGI} - \lambda_{KG} \left(\frac{KG_{-1} \cdot PGI_{-1}}{YD_{-1} \cdot PY_{-1}} - \frac{\overline{\mathbf{KG}} \cdot \overline{\mathbf{PGI}}}{\overline{\mathbf{YD}} \cdot \overline{\mathbf{PY}}} \right) \cdot YP \tag{G.16}$$

$$\begin{aligned} \mathrm{dlog}(G\_I) &= 0.854 \cdot \mathrm{dlog}(G\_I_{-1}) + (1 - 0.854) \cdot \mathrm{dlog}(G\_I^*) - \\ &\quad -0.107 \cdot \left(\log(G\_I^*_{-1}) - \log(G\_I_{-1})\right), \end{aligned} \tag{G.17}$$

where PGI denotes the government investment deflator.

Financial transfers to households (G_FTRAN) is determined as a residual of the aimed budget deficit (G_BAL^*), and the other income and expenditure items. In the short run financial transfers are indexed to inflation:

$$\begin{aligned} G\_FTRAN^* &= G\_INC - G\_BAL^* - PC \cdot G\_MAT - G\_NAT \\ &\quad -PGI \cdot G\_I - G\_COMP + INC\_GFA, \end{aligned} \tag{G.18}$$

$$G\_FTRAN = G\_FTRAN_{-1} \cdot \frac{CPI_{-1}}{CPI_{-2}} - \lambda_{G\_FTRAN} \left(\frac{G\_FTRAN_{-1} - G\_FTRAN^*_{-1}}{PY_{-1} \cdot YD_{-1}} \right) \tag{G.19}$$

The total expenditure of the government (without interest payment) is the sum of the five items listed below:

$$G\_EXP = PC \cdot G\_MAT + G\_NAT + PGI \cdot G\_I + G\_COMP + G\_FTRAN. \tag{G.20}$$

The interest payment after the net financial asset of the government is (INC_GFA):

$$INC\_GFA = RNOM\_GFA \cdot GFA_{-1}, \tag{G.21}$$

where GFA denotes the net financial asset of the government, $RNOM\_GFA$ is the effective interest rate of government debt.

2.7.4 Budget balance

Primary balance of the government is calculated as the balance of total incomes and expenditures (without interest payment):

$$G\_PBAL = G\_INC - G\_EXP. \tag{G.22}$$

Adding the interest payment to primary balance we get the total balance:

$$G\_BAL = G\_PBAL + INC\_GFA. \tag{G.23}$$

2.8 Balance sheet of the households [H]

Personal disposable income of the households (PDI) is the sum of gross wage income (INC_LAB) less private income tax (TAX_PRIV), financial transfer from government (G_FTRAN), a certain share of foreign transfer (H_FORTR) and other personal income (OPI) from firms. Latter one incorporates interest income of financial asset accumulated by the households:

$$PDI = INC\_LAB - TAX\_PRIV + G\_FTRAN + H\_FORTR + OPI, \tag{H.1}$$

The gross income earned in private and government sector is denoted by INC_LABP and INC_LABG, respectively, and it is calculated as the product of (average) wage and number of employed:[12]

$$INC\_LABP = WP \cdot EP \cdot \frac{3}{1000000} \tag{H.2}$$

$$INC\_LABG = WG \cdot EG \cdot \frac{3}{1000000} \tag{H.3}$$

[12] Factor 3/1000000 corrects the different unit and frequency of labor and wage statistics.

Total gross wage bill (INC_LAB) is a total sum of gross wage bill earned in the private and government sector:

$$INC\_LAB = INC\_LABP + INC\_LABG \tag{H.4}$$

Households get transfers from abroad (H_FORTR, which is φ_{FORTR} share of total transfers from abroad ($FORTR$):

$$H\_FORTR = \varphi_{FORTR} \cdot NEER \cdot FORTR. \tag{H.5}$$

Personal disposable income finances consumption and housing investment. Unspent income becomes households' saving (H_SAV):

$$H\_SAV = PDI - PC \cdot H\_C - PHI \cdot H\_I. \tag{H.6}$$

2.9 Balance sheet of the firms [I]

The firms earned gross profit equals to output less wage bills and it is subject to profit tax:

$$INC\_KC = (1 - \tau_{PROF}) \cdot (YPD \cdot PYP - (1 + \tau_{SSCP}) \cdot INC\_LABP) \tag{I.1}$$

Corporate sector includes both financial and non financial corporations. Besides its production and investment activities, it accumulates financial wealth (CFA) and intermediate capital and capital income among other sectors. The net saving of the firms is:

$$C\_SAV = INC\_KC - TAX\_CREST - PCI \cdot C\_I - PDS \cdot DS - OPI - INC\_GFA - INC\_FFA \tag{I.2}$$

where TAX_CREST is the net value of other government taxes and subsidies, OPI denotes other personal incomes of the households (mainly capital and interest income) paid to households, INC_GFA is the interest payment of the government, INC_FFA is interest income of foreigners. Firms sets income paid to households (OPI) such that it ensures obtaining an equilibrium (exogenously set) $\left(\frac{\overline{\mathbf{CFA}}}{\overline{\mathbf{PY}} \cdot \overline{\mathbf{Y}}}\right)$ ratio in the long run:

$$OPI^* = \frac{\overline{\mathbf{OPI}}}{\overline{\mathbf{PY}} \cdot \overline{\mathbf{Y}}} - \lambda_{CFA} \cdot \left(\frac{CFA_{-1}}{YD_{-1} \cdot PY_{-1}} - \frac{\overline{\mathbf{OPI}}}{\overline{\mathbf{PY}} \cdot \overline{\mathbf{Y}}} \right) \cdot PYP \cdot YP \tag{I.3}$$

$$OPI = (OPI_{-1} + H\_FORTR_{-1}) \cdot \frac{INC\_LABP}{INC\_LABP_{-1}} - H\_FORTR - \lambda_{OPI} \cdot (\log(OPI_{-1}) - \log(OPI^*_{-1})) \tag{I.4}$$

where $\frac{\overline{\mathbf{OPI}}}{\overline{\mathbf{PY}} \cdot \overline{\mathbf{Y}}}$ is the other personal income share consistent with equilibrium $\frac{\overline{\mathbf{CFA}}}{\overline{\mathbf{PY}} \cdot \overline{\mathbf{Y}}}$ ratio.

2.10 Balance sheet of foreign sector [J]

Current account incorporates net export, net income transfers and transfers from abroad (mainly from EU funds):

$$CA = PX \cdot NX - INC\_FFA + NEER \cdot FORTR. \tag{J.1}$$

The (interest) income of net foreign financial wealth denominated in Forint is a function of net foreign assets (FFA), nominal exchange rate ($NEER$) and foreign interest rate ($RNOM\_FFA$):

$$INC\_FFA = RNOM\_FFA \cdot NEER \cdot FFA_{-1}. \tag{J.2}$$

We assume that transfers from abroad ($FORTR$) to GDP ratio is gradually decreases and approaches its equilibrium ratio $\left(\frac{\overline{\mathbf{FORTR}}}{\overline{\mathbf{YP}} \cdot \overline{\mathbf{PYP}}}\right)$:

$$\frac{FORTR}{YP \cdot PYP} = (1 - 0.900) \cdot \frac{\overline{\mathbf{FORTR}}}{\overline{\mathbf{YP}} \cdot \overline{\mathbf{PYP}}} + 0.900 \cdot \frac{FORTR_{-1}}{YP_{-1} \cdot PYP_{-1}} \tag{J.3}$$

2.11 Accumulation of stocks [K]

Capital accumulation is calculated in a standard rule:

$$KC = C\_I + (1 - \delta_{KC}) \cdot KC_{-1} \tag{K.1}$$

$$KH = H\_I + (1 - \delta_{KH}) \cdot KH_{-1} \tag{K.2}$$

$$KG = G\_I + (1 - \delta_{KG}) \cdot KG_{-1}, \tag{K.3}$$

where KC is the firms' capital stock, KG denotes the government capital stock and KH is the housing stock.

Change in net financial wealth of the household (ΔHFA) is the sum of households' saving (H_SAV) and revaluation of previous period financial wealth:

$$\Delta HFA = H\_SAV + HFA\_REVAL, \tag{K.4}$$

where the revaluation term is expressed as a percentage of net financial wealth:

$$\frac{HFA\_REVAL}{HFA(-1)} = 0.011 - 0.352 \cdot HFA\_DEV\_RATIO \cdot \frac{\Delta NEER}{NEER(-1)}, \tag{K.5}$$

where HFA_DEV_RATIO is the ratio of households' debt denominated in foreign currency relative to their total debt.

Change in net financial wealth of the government (i.e. amount of newly issued government bond) is the sum of actual deficit and the revalutation:

$$\Delta GFA = G\_BAL + GFA\_REVAL, \tag{K.6}$$

where the revaluation term GFA_REVAL is expressed as a percentage of net financial wealth of the government:

$$\frac{GFA\_REVAL}{GFA(-1)} = -0.002 + 1.329 \cdot GFA\_DEV\_RATIO \cdot \frac{\Delta NEER}{NEER(-1)}, \tag{K.7}$$

where GFA_DEV_RATIO is the ratio of government debt denominated in foreign currency relative to total government debt.

Change in net financial wealth of the rest of the world (ΔFFA) is the sum of current account (CA) and revaluation (FFA_REVAL)

$$\Delta FFA = -\frac{CA}{NEER} + FFA\_REVAL, \tag{K.8}$$

where the revaluation of the net financial wealth of the foreign sector is:

$$\frac{FFA\_REVAL}{FFA(-1)} = 0.005 - 0.859\frac{\Delta NEER}{NEER(-1)}. \tag{K.9}$$

Change in net financial asset of corporations (ΔCFA) is the sum of corporate saving and revaluation of previously accumulated asset (CFA_REVAL). Latter term is residual ensuring that the sum of the four sectors' revaluation equals to 0:

$$\Delta CFA = C\_SAV + CFA\_REVAL, \tag{K.10}$$

$$CFA\_REVAL = -NEER \cdot FFA\_REVAL - HFA\_REVAL - GFA\_REVAL, \tag{K.11}$$

where FFA_REVAL, HFA_REVAL and GFA_REVAL are the revaluations of net foreign, households and government financial asset, respectively.

2.12 Deflators of demand components [L]

In the DELPHI model there are two types of deflator: (1) deflators of demand components each set by a behavioral equation, (2) deflators of aggregates (GDP, private and government value added) which are computed as the ratio of proper current price value to chained indexed volume. The second type of deflators is presented in section 2.14.

Consumption deflator is:

$$PC^* = PF \cdot \frac{NEER}{REER\_EQC} \tag{L.1}$$

$$\text{dlog}(PC) = \text{dlog}(CPI) - 0.010 \cdot (\log(PC_{-1}) - \log(PC^*_{-1})) \tag{L.2}$$

Government consumption deflator is:

$$PG^* = PF \cdot \frac{NEER}{REER\_EQG} \tag{L.3}$$

$$\begin{aligned}\text{dlog}(PG) &= (1-0.280-0.348)\cdot(1-\text{dlog}(REER\_EQG))\cdot\log(1+\overline{\pi})+ \\ &+0.280\cdot\text{dlog}(PG_{-1})+0.348\cdot\text{dlog}(CPI)-0.041\cdot(\log(PG_{-1})-\log(PG^{*}_{-1}))\end{aligned} \tag{L.4}$$

Housing investment deflator is:

$$PHI^{*}=PF\cdot\frac{NEER}{REER\_EQHI} \tag{L.5}$$

$$\begin{aligned}\text{dlog}(PHI) &= (1-0.380-0.029-0.206)\cdot\text{dlog}\left(PF\cdot\frac{NEER}{REER\_EQHI}\right)+ \\ &+0.380\cdot\text{dlog}(PHI_{-1})+0.029\cdot\text{dlog}(PF\cdot NEER)+0.206\cdot\text{dlog}(ULC)- \\ &-0.010\cdot(\log(PHI_{-1})-\log(PHI^{*}_{-1}))\end{aligned} \tag{L.6}$$

Government investment deflator is:

$$PGI^{*}=PF\cdot\frac{NEER}{REER\_EQGI} \tag{L.7}$$

$$\begin{aligned}\text{dlog}(PGI) &= (1-0.350+0.007-0.302)\cdot\log(1+\pi)+ \\ &+0.350\cdot\text{dlog}(PGI_{-1})-0.007\cdot\text{dlog}(PF\cdot NEER)+0.302\cdot\text{dlog}(ULC)- \\ &-0.010\cdot(\log(PGI_{-1})-\log(PGI^{*}_{-1}))\end{aligned} \tag{L.8}$$

Corporate investment deflator is:

$$PCI^{*}=PF\cdot\frac{NEER}{REER\_EQCI} \tag{L.9}$$

$$\begin{aligned}\text{dlog}(PCI) &= (1-0.250-0.121-0.349)\cdot(1-\text{dlog}(REER\_EQCI)\cdot\log(1+\overline{\pi})+ \\ &+0.250\cdot\text{dlog}(PCI_{-1})+0.121\cdot\text{dlog}(PF\cdot NEER)+0.349\cdot\text{dlog}(ULC)- \\ &-0.026\cdot(\log(PCI_{-1})-\log(PCI^{*}_{-1}))\end{aligned} \tag{L.10}$$

Total investment deflator is:

$$PITOT=\frac{PCI\cdot C\_I+PGI\cdot G\_I+PHI\cdot H\_I}{C\_I+G\_I+H\_I} \tag{L.11}$$

Export deflator is:

$$PX^{*}=\frac{PF\cdot NEER}{REER\_EQX} \tag{L.12}$$

$$\begin{aligned}\text{dlog}(PX) &= (1-0.120-0.496)\cdot\log(1+INF\_)+0.120\cdot\text{dlog}(PX_{-1})+ \\ &+0.496\cdot\text{dlog}(PF\cdot NEER)-0.270\cdot(\log(PX_{-1})-\log(PX^*_{-1})\end{aligned} \tag{L.13}$$

Import deflator is:

$$PM^* = PF\cdot\frac{NEER}{REER\_EQM} \tag{L.14}$$

$$\begin{aligned}\text{dlog}(PM) &= (1-0.160-0.510-0.059)\cdot\log(1+\overline{\pi})+0.160\cdot\text{dlog}(PM_{-1})+ \\ &+0.510\cdot\text{dlog}(PF\cdot NEER)+0.059\cdot\text{dlog}(P\_OIL\cdot NEER)- \\ &-0.200\cdot(\log(PM_{-1})-\log(PM^*_{-1})\end{aligned} \tag{L.15}$$

Changes in inventories deflator is:

$$PDS = PDS_{-1}\cdot\left[\frac{PC}{PC_{-1}}-0.05\cdot\left(\frac{PDS_{-1}}{PY_{-1}}-1\right)\right] \tag{L.16}$$

2.13 Value added at current prices [M]

In the model there are three value added variables. Value added of private and government sector (YPD_NOM and YG_NOM) are measured at factor price, while the total value added produced in the economy (YD_NOM) i.e. GDP is measured at market price:

$$\begin{aligned}YPD\_NOM &= PC\cdot H\_C+PCI\cdot C\_I+PGI\cdot G\_I+PHI\cdot H\_I+PDS\cdot DS+ \\ &+PX\cdot X-PM\cdot M+PC\cdot G\_MAT+G\_NAT-TAX\_CPAY-TAX\_VAT\end{aligned} \tag{M.1}$$

$$YG\_NOM = G\_COMP+INC\_KG \tag{M.2}$$

$$YD\_NOM = PC\cdot H\_C+PG\cdot G\_C+PCI\cdot C\_I+PGI\cdot G\_I+PHI\cdot H\_I+PDS\cdot DS+PX\cdot X-PM\cdot M. \tag{M.3}$$

2.14 Chain indexation [N]

2.14.1 Chain indices

To replicate national account aggregation we use chain indexing to calculate GDP from its components. To do so we calculate following chain indices:

$$PC_{CHAIN} = DQ1 \cdot \frac{\sum_{i=-1}^{-4} PC_i \cdot H\_C_i}{\sum_{i=-1}^{-4} H\_C_i} + (1 - DQ1) \cdot PC_{CHAIN-1} \tag{N.1}$$

$$PG_{CHAIN} = DQ1 \cdot \frac{\sum_{i=-1}^{-4} PG_i \cdot G\_C_i}{\sum_{i=-1}^{-4} G\_C_i} + (1 - DQ1) \cdot PG_{CHAIN-1} \tag{N.2}$$

$$PI_{CHAIN} = DQ1 \cdot \frac{\sum_{i=-1}^{-4} PITOT_i \cdot I_i}{\sum_{i=-1}^{-4} I_i} + (1 - DQ1) \cdot PI_{CHAIN-1} \tag{N.3}$$

$$PDS_{CHAIN} = DQ1 \cdot \frac{\sum_{i=-1}^{-4} PDS_i \cdot DS_i}{\sum_{i=-1}^{-4} DS_i} + (1 - DQ1) \cdot PDS_{CHAIN-1} \tag{N.4}$$

$$PX_{CHAIN} = DQ1 \cdot \frac{\sum_{i=-1}^{-4} PX_i \cdot X_i}{\sum_{i=-1}^{-4} X_i} + (1 - DQ1) \cdot PX_{CHAIN-1} \tag{N.5}$$

$$PM_{CHAIN} = DQ1 \cdot \frac{\sum_{i=-1}^{-4} PM_i \cdot M_i}{\sum_{i=-1}^{-4} M_i} + (1 - DQ1) \cdot PM_{CHAIN-1}, \tag{N.6}$$

$$PY_{CHAIN} = DQ1 \cdot \frac{\sum_{i=-1}^{-4} PY_i \cdot YD_i}{\sum_{i=-1}^{-4} YD_i} + (1 - DQ1) \cdot PY_{CHAIN-1}, \tag{N.7}$$

$$PYP_{CHAIN} = DQ1 \cdot \frac{\sum_{i=-1}^{-4} PYP_i \cdot YP_i}{\sum_{i=-1}^{-4} YP_i} + (1 - DQ1) \cdot PYP_{CHAIN-1}, \tag{N.8}$$

$$PYG_{CHAIN} = DQ1 \cdot \frac{\sum_{i=-1}^{-4} PYG_i \cdot YG_i}{\sum_{i=-1}^{-4} YG_i} + (1 - DQ1) \cdot PYG_{CHAIN-1}, \tag{N.9}$$

where $DQ1$ is a time series with value 1 in the first quarter of each year and value 0 otherwise.

2.14.2 Chain indexed aggregate volumes

We aggregate the private, government and total value added with chain indices. According the chain indexation rule the actual year chain index can be calculated from previous year deflators, and using actual year chain index one can calculate aggregates for the actual year. Using the actual year aggregates and knowing the actual year aggregates at current prices one can calculate the actual year deflators that can be used to chain indexing for the next year data.

The volume of the government value added (YG) is:

$$YG = \frac{PG_{CHAIN} \cdot \left(G\_C - \frac{G\_NAT+TAX\_CPAY}{PG}\right) - PC_{CHAIN} \cdot G\_MAT}{PYG_{CHAIN}} \tag{N.10}$$

The volume of the private value added (YPD) is:

$$\begin{aligned} YPD \;=\; & \frac{PC\_CHAIN \cdot H\_C + PI\_CHAIN \cdot I + PDS\_CHAIN \cdot DS}{PYP\_CHAIN} + \\ & + \frac{PX\_CHAIN \cdot X - PM\_CHAIN \cdot M}{PYP\_CHAIN} + \\ & - \frac{PC_{CHAIN} \cdot \left(G\_MAT - \frac{TAX\_VAT}{PC}\right) + PG_{CHAIN} \cdot \left(\frac{G\_NAT-TAX\_CPAY}{PG}\right)}{PYP\_CHAIN} \end{aligned} \tag{N.11}$$

The volume of GDP (YD) is:

$$\begin{aligned} YD \;=\; & \frac{PC\_CHAIN \cdot H\_C + PG\_CHAIN \cdot G\_C + PI\_CHAIN \cdot I}{PY\_CHAIN} + \\ & + \frac{PDS\_CHAIN \cdot DS + PX\_CHAIN \cdot X - PM\_CHAIN \cdot M}{PY\_CHAIN} \end{aligned} \tag{N.12}$$

In the previous equations, I denotes the sum of housing, government and corporate investments:

$$I = C\_I + G\_I + H\_I \tag{N.13}$$

2.14.3 Deflators calculated from chain indexed aggregate

Deflators calculated from chain indexed aggregate

The deflators of GDP, private and government value added are the ratios of proper current price values to chain indexed volumes:

$$PY = \frac{YD\_NOM}{YD} \tag{N.14}$$

$$PYP = \frac{YPD\_NOM}{YPD} \tag{N.15}$$

$$PYG = \frac{YG\_NOM}{YG} \tag{N.16}$$

2.15 Further identities [O]

Taking account the national account identities consequently, the model automatically fulfils some further identities listed below.

Sum of the private and government value added at factor prices and indirect taxes gives the total value added at current market prices (GDP):

$$YD\_NOM = YPD\_NOM + YG\_NOM + TAX\_VAT. \tag{O.1}$$

The value added can be sum up from income side as well:

$$\begin{aligned} YD\_NOM &= INC\_LABP + INC\_LABG + INC\_KC + INC\_KG + \\ &\quad + TAX\_SSC + TAX\_PROF + TAX\_VAT \end{aligned} \tag{O.2}$$

$$YPD\_NOM = (1 + \tau_{SSCP}) \cdot INC\_LABP + INC\_KC + TAX\_PROF \tag{O.3}$$

$$YG\_NOM = G\_COMP + INC\_KG. \tag{O.4}$$

Current account equals the sum of the net saving of the domestic sectors:

$$CA = H\_SAV + G\_BAL + C\_SAV. \tag{O.5}$$

Net foreign debt equals the sum of the net financial wealth of the domestic sectors:

$$FFA = -\frac{HFA + GFA + CFA}{NEER}. \tag{O.6}$$

3 Monetary transmission channels in the model

The DELPHI model has four monetary variables: (short term) interest rate, exchange rate, net credit flow, and inflation expectation. The first three variables are exogenous. It means that simulation of a monetary shock needs exogenously given shocks. In this section we describe one by one how these variables affect economy.

3.1 Interest rate channel

Interest rate has a direct effect on investment decision and interest payments of financial wealth. In case of interest rate decrease, the user cost of capital decreases starting an investment boom. Increasing demand for goods raises prices to a very limited extent because it does not affect consumption decision directly.

3.2 Exchange rate channel

The exchange rate affects the economy via two channels. A weaker exchange rate yields higher export and lower import, thus it increases net export. On the other hand, it revaluates the financial wealth denominated in foreign currency and financial transfers from abroad that lowers consumption. In short run, exchange rate depreciation increases the GDP, however, in the long run, GDP remains unchanged.

The exchange rate has direct effect on consumer prices. Lower consumer prices yield higher real income, thus households' consumption increases. The increasing demand widens the output gap which as a second round effect increases the prices to a limited extent.

3.3 Credit channel

An increase in net credit flow increases the households' consumption. However, a permanent net credit shock has temporary effect, because households reallocate their consumption intertemporary only and in the long run maintain the desired wealth to income and hence consumption to income ratio. Higher demand has positive but moderate effect on inflation.

3.4 Expectation channel

Inflation expectation directly affects consumer price and wage inflation. As the nominal wages are stickier than prices are, thus a raise in inflation expectation decreases real wages that yields lower consumption.

4 Impulse response functions

In this section we review the impulse response functions of three monetary and three real shocks. We consider the following shocks:

1. Nominal interest rate shock
2. Exchange rate shock
3. Net credit flow shock
4. Foreign demand shock

5. Fiscal expenditure shock

6. Productivity shock.

For each shock we present the difference from baseline scenario. In case of GDP, households' consumption, corporate investment, export, import and private employment the differences are expressed in percentage points. In case of private wage, consumer prices and core inflation the year on year differences in percentage points are presented. In case of net export the difference of percentage point change of export and import is depicted. Impulse response function can be found in the appendix.

4.1 Nominal interest rate shock

We investigate the effect of a 1 percentage point higher nominal interest rate lasting for 1 year. In the DELPHI model, exchange rate is not linked to interest rate (i.e. there is no UIP like relation), thus change in interest rate does not alter exchange rate. Therefore the impulse response functions of interest rate shock reflect the interest rate channel solely.

The nominal interest rate increase has direct negative effect on investment (see Figure 1.). As a consequence of decreasing demand firms lower labor demand yielding higher unemployment and lower wages. Decreasing labor income decreases households' consumption. Import also decreases as a consequence of lower investment and consumption, however, overall effect on GDP is negative. Inflation is affected by two contrary effects. On one hand, drop in demand results in negative output gap that has disinflationary effect. On the other hand, labor market adjustment yields higher ULC that increases inflation. However the net effect on inflation turns to be negative.

4.2 Exchange rate shock

We explore the effect of a permanent 1 percent exchange rate depreciation. Exchange rate affects real economy via two channels. On one hand, weaker exchange rate increases export and decreases import. The net export remains higher on the whole horizon compared to baseline scenario (see Figure 2.). Growth in export (ösztönöz) excess investment. On the other hand, there is a balance sheet channel. Weaker exchange rate depreciates the financial wealth of the households that decreases consumption. After exchange rate depreciation GDP increases for some period, but afterwards these two channels broadly off set each other.

Exchange rate depreciation passes-through into consumer prices giving an inflationary impetus. To a modest extent, the positive output gap fuelled by increased demand also raises inflation. Higher inflation is also reflected in higher wage increase. Employment increases due to higher demand.

4.3 Credit shock

We investigate the effects of a permanent 1 percentage point increase of cyclical component of the net credit flow. It is a proxy for easing/tightening of credit supply shock.) In the short run, positive credit flow shock increases consumption and import (see Figure 3.). However, credit shock has no long run effect, as the households reallocate consumption intertemporary only. As a second round effect, inflation rate, wage inflation and employment increase to a modest extent due to higher demand.

4.4 Foreign demand shock

We describe the propagation mechanism of a permanent 1 percent increase in a foreign demand. Higher foreign demand boosts export that requires more corporate investment (see Figure 4.) Due to high import content of export and investment net export strengthen slightly and only temporary. Higher demand leads to increased labor demand that raises both employment and wages. Increased wage income boosts consumption as well. Though, the increase in consumption is modest compared to increase of export and investment. GDP increases permanently. Higher demand yields positive output gap and wages increase that generate inflation impetus as well.

4.5 Fiscal expenditure shock

We investigate the effect of a permanent increase in government expenditure on purchased goods and services by 1 percent of the nominal GDP. Increased government expenditure increases domestic demand permanently (see Figure 5). This yields higher labor demand that raises both employment and wages. Increased wage income boosts consumption as well. Though, the increase in consumption is modest compared to increase of export and investment. GDP increases permanently. Higher demand yields positive output gap and wages increase that generate inflation impetus as well. As the nominal interest rate remains unchanged, the real interest rate falls that boosts investment. Higher investment, government and households' consumption raise import. All in all, GDP permanently increases to a considerable extent.

4.6 Productivity shock

We explore the effect of a permanent 1 percent increase in productivity. A TFP shock affects the corporate sector primary. Higher productivity makes investment more profitable, hence corporate investment increases. Labor demand decreases, as after a positive productivity shock, producing an output requires less labor, however, the more productive labor is honored by higher wages (see Figure 6). Increased labor income boosts households' consumption. Increased consumption and investment yield increase import and GDP. Higher productivity raises the level of potential output that yields negative output gap that diminishes inflation.

5 Direction of further model development

For monetary policy decision making, it is more favorable to use macro models taking into account that economic agents' today decisions is affected by their expectation. The current version of DELPHI model has no forward looking variables. Incorporating forward looking variable in a large scale non linear model is a challenge as from one hand it requires model having appropriate long run (consistency, well defined stable balance growth path), on the other hand solving a non linear model with model consistent expectation is difficult. Nevertheless, current model version has favorable long run properties, so from this point of view, incorporating forward looking variables seems promising. There are at least two sets of variables that should have forward looking components: (1) prices, (2) monetary variables (interest rate rule depending on expected inflation, endogenous exchange rate assuming UIP).

Beside this huge step of planned model development, we regularly update the model. It means that we confront the expert knowledge with the impulse response function of the DELPHI model quarter by quarter. Nevertheless, we are intent not only to incorporate knowledge accumulated outside from the model, but to study the model properties more deeply also: as a next milestone we want to investigate the forecasting performance of the DEPLHI model in detail.

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Figure 1. Impulse response functions of a 1 percentage point higher nominal interest rate lasting for 1 year.

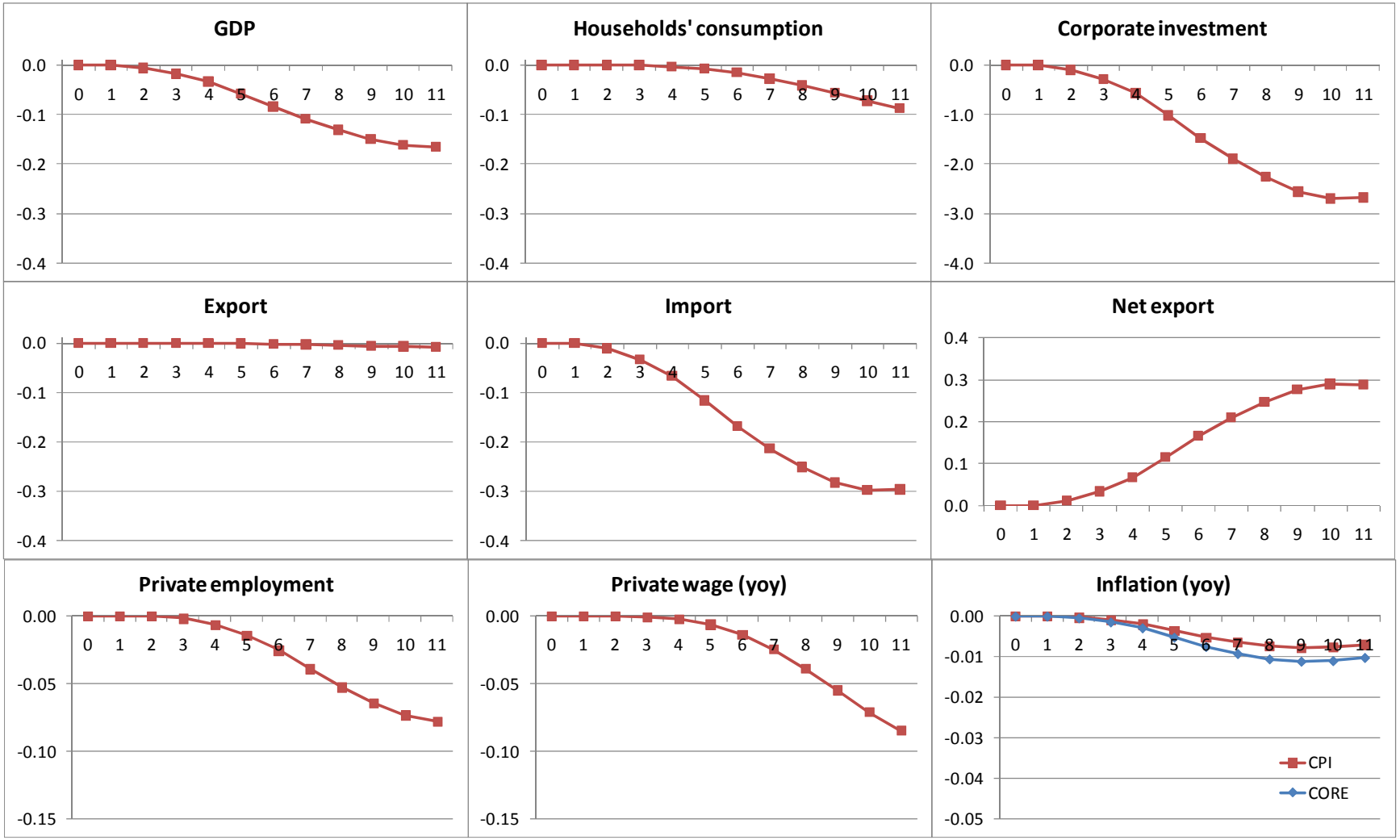


Figure 2. Impulse response functions of a permanent 1 percent exchange rate depreciation.

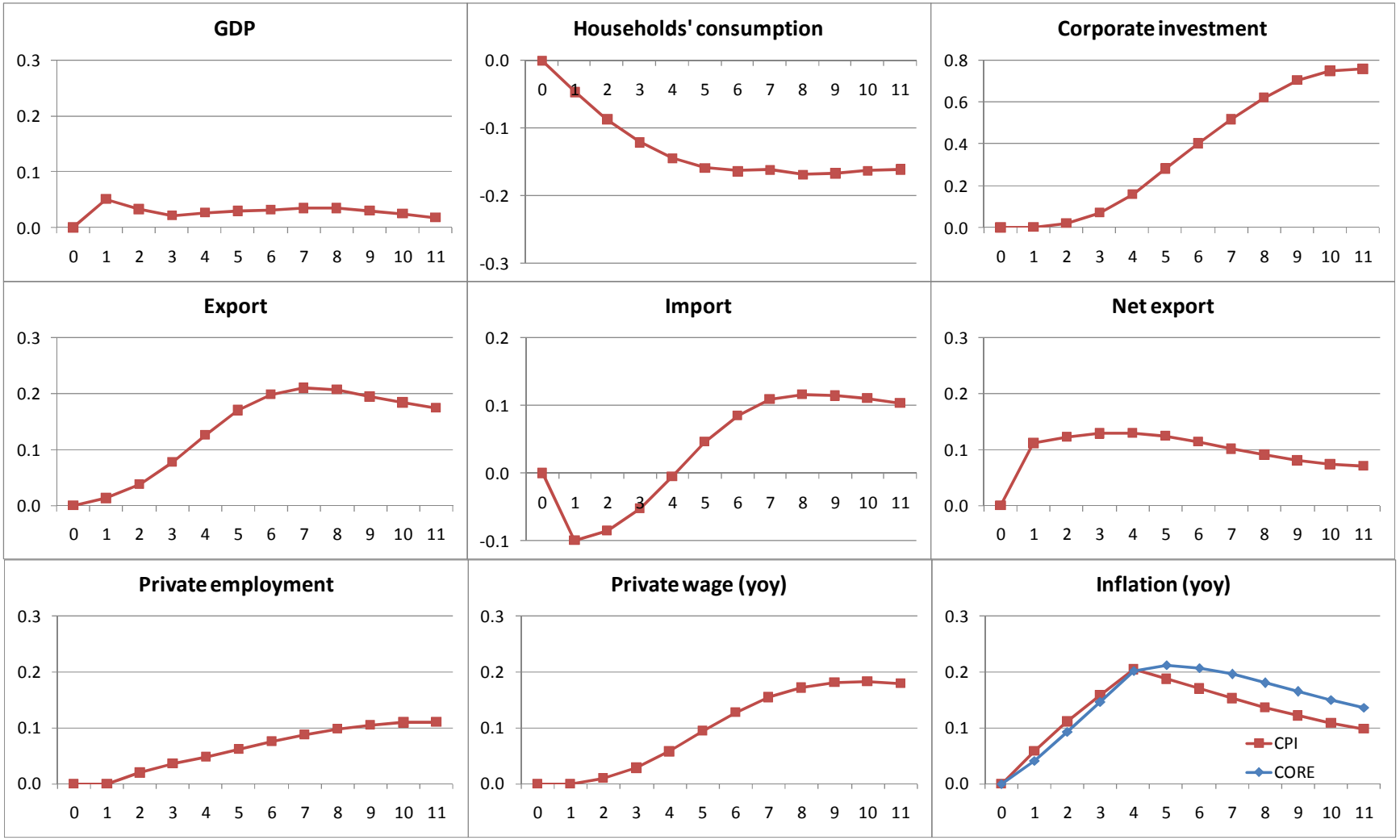


Figure 3. Impulse response functions of a permanent 1 percentage point increase of cyclical component of the net credit flow.

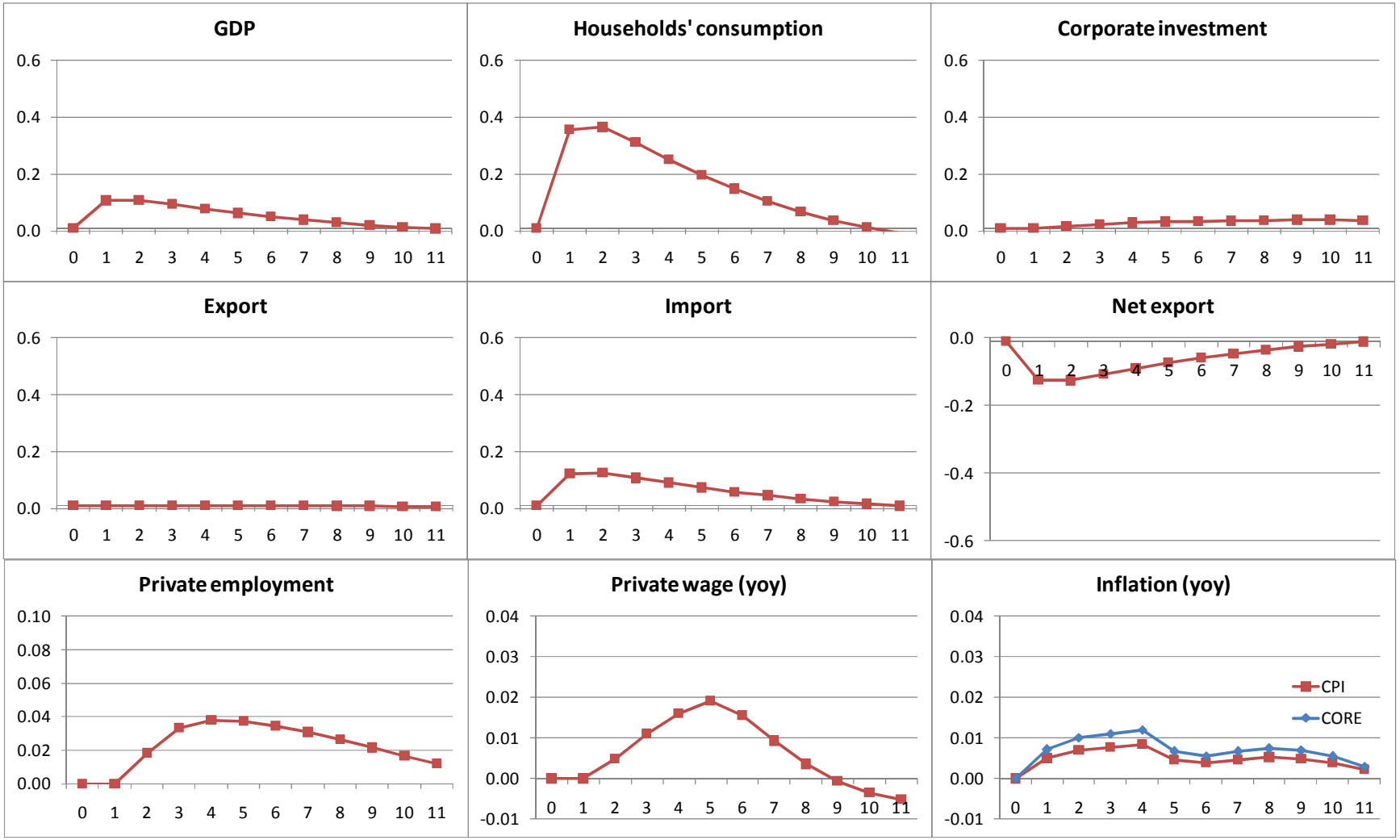


Figure 4. Impulse response functions of a permanent 1 percent increase in a foreign demand.

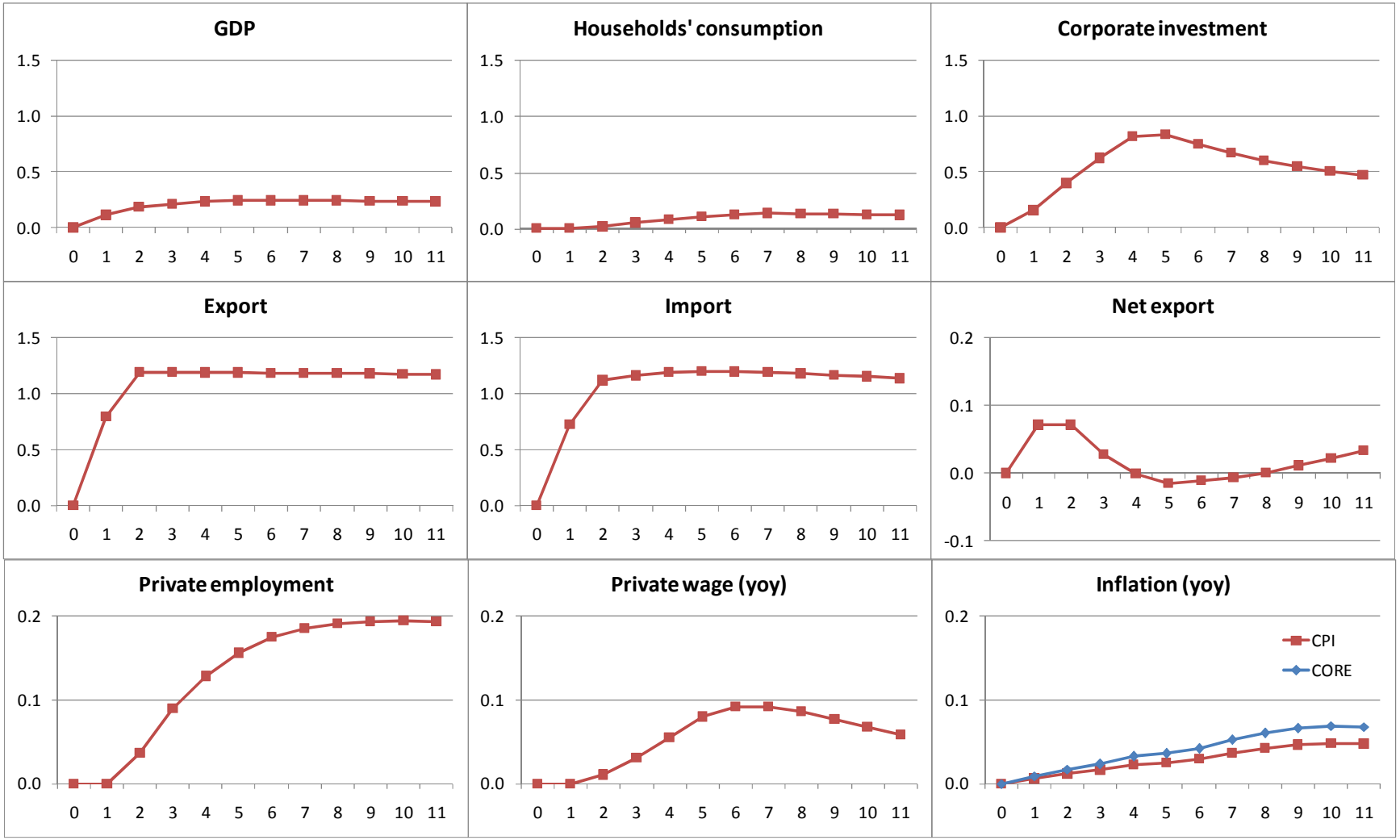


Figure 5. Impulse response functions of a permanent increase in government expenditure on purchased goods and services by 1 percent of the GDP.

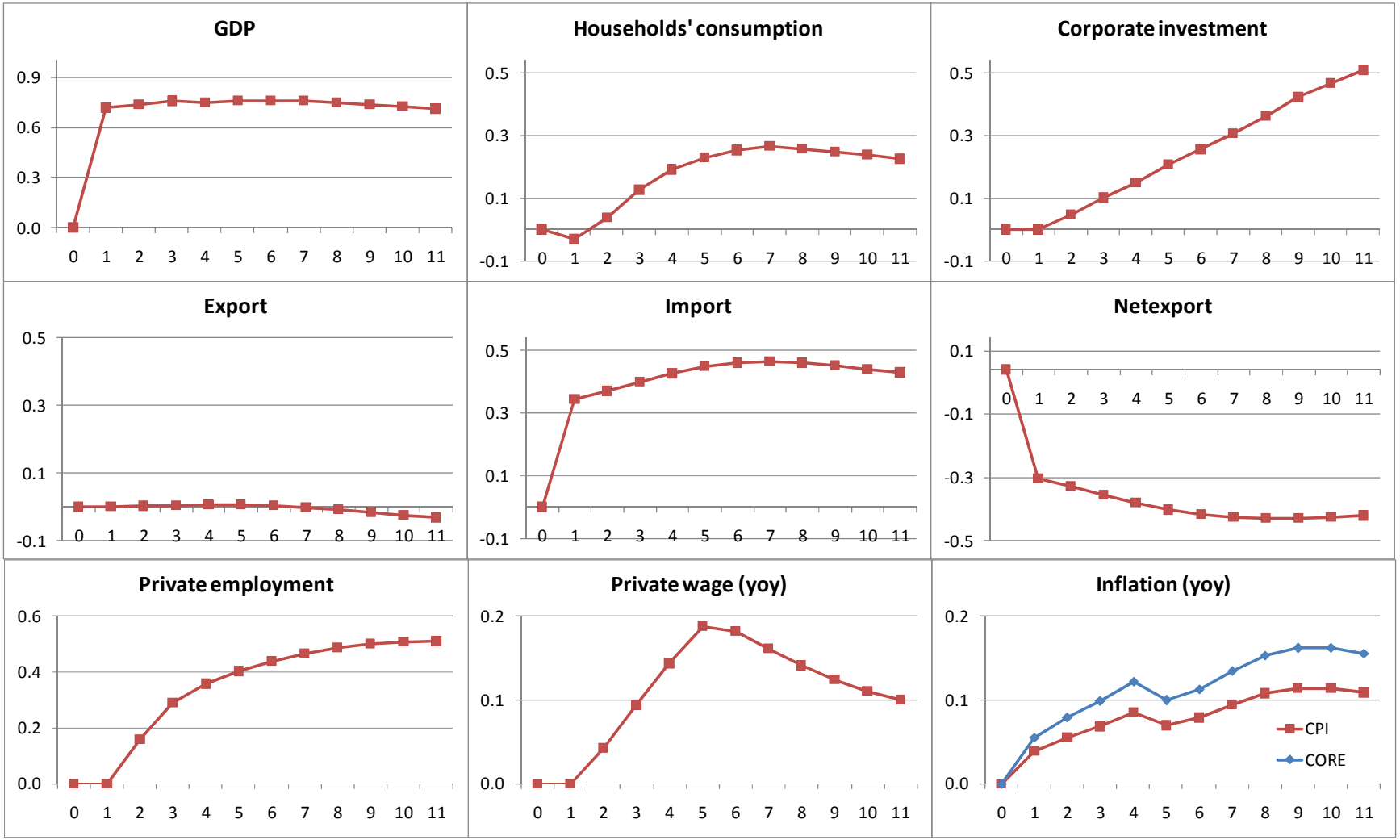
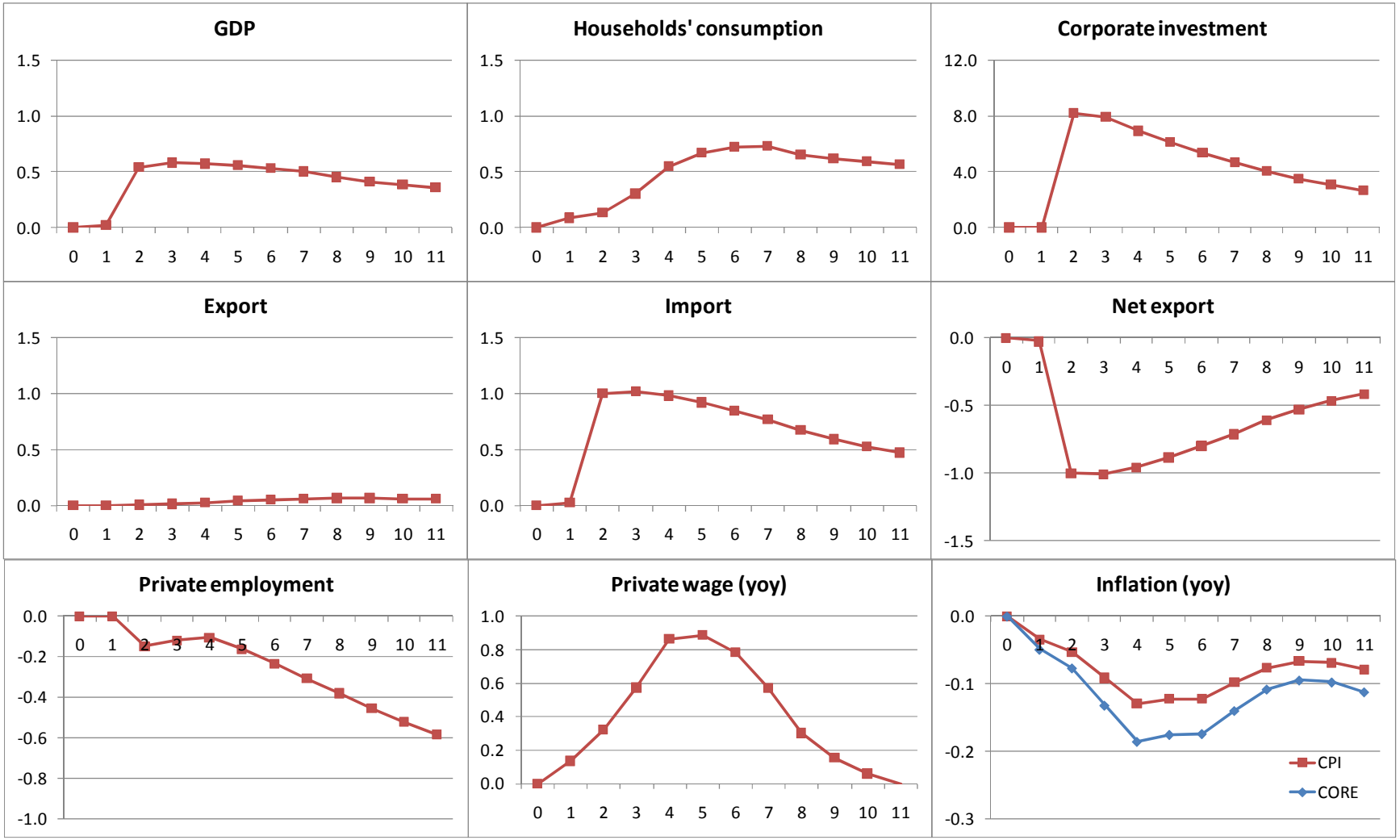


Figure 6. Impulse response functions of a permanent 1 percent increase in productivity.



List of variables

Notation	Description	Unit	Type	Equation	Deflator	Remark
C_I	Corporate investment		R	(B.6) (B.7)	PCI	
C_SAV	Corporate sector's saving		N	(I.2)		
CA	Current account		N	(J.1)		
CFA	Net financial assets of corporations		N	(K.10)		Id
CFA_REVAL	Revaulation of net financial assets of corporations		N	(K.11)		
CORE	Core inflation		N	(C.8)		
COREVAI	Core inflation excluding changes in tax		N	(C.1) (C.2)		
CPI	Consumer price index		N	(C.10)		
DS	Changes in inventories		R	(E.6)	PDS	
EG	Number of employed in government sector (LFS statistics)			(B.3) (B.4)		
EP	Number of employed in private sector (LFS statistics)			(B.1) (B.2)		
EPRATIOTR	Ratio of number of employees in private and government sector		R			Ex
FFA	Net financial assets of foreigners		N	(K.8)		Id
FFA_REVAL	Revaluation of net financial assets of foreigners		N	(K.9)		
FORTR	Net transfers from abroad		N	(J.3)		Ex
G_BAL	Budget balance		N	(G.1) (G.23)		
G_C	Government consumption		R	(E.5)	PG	
G_EXP	Government expenditures (without interest payment)		N	(G.20)		Id
G_FORTR	Transfers to government from abroad		N	(G.8)		
G_FTRAN	Financial transfers to households		N	(G.18) (G.19)		
G_I	Government investment		R	(G.16) (G.17)	PGI	
G_INC	Government incomes		N	(G.9)		Id
G_MAT	Purchased goods and services		R	(G.12) (G.13)	PC	
G_NAT	Transfers to households in kind		N	(G.14) (G.15)		
G_PBAL	Primary budget balance		N	(G.22)		Id
GAP	Output gap (%)		R	(A.9)		Id
GFA	Net financial assets of government (= - debt)		N	(K.6)		Id
GFA_DEV_RATIO	Ratio of foreign currency denominated debt in gov. assets %		R			Ex
GFA_REVAL	Revaluation of the net financial assets of government		N	(K.7)		
H_C	Households consumption		R	(E.1) (E.2)	PC	
H_FORTR	Transfers to households from abroad		N	(H.5)		
H_I	Housing investment		R	(E.3) (E.4)	PH	
H_SAV	Households saving		N	(H.6)		Id
HFA	Net financial assets of households		N	(K.4)		Id

Notation	Description	Unit	Type	Equation	Deflator	Remark
HFA_DEV_RATIO	Ratio of foreign currency denominated debt in househ. assets %		R			Ex
HFA_REVAL	Revaluation of the net assets of households		N	(K.5)		
I	Total investment		R	(N.13)	PITOT	Id
INC_FFA	Interest earnings of foreigners		N	(G.21)		
INC_GFA	Interest payment of the government		N	(J.2)		
INC_KC	Profit income in private sector		N	(I.1)		
INC_KG	Profit income in government sector		N	(G.10)		
INC_LAB	Total gross labor income		N	(H.2)		
INC_LABG	Gross labor income in government sector		N	(G.11) (H.3)		
INC_LABP	Gross labor income in private sector		N	(H.4)		
INF	Inflation (%)		R	(D.4)		Id
KC	Capital stock		R	(K.1)	PCI	Id
INF_EXP	Inflation expectation		N	(??)		
KG	Government capital stock		R	(K.3)	PGI	Id
KH	Houing stock		R	(K.2)	PH	Id
KPREM_HP	Risk premium (HP trend)		R			Ex
LF	Number of active population		R			Ex
LFTR	Number of active population (trend)		R			Ex
M	Import		R	(F.3)	PM	
NCORE	Non core inflation		N	(C.6) (C.9)		
NEER	Exchange rate		N	(D.1)		Id
NX	Net export		N	(F.4)		Id
OPI	Other personal income		N	(I.3) (I.4)		
P_OIL	Oil price (Brent in EUR)		N	(C.14)		
PC	Consumption deflator		N	(L.1) (L.2)		Id
PC_CHAIN	Chain index of households' consumption		N	(N.1)		Id
PCI	Corporate investment deflator		N	(L.10) (L.9)		Id
PDI	Personal disposable income		N	(H.1)		Id
PDS	Changes in inventories deflator		N	(L.16)		Id
PF	Foreign price level		N	(C.15)		Ex
PG	Government consumption deflator		N	(L.3) (L.4)		Id
PG_CHAIN	Chain index of government consumption		N	(N.2)		Id
PGI	Government investment deflator		N	(L.7) (L.8)		Id
PHI	Housing investment deflator		N	(L.5) (L.6)		Id

Notation	Description	Unit	Type	Equation	Deflator	Remark
PI_CHAIN	Chain index of total investment		N	(N.3)		Id
PITOT	Total investment deflator		N	(L.11)		Id
PM	Import deflator		N	(L.14) (L.15)		Id
PM_CHAIN	Chain index of import		N	(N.6)		Id
PX	Export deflator		N	(L.12) (L.13)		Id
PX_CHAIN	Chain index of export		N	(N.5)		Id
PY	GDP deflator		N	(N.14)		Id
PY_CHAIN	Chain index of GDP		N	(N.7)		Id
PYG	Government value added deflator		N	(N.16)		
PYP	Private value added deflator		N	(N.15)		Id
QE	Excess rate of return on investment (Tobin-q)		N	(B.8)		
R	Real interest rate		R			Ex
REER_EQC	Equilibrium real exchange rate of consumption		R			Ex
REER_EQCI	Equilibrium real exchange rate of corporate investment		R			Ex
REER_EQG	Equilibrium real exchange rate of government consumption		R			Ex
REER_EQGI	Equilibrium real exchange rate of governmetn investment		R			Ex
REER_EQHI	Equilibrium real exchange rate of housing investment		R			Ex
REER_EQM	Equilibrium real exchange rate of import		R			Ex
REER_EQX	Equilibrium real exchange rate of export		R			Ex
RNOM	Interest rate (annualized 3 month interbank rate)		N	(D.2)		Id
RNOM_FFA	Interes rate on foreing assets		N	(D.8)		Id
RNOM_GFA	Interes rate on government assets		N	(D.7)		Id
RULC	Real unit labor cost		R	(F.2)		
TARGET	Inflation target		N			Ex
TAX_CPAY	Copayments		N	(G.7)		
TAX_PRIV	Income tax		N	(G.2)		
TAX_PROF	Profit tax		N	(G.5)		
TAX_SSC	Social contribution		N	(G.3)		
TAX_VAT	Value added tax and excise duties		N	(G.4)		
TFP	Total factor productivity		R			Ex
U	unemployment rate		R	(B.5)		Id
ULC	Unit labor cost		N	(C.3)		Id
UTR	Unemployment (trend)		R			Ex
VAI	Price effec of changes in tax		R			Ex
WG	Gross average wage in government sector		N	(C.20) (C.21)		

Notation	Description	Unit	Type	Equation	Deflator	Remark
WP	Gross average wage in private sector		N	(C.18) (C.19)		
X	Export		R	(F.1)	PX	
Y_EU	Output of EU members' average		R			Ex
YD	GDP		R	(N.12)	PY	
YF	Foreign demand		R			Ex
YD_NOM	GDP at current prices			(M.3)		
YG	Value added in government sector		R	(A.3) (N.10)	PYG	
YP	Potential output in private sector		R	(A.1)	PYP	
YP_TOT	Potential output		R	(A.6)		Id
YPD	Value added in private sector		R	(N.11)	PYP	